Homework 4

IMPORTANT
• Plagiarism: Homework may be discussed with other students, but all homework is to be completed individually.
• Typeset all of your answers whenever possible. Illegible handwriting may get no points, at the discretion of the graders.
• Deposit hard copy of your answers in class at 1:30pm on Thursday, February 27th. For ease of grading, please
  − solve each of the 5 questions on a separate page. If you need more pages for one question, please staple them together.
  − Type the full info on each page: your name, Andrew ID, course #, Homework #, Question # on each of the 5 pages.
• Late homeworks: If you are turning your homework in late, please email it
  − to all TAs
  − with the subject line exactly 15-415 Homework Submission (HW 4)
  − and the count of slip-days you are using.

For your information:
• Graded out of 100 points; 5 questions total
• Rough time estimate: ≈ 2-4 hours

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Question 1: ISAM Tree.................................[10 points]
Submit on separate page

For the following sub-questions, consider the ISAM tree structure with order of $d = 1$
(i.e. there are at most 2 keys per node, and at most 3 pointers to children).

![ISAM tree structure](image)

(a) [10 points] show the new structure after inserting keys 26, 29 and 31.
Question 2: B+ Tree Bulk-Loading ..................... [15 points]
Submit on separate page

Consider an empty B+ Tree with order d = 2 (i.e. there are at most 4 keys per node, and at most 5 pointers to children). Bulk load the B+ tree with data entries with integers from 1 to 70 (i.e. 1,2,3,...,70) so that each leaf is full, using the algorithm outlined in Section 10.8.2 (Page 360) of the textbook.

(a) [5 points] What is the height of the resulting tree after inserting all the above keys? (i.e. The height of the tree in Figure 2 is three)

(b) [5 points] Using the bulk-load algorithm described in Section 10.8.2, what’s the maximum number of entries that can be inserted to the tree while keeping its height to be three?

(c) [5 points] Print out the content in the root node after inserting 1*, 2*, ..., 70*.

Homework 4 continues...
Question 3: B+ Trees ........................................... [20 points]
Submit on separate page

For the following sub-questions, consider the B+ tree structure with order of $d = 1$ (i.e. there are at most 2 keys per node, and at most 3 pointers to children).

Figure 2: B+ tree structure for Question 3

(a) Starting from the original B+ tree in Figure 2, we insert the record 26* with a key 26.
   i. [5 points] From the choices in Figure 3, which is the resulting tree if we use the default "no redistribution" strategy: A, B, C, D, or neither? If neither, draw the correct tree.
   ii. [5 points] From the choices in Figure 3, which is the resulting tree if we use the "redistribution with left sibling" strategy: A, B, C, D, or neither? If neither, draw the correct tree.

(b) [10 points] Starting from the original B+ tree in Figure 2, show the resulting tree after deleting record 52 if we can borrow node from the left sibling.
Figure 3: Figure for part (a)

Homework 4 continues...
Question 4: Extendible Hashing and Linear Hashing. [30 points]
Submit on separate page

Assume we have the following records where we indicate the hashed key in parenthesis (in binary)

a [11001]
b [00111]c [00101]
d [00110]e [10101]
f [01000]g [00011]h [11110]i [00001]

(a) Consider an Extendible Hashing structure where buckets can hold up to three records. Initially the structure is empty (only one empty bucket). Consider the result after the records above have been inserted in the order shown, using the highest-bits for the hash function. The directory doubles in size after very few overflows.

i. [5 points] What will be the global depth of the resulting directory?
ii. [5 points] How many buckets will we have?
iii. [5 points] List all the elements in the bucket which contains the element ”a” (including ”a”). What is the local depth of this bucket?

(b) For the same records above, consider a linear hashing structure where buckets can hold up to three records. Initially the structure is empty (only one empty bucket). Show the linear hashing structure after these records (in the order shown above) have been inserted. Assume that we split whenever the new key goes into a full bucket (which may or may not be the split bucket).

i. [5 points] How many buckets will we have (overflow pages do not count)?
ii. [5 points] List all the elements in the bucket which contains the element ”d” (including ”d”). If there is overflow page for the bucket, also list the elements in the overflow page(s).
iii. [5 points] List all the elements in the bucket which contains the element ”a” (including ”a”). If there is overflow page for the bucket, also list the elements in the overflow page(s).
Question 5: External Sorting ......................... [25 points]

Submit on separate page

Suppose you have a file with \( N = 5 \times 10^7 \) pages.

(a) [10 points] What is the total I/O cost of sorting the file using the two-way merge sort (with three buffer pages)?

(b) Now suppose we have 129 buffer pages and we are using the external sorting algorithm discussed in chapter 13.3 (p.424).
   i. [5 points] How many runs will you produce in the first pass?
   ii. [5 points] Now assume that we have a disk with an average seek time of 10ms, average rotation delay of 5ms and a transfer time of 1ms for each page. Assuming the cost of reading/writing a page is the sum of those values (i.e. 16ms) and do not distinguish between sequential and random disk-access - any access is 16ms, what is the total running time to sort the file?
   iii. [5 points] With 129 buffer pages, what’s the maximum size of the file (in number of pages) that we can sort with 3 passes?