15-826: Multimedia Databases and Data Mining

Lecture #6: Spatial Access Methods
Part III: R-trees

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Must-read material

• Textbook, Chapter 5.2
• Ramakrishnan+Gehrke, Chapter 28.6

Outline

Goal: ‘Find similar / interesting things’
• Intro to DB
• Indexing - similarity search
• Data Mining
Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - problem dfn
  - z-ordering
- R-trees
  - ...
- text
  - ...

Indexing - more detailed outline

- R-trees
  - main idea; file structure
  - algorithms: insertion/split
  - deletion
  - search: range, nn, spatial joins
  - performance analysis
  - variations (packed; hilbert;...)

Reminder: problem

- Given a collection of geometric objects (points, lines, polygons, ...)
- organize them on disk, to answer spatial queries (range, nn, etc)
R-trees

- z-ordering: cuts regions to pieces -> dup. elim.
- how could we avoid that?
- Idea: try to extend/merge B-trees and k-d trees

(first attempt: k-d-B-trees)

- [Robinson, 81]: if $f$ is the fanout, split point-set in $f$ parts; and so on, recursively

(first attempt: k-d-B-trees)

- But: insertions/deletions are tricky (splits may propagate downwards and upwards)
- no guarantee on space utilization
R-trees

• [Guttman 84] Main idea: allow parents to overlap!
  – => guaranteed 50% utilization
  – => easier insertion/split algorithms.
  – (only deal with Minimum Bounding Rectangles - MBRs)

Antonin Guttman
[http://www.baymoon.com/~tg2/]

R-trees

• eg., w/ fanout 4: group nearby rectangles to parent MBRs; each group -> disk page

A
B
C
D
E
F
G
H
I
J
R-trees

- eg., w/ fanout 4:

R-trees

- eg., w/ fanout 4:

R-trees - format of nodes

- \{(MBR; obj-ptr)\} for leaf nodes
R-trees - format of nodes

- \{\text{MBR; node-ptr}\} for non-leaf nodes

\[
\begin{array}{c|c}
\text{x-low; x-high} & \text{node ptr} \\
\hline
\text{y-low; y-high} & \\
\end{array}
\]

R-trees - range search?

R-trees - range search?
R-trees - range search

Observations:
• every parent node completely covers its ‘children’
• a child MBR may be covered by more than one parent - it is stored under ONLY ONE of them. (ie., no need for dup. elim.)

R-trees - range search

Observations - cont’d
• a point query may follow multiple branches.
• everything works for any dimensionality

Indexing - more detailed outline

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  – variations (packed; hilbert;...
R-trees - insertion

- eg., rectangle 'X'

R-trees - insertion

- eg., rectangle 'X'

R-trees - insertion

- eg., rectangle 'Y'
R-trees - insertion

- eg., rectangle ‘Y’: extend suitable parent.
- Q: how to measure ‘suitability’?

A: by increase in area (volume) (more details: later, under ‘performance analysis’)
- Q: what if there is no room? how to split?
R-trees - insertion

• eg., rectangle ‘W’

R-trees - insertion

• eg., rectangle ‘W’ - focus on ‘P1’ - how to split?

R-trees - insertion

• eg., rectangle ‘W’ - focus on ‘P1’ - how to split?

• (A1: plane sweep, until 50% of rectangles)
  • A2: ‘linear’ split
  • A3: quadratic split
  • A4: exponential split
R-trees - insertion & split

• pick two rectangles as ‘seeds’;
• assign each rectangle ‘R’ to the ‘closest’ ‘seed’

Q: how to measure ‘closeness’?

A: by increase of area (volume)
R-trees - insertion & split

• pick two rectangles as ‘seeds’;
• assign each rectangle ‘R’ to the ‘closest’ ‘seed’

• smart idea: pre-sort rectangles according to delta of closeness (ie., schedule easiest choices first!)
R-trees - insertion - pseudocode

- decide which parent to put new rectangle into (‘closest’ parent)
- if overflow, split to two, using (say,) the quadratic split algorithm
  - propagate the split upwards, if necessary
- update the MBRs of the affected parents.

