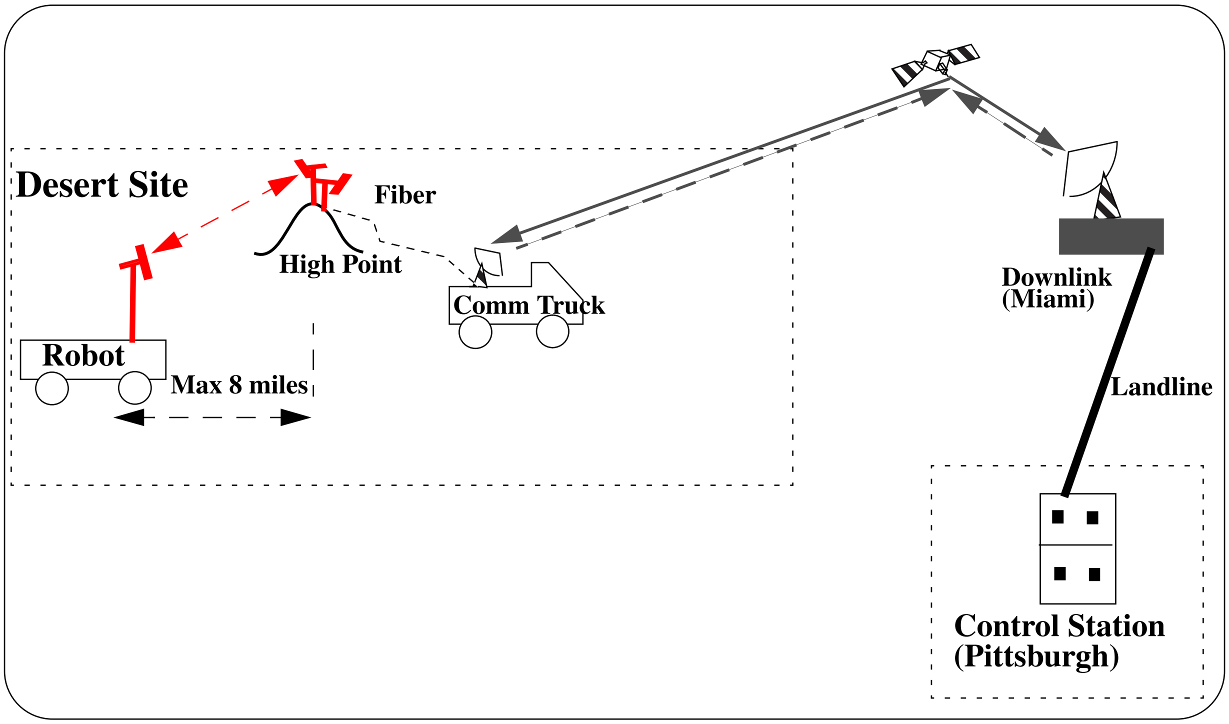
ATACAMA FIELD TRIALS- COMMUNICATION SYSTEM



- Deepak Bapna
- Jim Teza
- Eric Rollins ***(Antenna Pointing Device- Mechanical)***
- Alex Foessel ***(Site Logistics, Antenna Pointing Software)***
- Mark Maimone ***(Protocols)***

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Communication System

9 Requirements

We need to transmit about 2 Mbps of data (Table 1) from the Nomad to the control site in Pittsburgh. Ideally we would like to transmit from the rover directly to satellite and from there to Pittsburgh. The size and mass of the antenna dish and the transmitter equipments deems this scenario infeasible for this category of robot. The next option then is to transmit to a nearby relay station and from there transmit to the satellite. Since the satellite dish (3.4 m, C-band, approx. 300 lbs) and the transmitter equipments are bulky it is not convenient to move them often. Additionally, once the satellite dish is moved, it takes time to realign and be ready for the transmission. Due to the logistics reasons, we do not want to move the satellite equipments more than once a week. That means that the rover should be able to communicate with the relay station over distances of 8-10 miles. The main requirements for the communication system thus are:

- Control from remote site (non line-of-sight).
- 2 Mbps from the rover to the control station during times the rover is operational.
- 500 kbps from control station(s) to the rover during times the rover is operational.
- Communication range up to 8 miles between the rover and the relay station.
- Uninterrupted command and control.

