Reminders

- Weight: 5% of the homework grade.
- Out of 100 points.
- Lead TA: B. Aditya Prakash

Notes:
- Rough time-estimate: 2~4 hours (about 30-60mins per question).
- You are encouraged to type your answers. Illegible handwriting may get no points, at the discretion of the grader.
- Whenever you are making an assumption, please state it clearly.

Question 1 [5 points]
Replace the FD $AB \rightarrow CD$ by the smallest equivalent set of simple FDs. A FD $X \rightarrow Y$ is simple if $Y$ is a single attribute.

Question 2 [20 points, 5 points each]
Table 1 shows an instance of relation R(A, B, C, D, E). In future, the instance can change by insertion, deletion, and update operations but the schema will remain the same.

Do the following dependencies hold, choose between (A) “holds”; (B) “does not hold”; (C) “cannot say”.

(A) If it holds, give the proof
(B) if it does not hold, give tuple-id(s) which violate the dependency
(C) if you cannot say, then give a SQL query that checks whether the given FD holds.

Assume that no record has NULL values.

2.1 $A \rightarrow CD$
2.2 $AC \rightarrow B$
2.3 $AB \rightarrow CD$
2.4 $BD \rightarrow CE$
Table 1: Instance of R

| T1 | 4 | 7 | 22 | 48 | 1 |
| T2 | 5 | 6 | 22 | 49 | 7 |
| T3 | 5 | 7 | 24 | 53 | 9 |
| T4 | 3 | 2 | 10 | 23 | 8 |
| T5 | 6 | 0 | 12 | 30 | 3 |
| T6 | 3 | 2 | 10 | 23 | 0 |
| T7 | 2 | 3 | 10 | 22 | 9 |
| T8 | 5 | 1 | 12 | 29 | 1 |
| T9 | 5 | 6 | 22 | 49 | 2 |

**Question 3 [40 points]**

Consider the relation schema R(A, B, C, D, E, F) with functional dependencies AC → B, BD → F and F → CE.

Q3.1 List all the primary keys of R. [5 points]
Q3.2 Find the attribute closure of {A}+. [5 points]
Q3.3 Find the attribute closure of {A, C}+. [5 points]
Q3.4 Is R in 3NF? Why or why not? [5 points]
Q3.5 Which one FD (if any) of R violates BCNF? If any, decompose R into 2 relations R1 and R2 using this FD. [5 points]
Q3.6 Suppose we project R onto S(A, C, D, E). Give one non-trivial FD that holds in S. [5 points]
Q3.7 Out of the 6 subsets of five out of the six attributes (A, B, C, D, E, F), how many and which ones are in BCNF w.r.t. to the given FD’s? [10 points]

**Question 4 [15 points, 5 points each]**

Consider the following set S of functional dependencies: \( S \equiv A \rightarrow B, \ AB \rightarrow C, \ AC \rightarrow B \) and \( B \rightarrow C \).

Q4.1 Given \( A \rightarrow B \) and \( B \rightarrow C \) prove that \( AB \rightarrow C \) holds.
Q4.2 Again, given \( A \rightarrow B \) and \( B \rightarrow C \) prove that \( AC \rightarrow B \) holds.
Q4.3 Give a minimal cover for the set S.
   *Hint: Keep in mind the proofs above*

**Question 5 [15 points, 5 points each]**

You are given the functional dependencies set \( S \equiv AB \rightarrow C, \ C \rightarrow B \) and \( C \rightarrow D \).

Q5.1 Given S, is the relation R1(A, B, C) in 3NF? Why or why not?
Q5.2 Given $S$, give the strongest normal form (BCNF, 3NF, 2NF, 1NF) **not** violated by the relation $R_2(C, D)$.

Q5.3 Decompose the relation $R(A, B, C, D)$ into a collection of BCNF relations.