

# BEHAVIORAL EQUIVALENCE RELATIONS INDUCED BY PROGRAMMING LOGICS

Stephen D. Brookes  
Carnegie-Mellon University  
Pittsburgh, Pa.

William C. Rounds  
University of Michigan  
Ann Arbor, Mi.

## 1.0. Abstract.

In this paper we compare the descriptive power of three programming logics by studying the elementary equivalence relations which the logics induce on nondeterministic state-transition systems. In addition, we compare these relations with other natural state-equivalence relations for nondeterministic systems. We find that the notions of *bisimilarity* (Park [P], Ogden [O]) and *observation equivalence* (Milner [M]) are very strong equivalences compared with those induced by the logics. These three comprise *regular trace logic* (RTL), *propositional dynamic logic* (PDL), and *Hennessy-Milner logic* (HML). Regular trace logic is a new logic which can be used to give behavioral specifications for concurrent systems (*e.g.* Wolper [W], but with significant differences). It is a way of formalising those properties of programs which have been given informally in terms of path expressions [CH]. The model theory and axiomatics of this logic are interesting in their own right. Propositional dynamic logic is well-known; our treatment differs from the standard one only in that we regard the modalities as specifying intended behavior instead of being programs. Hennessy-Milner logic is a simplified modal logic which those authors used as a characterisation of their notion of observation equivalence, which we call weak observation equivalence in this paper. We also include a brief treatment in this context of two other natural equivalences for nondeterministic systems: *failure* equivalence [HBR] and *trace* equivalence [H], both of which are weaker than the relations induced by the logics but can be characterised using appropriate logical subsets.