Following table shows the data rate requirements

Item	Data rate	Data rate with Compression	Comments	Net Data Rate
Uplink				
Rover Status	20 int at 4 Hz	1280 bps	Continuous	2 Mbps
Panospheric Camera	2k x 2k @10 bit, 6-8 Hz = 32 Mbps	1.88 Mbps (at 170:1 compression)	Continuous	
Front Camera	1k X 1k @ 8 bit		Occasionally	
Back Camera	1k X 1k @ 8 bit		Occasionally	
Stereo Cameras			For Debugging only	
Downlink				

Item	Data rate	Data rate with Compression	Comments	Net Data Rate
Command & Control	20 int + 20 floats at 4 Hz.	3840 bps	Continuous	500 Kbps available
Camera Control			Which camera, Compression Ratio, FOV	
Downloading code	-	-	Occasional	

Table 1 - Data Rate Requirements

We looked at various options to meet these requirements using omnidirectional antennas and it became clear that directional antennas are required. This adds challenge of keeping the antenna on the mobile robot aligned to the antenna on the control station. We could not find any off-the-shelf device that would achieve the pointing requirements needed and so we have to design and build an antenna pointing device ourself. The detailed design, budgets and the schedule for communication system, including the pointing device are described in the following sections.

10 Configuration

The overview of the communication system is shown in Figure 1. The robot talks to a relay station, located on a high point (local hill or tower) using a wireless bridge and a radio. The relay station communicates with the communication truck using two fiber lines (one each for ethernet bridge and the radio). The communication truck carries the satellite equipments (modem and the dish) and also acts as the local control station. The fiber lines goes into a router which is connected to the local ethernet as well as the satellite modem (RS 449). The information is transmitted to satellite using a 3.4 m C-band dish. This information is downlinked at the reciever station in Maimi using a 9.3 m C-band dish. From here it is send to science center using land line.

The return link is similar. The information is sent to Miami using the landline. From there it is send to the satellite truck using the satellite. Then to relay station using the fiber modems and to the rover using the wireless modem.

The main components of the communication system are:

Rover

- High bandwidth ethernet bridge (4 Mbps throughput max)
- High gain planar antenna (16.5 dB planar antenna)
- Low bandwidth radio (115 Kbps max)
- Multiplexer (multiplex two serial lines)

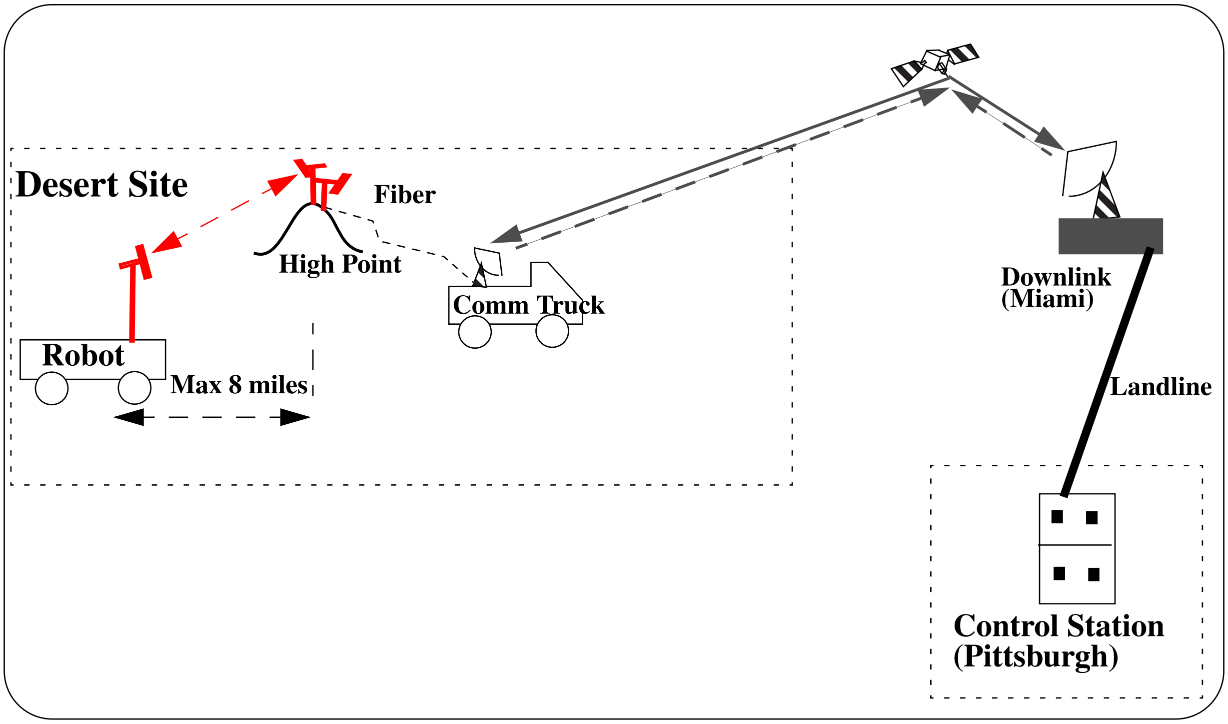

Figure 1 - ***Communication System Configuration***

• Antenna pointing device (to point the high gain antennas)

