Using Fixed-Point Semantics to Prove Retiming Lemmas

STEPHEN BROOKES
School of Computer Science, Carnegie Mellon University, Pittsburgh, PA 15213

Abstract. Algorithms designed for VLSI implementation are commonly described by directed graphs, in which the nodes represent functional units and the arcs indicate communication links. We give a denotational semantics for such a graph in terms of the least fixed point of a set of (mutually recursive) function definitions, describing the outputs produced at each node as a function of time. This semantics is consistent with the conventional clocked operational semantics of the system. A retiming is a systematic modification of the internode delays of a design, often used to convert an algorithm design into a systolic form. The utility of such retimings in optimizing the behavior of designs is well known. We use fixed-point semantics to provide simple proofs of the correctness of certain retiming transformations from the literature and to justify other design transformations such as pipelining.

Keywords: systematic arrays, fixed-point semantics, pipelining, retiming