R-trees - insertion - observations

- many more split algorithms exist (next!)

Indexing - more detailed outline

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R-trees - deletion

- delete rectangle
- if underflow
  - ??

R-trees - deletion

- delete rectangle
- if underflow
  - temporarily delete all siblings (!);
  - delete the parent node and
  - re-insert them

R-trees - deletion

- variations: later (eg. Hilbert R-trees w/ 2-to-1 merge)
Indexing - more detailed outline

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R-trees - range search

pseudocode:
check the root
for each branch,
  if its MBR intersects the query rectangle
    apply range-search (or print out, if this is a leaf)

R-trees - nn search
R-trees - nn search

• Q: How? (find near neighbor; refine...)

R-trees - nn search

• A1: depth-first search; then, range query
R-trees - nn search

• A1: depth-first search; then, range query

\begin{center}
\begin{tikzpicture}

\node at (0,0) {\includegraphics[width=0.5\textwidth]{r-trees.png}};
\end{tikzpicture}
\end{center}

• A2: [Roussopoulos+, sigmod95]:
  – priority queue, with promising MBRs, and their best and worst-case distance
  • main idea:

\begin{center}
\begin{tikzpicture}

\node at (0,0) {\includegraphics[width=0.5\textwidth]{r-trees.png}};
\end{tikzpicture}
\end{center}

consider only P2 and P4, for illustration

\begin{center}
\begin{tikzpicture}

\node at (0,0) {\includegraphics[width=0.5\textwidth]{r-trees.png}};
\end{tikzpicture}
\end{center}
R-trees - nn search

- best of P4
- worst of P2

$\Rightarrow$ P4 is useless for 1-nn

• what is really the worst of, say, P2?
R-trees - nn search

• what is really the worst of, say, P2?
• A: the smallest of the two red segments!

R-trees - nn search

• variations: [Hjaltason & Samet] incremental nn:
  – build a priority queue
  – scan enough of the tree, to make sure you have the k nn
  – to find the (k+1)-th, check the queue, and scan some more of the tree
• ‘optimal’ (but, may need too much memory)

Indexing - more detailed outline

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R-trees - spatial joins

**Spatial joins**: find (quickly) all counties intersecting lakes
R-trees - spatial joins

Assume that they are both organized in R-trees:

for each parent P1 of tree T1
for each parent P2 of tree T2
if their MBRs intersect,
   process them recursively (i.e., check their children)
R-trees - spatial joins

Improvements - variations:
- [Seeger+, sigmod 92]: do some pre-filtering; do plane-sweeping to avoid $N_1 \times N_2$ tests for intersection
- [Lo & Ravishankar, sigmod 94]: ‘seeded’ R-trees (FYI, many more papers on spatial joins, without R-trees: [Koudas+ Sevcik], e.t.c.)

Indexing - more detailed outline

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R-trees - performance analysis

• How many disk (=node) accesses we’ll need for
  – range
  – nn
  – spatial joins
• why does it matter?
R-trees - performance analysis

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R-trees - performance analysis

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R-trees - performance analysis

• motivating question: on, e.g., split, should we try to minimize the area (volume)? the perimeter? the overlap? or a weighted combination? why?
R-trees - performance analysis

• How many disk accesses for range queries?
  – query distribution wrt location?
  – “ ” wrt size?

- query distribution wrt location: uniform, (biased)
- “ ” wrt size: uniform

• easier case: we know the positions of parent MBRs, eg:
R-trees - performance analysis

• How many times will P1 be retrieved (unif. queries)?

P1

x1

x2

R-trees - performance analysis

• How many times will P1 be retrieved (unif. POINT queries)?