The high bandwidth (BR2040 from Aironet, 4 Mbps max.) ethernet bridge is to support the imagery requirements. There are three high resolution cameras (Panospheric camera, front camera and the back camera) mounted on the robot. The three cameras are attached to a 3Way Mux which is controlled via an RS232 connection to the imaging computing. This Mux determines which of the cameras will be providing the image that is framegrabbed and which camera is receiving the control signals. The output of the Mux goes to the framegrabber board which places the image into the memory of the pc. This is then compressed and sent to real-time computer over the ethernet and then to the wireless ethernet bridge. This bridge communicates with the bridge on the relay station using high gain antennas and pointing mechanism (discussed in detail later).

The Low bandwidth radio (115 Kbps) is used for status/command/and control. This radio operates on lower frequency (900 MHz) compared to the ethernet bridge and can take some Non-LOS. Since it has much lower bandwidth it doesn't need high gain antennas either. Thus it is more reliable than the bridge and so is used for crucial status/command and control. It will also be used for the communication required by the DGPS unit. The command/control and DGPS streams are combined using a mux.

Relay station/Satellite Link

• High bandwidth ethernet bridge
• High gain planar antenna
• Low bandwidth radio

- Antenna pointing device and computer with motion control board
- DGPS (base unit)
- Multiplexer
- 2 Fiber modems (one ethernet and one serial)
- Generator

The bridge and the radio on the relay station communicate with the corresponding units on the rover. The relay station communicates with the communication van using fiber modems and fibers. Seperate fiber modems are used for the bridge and the radio. The high gain antenna on the control station also needs to be pointed towards the rover. But this is much simpler device compared to the one we need on the rover.

Control Truck

- 2 fiber Modems
- Router (CISCO 2511)
- Satellite Transmitter
- 3.4 m C-band satellite dish

The RF modem will use a PPP or SLIP protocol. This will provide the guaranteed end-to-end transport layer needed for the robot telemetry-operator commands. Thus a PPP driver for the Real Time 68030 will be needed[1]. The Real Time computer will have two IP addresses; one to handle the PPP data coming over the serial line (RF modem), and one to enable addressing on Nomad's onboard ethernet (it might be prudent to devote the 68030 board to handling these two IP connections[2]). Relay Station GPS data, which must be made available to Nomad to enable differential GPS computations, can be communicated over the RF modem link via a multiplexer. This will allow the data to be communicated without further burdening the Real Time computer, though it will require the addition of a Mux at the relay station and one onboard Nomad. The Arlan bridge at the relay station will effectively isolate any bad packets resulting from poor high speed communication with Nomad. Thus it is ok to have the Arlan on the same ethernet as the rest of the Command Truck local network. We will use a router (CISCO 2511)[3] to combine ethernet and serial link and convert it to RS449 (or V.35) serial that goes into satellite modem. A router would be used at the Pittsburgh end to recover the serial and ethernet steram. By simply adding a modem to the stateside network, we will be able to monitor and debug networking problems during the mission.

Panospheric data will be broadcast using IP Multicasting. That is, the panospheric compression box will send out compressed data with a multicast destination address. Freeware code for this is available in the VAT (Internet Video and Audio Teleconferencing software) toolkit, available from Lawrence Livermore. This data will be available in the Command Truck, and shipped stateside via an IP tunnel in the CISCO router on the Command Truck.

A detailed schematic showing connectivity of various sites is shown in Figure 2.

1. Paul says one is available for the latest release of VxWorks
2. We'll have to see how that influences laser scanner processing.
3. 1 Ethernet, 2 High Speed Serial, 8 115Kb Asynchronous ports

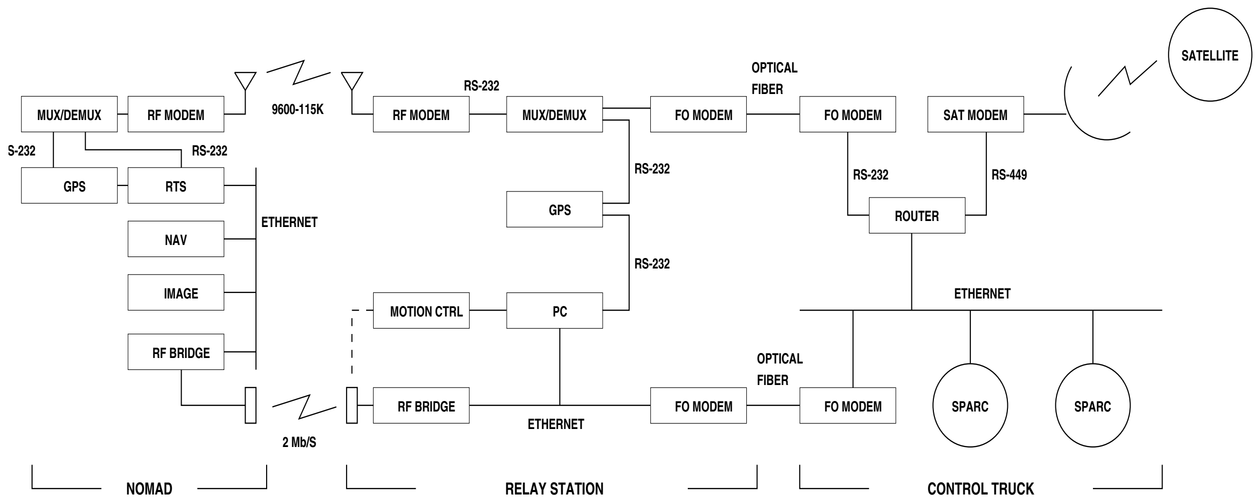

Figure 2 - ***Communication System- Connectivity Diagram***

Control Station

The satellite downlink is in Miami[1]. From there it is send to the control station at Pittsburgh Science Center using land line. On the stateside, the serial and ethernet is recovered using a similar router. The ethernet contains the compressed images and is decompressed before display. The serial stream contains the robot status and sensor data and is used for control of the rover. The command information is send back to the robot via serial line.

In case we decide to have multiple control stations we would need multiple landlines from Miami (satellite station). In addition each control station would need router and decompressing hardware.

The current configuration does not include an Internet link. However, such a link could be easily added to the stateside network via a gateway machine. We must make certain to keep Internet packets off of our local network, however, mainly because we expect the panospheric data to occupy much of the local network's bandwidth. And just by connecting to the Internet, bridge and routing table usage will explode (CMU's local net alone has several thousand nodes that will all try to register themselves).

11 Antenna Pointing Device

Antenna pointing is one of the most critical component of the communication system. To maintain the communication at the required data rate we need to keep the high gain antennas (on the rover and the relay station) pointed towards each other. Since the relay station is stationary, it is fairly easy to point the antenna located there. The main challenge is to keep the antenna on the rover pointed towards the antenna on the relay station in presence of the disturbances due to the rover motion. In this section we discuss the pointing of antenna from the rover.

1. We need a 9.3m C-band, RHCP dish to communicate with the satellite. We couldn't get one in Pittsburgh.

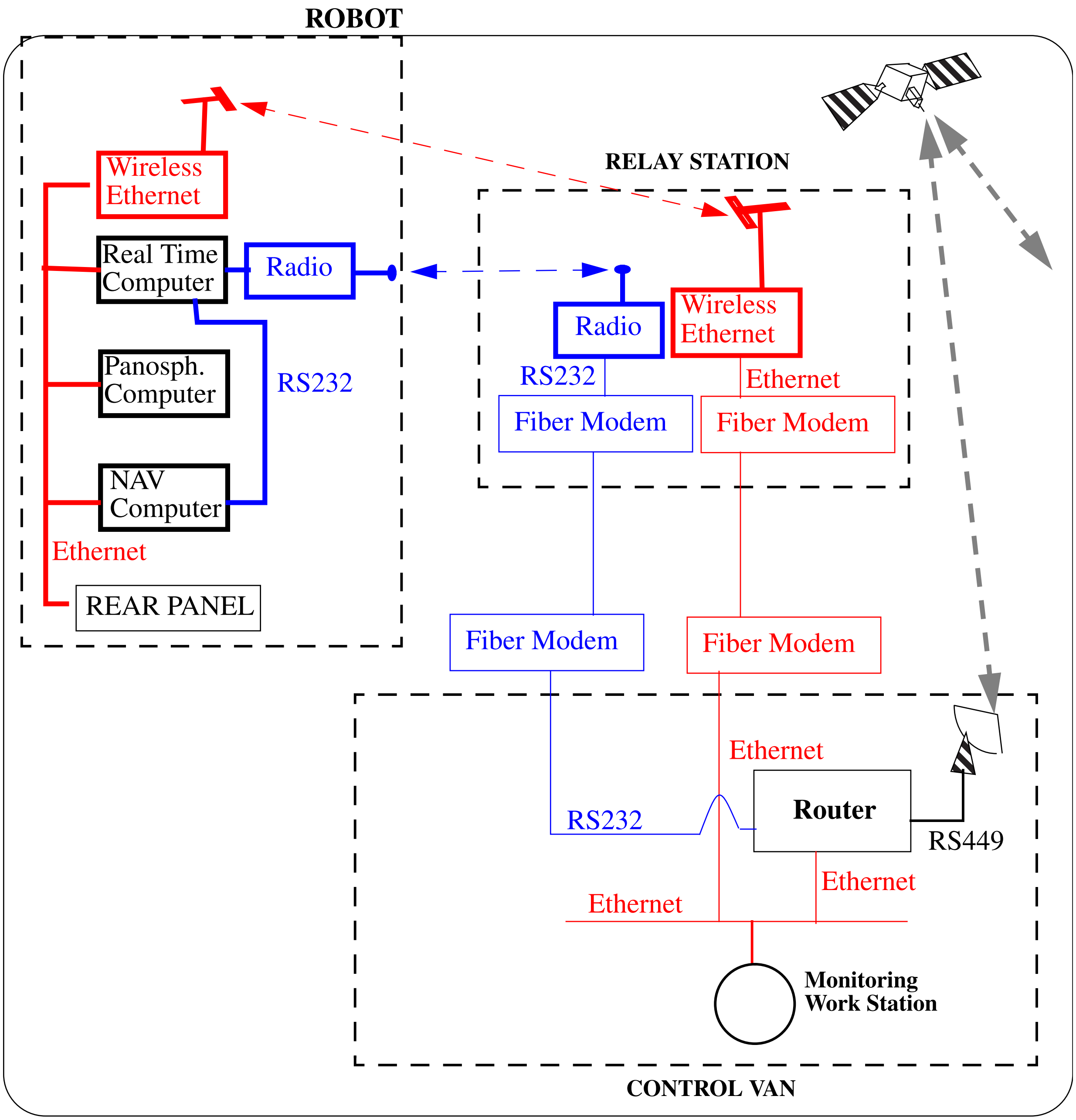

Figure 3 - ***Block Diagram***

11.1 Requirements

The pointing requirements are given in Table 2.

Item	***Value***	***Comments***
Elevation range	-60 to + 60 deg	
Azimuth range	360 deg	Continuous
Elevation Rate	60 deg/sec	
Azimuth Rate	30 deg/sec	
Elevation Accelera-tion	190 deg/sec^2	
Azimuth Acceleration	190 deg/sec^2	
Pointing Accuracy	2.5 deg	Aggregate
Pay Load Mass	2 kg	
Payload Size	27.3 x 34.0 x 2 cm	
Peak Torque		
Stall Torque		
Velocity Smoothness		
Stability		
Environmental		

Table 2 Pointing Requirements

11.2 Configuration

- Azimuth/Elevation Configuration
- Power for transmitter and tilt motor, encoder signal for tilt motor, command signal through slip ring.
- RF rotary joints to send RF signal from the transmitter to the antenna

11.3 Sensing

Following sensors will be used for antenna pointing: DGPS, encoders, compass, Inclinometer. IMU, if available can be used to improve the performance.

Figure 4 ***- Pointing Mechanism***

- DGPS: We will use DGPS to get the positions of the robot and the relay station in the inertial frame. One GPS unit will be mounted on the rover, while the second near the relay station. The GPS unit at the relay station will transmit differential corrections to the rover.
- Compass: A gyro compass will be used to get the magnetic azimuth of the vehicle. Compass reading might be improved based on GPS and IMU data.
- Inclinometers: Inclinometers are required to get the roll and pitch of the vehicle. The Azimuth digital gyro compass (ADGC) we have currently gives roll and pitch in addition to the azimuth.
- Encoders: Incremental encoders will be used to sense the motor positions on azimuth and elevation motors.
- IMU: IMU, if available might be used in various ways: it gives rates which are useful for controlling the antenna pointing device; the rates and accelerations can also be used in a feedforward fashion to improve poiniting performance.

DGPS, Compass, Inclinometers and Encoders are minimum sensors required for pointing control and are baseline sensors for the design. But they achieve only open loop pointing. There is no way to find the actual pointing error or evaluate the quality of pointing using these sensors. It would be useful (but not necesary) to have a feedback method. We are looking at several options to close the feedback loop, including:

- Laser Tracking:
- Visual tracking:
- RF Tracking:

11.4 Control

The block diagram of the controller is shown below. Given the reference (from kinematics box), it will point the antenna.

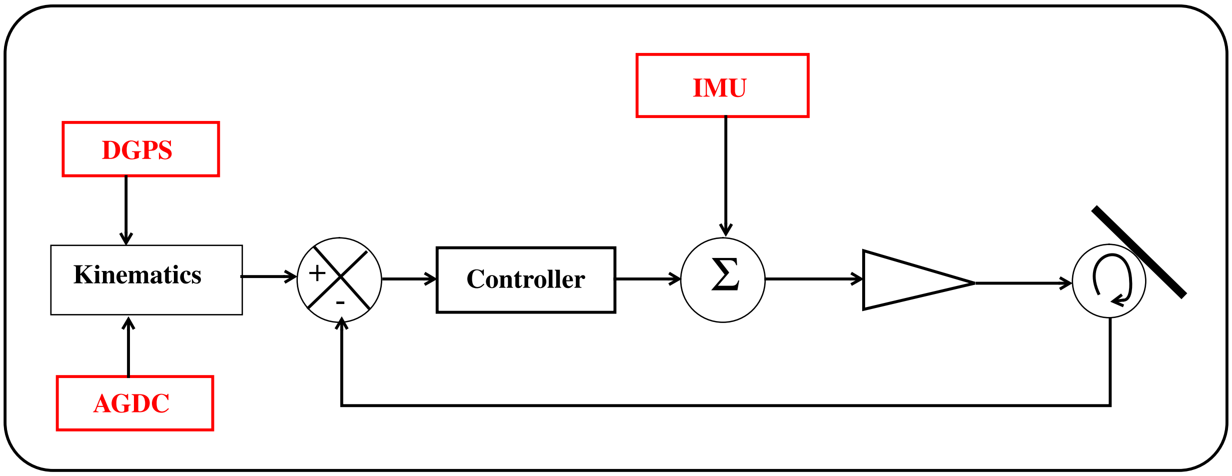

Figure 5 - *Antenna Pointing Control*

12 Budget

Item	Unit Price	Quantity	Total	Total w/ margin	Comments
ROVER					
Ethernet bridge	3000	1	3000	3300	BR2040-Aironet
16 dB Antenna	424.14	1	424.14	424.14	Ordered
Radio Modem	2000	1	2000	2200	Freewave/Pacific Crest
Multiplexer	1000	1	1000	1200	RAD
Cables/connectors	1000	1	1000	1100	

Item	Unit Price	Quantity	Total	Total w/ margin	Comments
			6424.14	**7024.24**	
Antenna Pointing Device					
Mechanism	3000	1	3000	3300	Machining
Slip Rings	1100	1	1100	1150	
RF Rotary Joint	500	1	500	550	
Pan motor	2000	1	2000	2300	Inland Motor
Tilt Motor	1200	1	1200	1400	Donation from BEI
DGPS	6000	1	-	-	Donation from Trimble
GyroCompass	2375	1	2375	2375	KVH Industries
Feedback Sensing	2000	1	2000	2400	Laser tracking
Encoders	5000	2	0	0	Donation- BEI
Cables/connectors	1000	-	1000	1100	
RELAY STATION					
Ethernet Bridge	3000	1	3000	3300	BR2040- Aironet
High gain antenna	427.14	1	427.14	427.14	Ordered; H+S 1324.19.0010
Radio Modem	2000	1	2000	2200	Freewave/Pacific Crest
DGPS	6000	1	-	-	Donation from Trimble
Antenna Pointing	2000	1	2000	2200	Use Existing Compo-nents
Fiber Modems	600	2	1200	1320	
Fiber	5 $/m	2000 m	10,000	11,000	Need to find something cheaper
Cables/connectors	1000	1	1000	1100	
CONTROL VAN /SATELLITE LINK					
Router	2500	1	2500	2750	

Item	Unit Price	Quantity	Total	Total w/ margin	Comments
Fiber Modems	600	2	1200	1320	
Link Time	34,000	1 month	34,000	35,000	Lyman Brothers, Fruit Heights, UT
Equipment rental	13,000	2 month	26,000	27,000	
Landline	5000	1 month	5000	5500	Referred by NASA Ames.
Consulting[a]			7000	8000	
Shipping and Handling	9000	-	9,000	9,000	
OTHER					
8 dB Antenna	65.00	2	0.00	0.00	Donation
RS 422-232 Converter	69.95	1	69.95	69.95	Gyro-compass testing
Laptop	4300	1	4300	4300	Ordered
Router	2500	1	2500	2750	Control Station
Cables	1000	1	1000	1100	
TOTAL					
If Ratler radio works for range...			- 4500	Good Chance	
Laptop			- 4300	Other Budget	

Table 3 Budgets

a. Installation, training

13 Status

- Tested high gain planar antenna (8dB gain) for long range. We were able to achieve 3.5 miles LOS communication with these antennas.
- Higher gain planar antennas (16dB gain) ordered. We expect to get range of 8-10 miles using the new antennas.

- Satellite vendor have been identified and the formal contract would be signed in a week or so.
- The design of the pointing mechanism is being finalized.
- The sensors needed are identified and some of the sensors (gyrostabilized compass, IMU) are acquired and being tested.
- The simulation of the pointing system are in progress and would be completed in next 2 weeks.

14 Milestones

- New Aironet Bridge and the new Antenna acquired.
- Antenna pointing mechanism (APM) design complete.
- Actuators ordered.
- Antenna pointing system assembled.
- All sensors for antenna pointing acquired.
- Essential kinematics/dynamics for APM done.
- Control shell simulations complete- Ready to go.
- New Bridge, radio and Antennas tested for datarate, range and non-LOS.

Table 4 Schedule

TASK	Start Date	End date	Comments
Radios, Bridges, etc.			
New Bridges/Antennas Acquired			
Testing of Bridges and Radios			
Mux spec-ed, identified and ordered			
Router Spec-ed, identified and ordered			
Fiber Modems sped-ed, identified and ordered			
Mux, Router amd Fiber Modems tested			
Protocols			
Integration on Rover			
Full-up link test			
Reduced Capabilities Mode			
Antenna Pointing			

Table 4 Schedule

TASK	Start Date	End date	Comments
Configuration			DONE
Kinemetics			DONE
Error Analysis			DONE
Sensors Identified			
Sensors- acquisition and testing			
Design			
Actuators identification, ordering, acquistion			
Actuators testing			
Mechanis- fabriction			
Slip rings- identification, ordering and acquisition			
Slip rings testing			
RF rotary joint- identification, ordering and testing			
RF rotary joint- testing			
Control Simulations			
APM Integration			
Testing			
Satellite Link			
Contract Signed		Jan 1, 1996	
Personnel Trained		March 15, 1996	
Full-up link test		March 31, 1996	

15 Features and Issues

Features:

- Data rate of 2 Mbps can be achieved continuously.
- Range of 8-10 miles can be achieved.

- Command and control is communicated via a lower bandwidth, higher reliability (non-LOS problems to a lesser extent) serial modem.
- Same configuration as Ratler.
- The high gain planar antennas are light weight and have wide beamwidth suitable for the application.
- Antenna pointing is required, but the accuracy needed is not low due to wide beamwidth.

Issues:

- Non-LOS is still a big issue. We couldn't identify a radio which would gives us non-LOS transmission over 8-10 miles range. The "freewave radio" might be OK, but hasn't been tested for more than 2 miles yet. In the worst case, we might to use few repeaters and/or choose the terrain accordingly.
- Closing the feedback loop for antenna pointing is important, but we don't have a proven/tested solution.

16 Future Work

- Test the "azimuth digital gyro compass".
- Test "freewave radio" for 8-10 miles LOS and non-LOS. If freewave doesn't work, "pacific crest" which is a lower frequency radio might be better. What are the other options?
- Acquire and test the "BR2040-EE" wireless ethernet bridge over longer range and various pointing offsets with the new antennas.
- Design, prototype and test the laser tracking and visual tracking for antenna pointing feedback.

Appendix C - Communication Permits

The United States Embassy in Santaigo has agreed to help us to obtain communications permission from the Ministerio de Transportes y Telecommunicaciones. There is no charge for these permits when used for research or science related projects. Through past experience, officials at the Embassy have suggested that these permit applications be completed as soon as possible. The Science Atache from the Embassy has forwarded the following information:

“The Ministry of Foreign Relations - Directorate of Special Policy _ informs the Embassy of the United States of America that the competent national agency has pointed out that in order to assign frequencies for temporary use it requires, at least, the following information:

Number of frequencies, indicating band(s). If specific frequencies are proposed, indicate substitute frequencies or frequency bands to find substitutes

Item	Frequency	Comments
1	5.8 GHz (C Band)	Transmit to satellite
2	3.475 GHz (C Band)	Recieve from Satellite
3	2.4-2.4835 GHz (S Band)	Direct Sequence Spread Spectrum Radio
4	902-928 MHz (L Band)	Spread Spectrum, GFSK

Table 5: Frequency Usage

Band width.

	Frequency	Bandwidth
1	5.8 GHz	3.0 MHz
2	3.475 GHz	600 KHz
3	2.4-2.4835 GHz	83.5 MHz
4	902-928 MHz	26.0 MHz

Table 6: Bandwidth

Area for mobile stations (Region, Province, or Municipality).

Location of fixed stations.

System configuration, diagram specifying simplex, semiduplex or duplex links

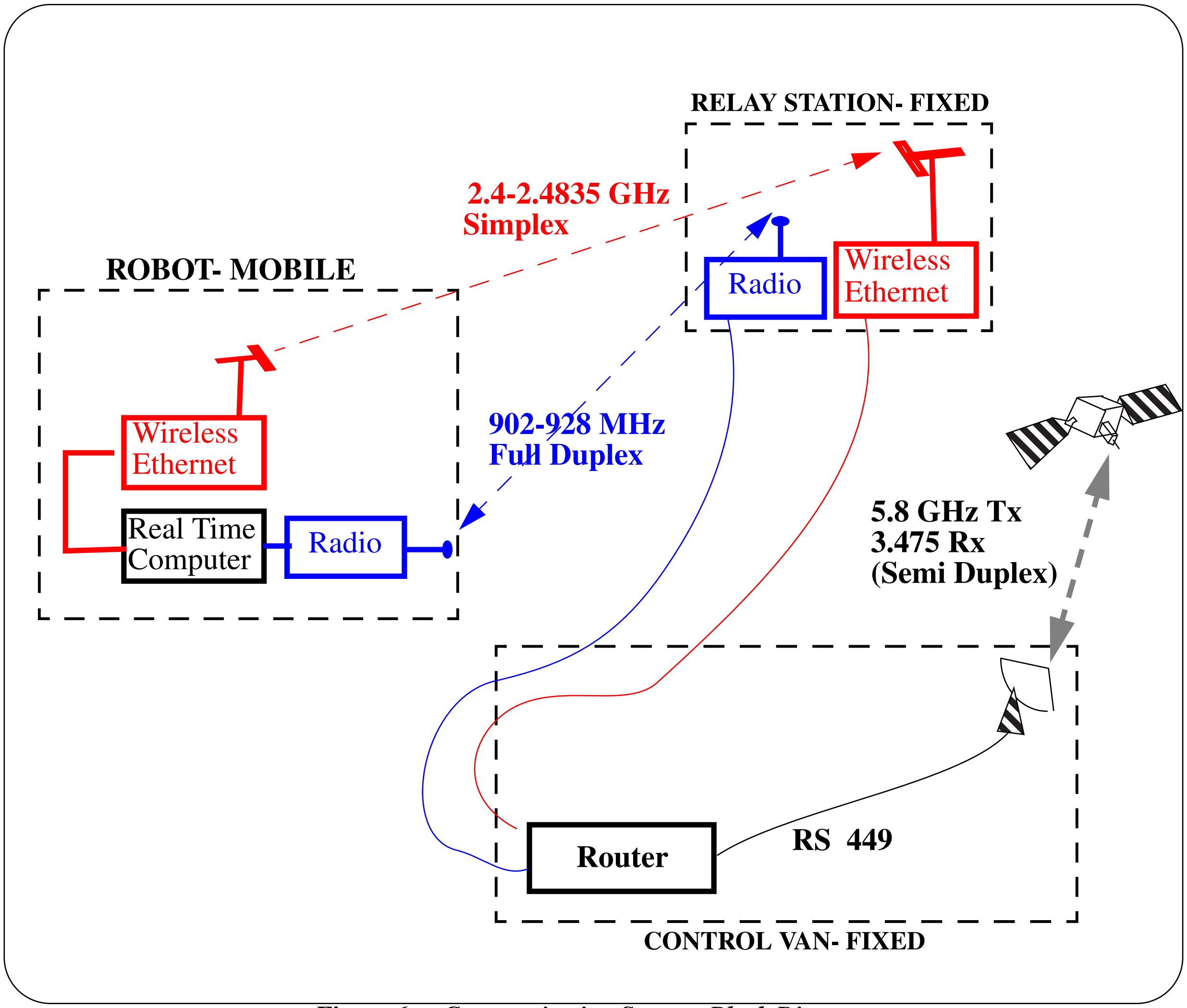

Figure 6 *- Communication System- