Programming Language Expressiveness and Circuit Complexity

Denis Dancanet          Stephen Brookes
School of Computer Science
Carnegie Mellon University
Pittsburgh, PA 15213
{dancanet,brookes}@cs.cmu.edu
Phone: (412) 268-3075, Fax: (412) 268-5576

Abstract

This paper is a continuation of the work begun in [5] on establishing relative intensional expressiveness results for programming languages. Language $L_1$ is intensionally more expressive than $L_2$, if $L_1$ can compute all the functions $L_2$ can, with at least the same asymptotic complexity. The question we address is: Does nondeterministic parallelism add intensional expressiveness? We compare deterministic and nondeterministic extensions of PCF, a simple functional programming language. We develop further the circuit semantics from our earlier work, and establish a connection between parallel PCF programs and boolean circuits. Using results from circuit complexity, and assuming hardware which can detect undefined inputs, we show that nondeterministic parallelism is indeed intensionally more expressive. More precisely, we show that nondeterministic parallelism can lead to exponentially faster programs, and also programs that do exponentially less work.

1 Introduction

We conduct an investigation of the relative intensional expressiveness of a deterministic and a nondeterministic parallel functional language. The languages we compare arose naturally from earlier work. The core functional language is a fragment of PCF [15], and the parallel primitive is query [6], a simple construct which allows several expressions to be evaluated at the same time. In [5] we established several intensional expressiveness results for parallel extensions of PCF, one of which was deterministic query. This raised the question if nondeterministic query is intensionally more expressive than the deterministic version. We answer it in the (qualified) positive here.

In order to be able to compare the two versions of query, we make a hardware assumption which is equivalent to being able to detect undefined inputs. This makes a subset of the programs using nondeterministic query return a deterministic result. The assumption is reasonable from a practical point of view and has been used in various studies of consensus problems in distributed systems [9].

The main idea behind our intensional separation results was suggested by our earlier use of circuits as a semantics [5]: PCF programs are equivalent to boolean circuits for a certain class of functions involving undefined inputs. This connection allows us to use strong results from complexity theory to establish intensional expressiveness results.

We develop our circuit semantics formally as a parallel complexity model, and extend it to take into account the query construct. We compare the size and depth of a circuit representing a parallel