P1

x1

x2

R-trees - performance analysis

• How many times will P1 be retrieved (unif. POINT queries)? A: $x1 \times x2$
R-trees - performance analysis

• How many times will $P_1$ be retrieved (unif. queries of size $q_1 \times q_2$)?

$P_1$ $x_1$

$0$ $1$

$q_1$ $q_2$ $x_2$
R-trees - performance analysis

• How many times will P1 be retrieved (unif. queries of size q1xq2)?

A: \((x_1+q_1) \times (x_2+q_2)\)

Thus, given a tree with \(N\) nodes \((i=1, \ldots, N)\) we expect

\[
\text{#DiskAccesses}(q_1,q_2) = \\
\sum (x_{i,1} + q_1) \times (x_{i,2} + q_2) \\
= \sum (x_{i,1} \times x_{i,2}) + \\
q_2 \times \sum (x_{i,1}) + \\
q_1 \times \sum (x_{i,2}) + \\
q_1 \times q_2 \times N
\]
**R-trees - performance analysis**

Thus, given a tree with \( N \) nodes (\( i = 1, \ldots, N \)) we expect:

\[
\#\text{DiskAccesses}(q_1, q_2) = \\
\sum (x_{i,1} + q_1) \times (x_{i,2} + q_2) \\
= \sum (x_{i,1} \times x_{i,2}) + q_2 \times \sum (x_{i,1}) + \\
q_1 \times \sum (x_{i,2}) + q_1 \times q_2 \times N
\]

- \('volume'\)
- \('surface area'\)
- \('count'\)

**Observations:**

- for point queries: only volume matters
- for horizontal-line queries: \( (q_2=0) \): vertical length matters
- for large queries (\( q_1, q_2 \gg 0 \)): the count \( N \) matters

**Observations (cont’ed)**

- overlap: does not seem to matter
- formula: easily extendible to \( n \) dimensions
- (for even more details: [Pagel +, PODS93], [Kamel+, CIKM93])

Berndt-Uwe Pagel
R-trees - performance analysis

Conclusions:
• splits should try to minimize area and perimeter
• ie., we want **few, small, square-like** parent MBRs
• rule of thumb: shoot for queries with $q_1=q_2 = 0.1$ (or =0.5 or so).

R-trees - performance analysis

• How many disk (=node) accesses we’ll need for
  – range
  – *nn*
  – spatial joins

R-trees - performance analysis

Range queries - how many disk accesses, if we just now that we have
- *N* points in *n*-d space?
A: ?
R-trees - performance analysis

Range queries - how many disk accesses, if we just now that we have
- $N$ points in $n$-d space?
A: can not tell! need to know distribution

What are obvious and/or realistic distributions?

A: uniform
A: Gaussian / mixture of Gaussians
A: self-similar / fractal. Fractal dimension $\sim$ intrinsic dimension
**R-trees - performance analysis**
Formulas for range queries and k-nn queries: use fractal dimension [Kamel+, PODS94], [Korn+ ICDE2000] [Kriegel+, PODS97]

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**R-trees - variations**
Guttman’s R-trees sparked much follow-up work
- can we do better splits?
  - what about static datasets (no ins/del/upd)?
  - what about other bounding shapes?
R-trees - variations

Guttman’s R-trees sparked much follow-up work
• can we do better splits?
  – i.e., defer splits?

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Popular

A: R*-trees [Beckmann+, SIGMOD90]

Norbert Beckmann
Hans Peter Kriegel
Ralf Schneider
Bernhard Seeger

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R-trees - variations

A: R*-trees [Beckmann+, SIGMOD90]
• defer splits, by forced-reinsert, i.e.: instead of splitting, temporarily delete some entries, shrink overflowing MBR, and re-insert those entries
• Which ones to re-insert?
• How many?
**R-trees - variations**

A: R*-trees [Beckmann+, SIGMOD90]

- defer splits, by forced-reinsert, i.e.: instead of splitting, temporarily delete some entries, shrink overflowing MBR, and re-insert those entries
- Which ones to re-insert?
- How many? A: 30%

**R-trees - variations**

Q: Other ways to defer splits?

A: Push a few keys to the closest sibling node (closest = ??)
R-trees - variations

R*-trees: Also try to minimize area AND perimeter, in their split. Performance: higher space utilization; faster than plain R-trees. One of the most successful R-tree variants.

Guttman’s R-trees sparked much follow-up work
• can we do better splits?

what about static datasets (no ins/del/udp)?
  – Hilbert R-trees
• what about other bounding shapes?

• what about static datasets (no ins/del/udp)?
• Q: Best way to pack points?
R-trees - variations

• what about static datasets (no ins/del/upd)?
• Q: Best way to pack points?
• A1: plane-sweep
  great for queries on ‘x’;
terrible for ‘y’
R-trees - variations
• A: plane-sweep on HILBERT curve!

R-trees - variations
• A: plane-sweep on HILBERT curve!
• In fact, it can be made dynamic (how?), as well as to handle regions (how?)

R-trees - variations
• Dynamic (‘Hilbert R-tree):
  – each point has an ‘h’-value (hilbert value)
  – insertions: like a B-tree on the h-value
  – but also store MBR, for searches
R-trees - variations

• Data structure of a node?

- LHV
- x-low, y-low
- x-high, y-high
- ptr
- h-value \geq LHV & MBRs: inside parent MBR
R-trees - variations

Guttman’s R-trees sparked much follow-up work
- can we do better splits?
- what about static datasets (no ins/del/upd)?
  - Hilbert R-trees - main idea
  - handling regions
  - performance/discussion
- what about other bounding shapes?

R-trees - variations

- What if we have regions, instead of points?
- I.e., how to impose a linear ordering (‘h-value’) on rectangles?

R-trees - variations

- What if we have regions, instead of points?
- I.e., how to impose a linear ordering (‘h-value’) on rectangles?
- A1: h-value of center
- A2: h-value of 4-d point
  (center, x-radius, y-radius)
- A3: ...

R-trees - variations

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R-trees - variations

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R-trees - variations

• with h-values, we can have deferred splits, 2-to-3 splits (3-to-4, etc)
• experimentally: faster than R*-trees (reference: [Kamel Faloutsos vldb 94])

R-trees - variations

Guttman’s R-trees sparked much follow-up work
• can we do better splits?
• what about static datasets (no ins/del/upd)?
• what about other bounding shapes?
R-trees - variations

- what about other bounding shapes? (and why?)
- A1: arbitrary-orientation lines (cell-tree, [Guenther])
- A2: P-trees (polygon trees) (MB polygon: 0, 90, 45, 135 degree lines)

R-trees - variations

- A3: L-shapes; holes (hB-tree)
- A5: SR-trees [Katayama+, SIGMOD97] (used in Informedia)

Indexing - Detailed outline

- spatial access methods
  - problem dfn
  - z-ordering
  - R-trees
- misc topics
  - grid files
  - dimensionality curse
  - metric trees
  - other nn methods
- text, ...
R-trees - conclusions

- Popular method; like multi-d B-trees
- Guaranteed utilization
- Good search times (for low-dim. at least)
- Informix (→ IBM-DB2) ships DataBlade with R-trees
- R* variation is popular

References

- Norbert Beckmann, Hans-Peter Kriegel, Ralf Schneider, Bernhard Seeger: The R*-Tree: An Efficient and Robust Access Method for Points and Rectangles. ACM SIGMOD 1990: 322-331

References

- Jagadish, H. V. (May 23-25, 1990). Linear Clustering of Objects with Multiple Attributes. ACM SIGMOD Conf., Atlantic City, NJ.
- Ibrahim Kamel, Christos Faloutsos: On Packing R-trees, CIKM, 1993
References, cont’d


Other resources

- Code, papers, datasets etc:  
  www.rtreeportal.org/

- Java applets and more info:  
  donar.umiacs.umd.edu/quadtree/points/rtrees.html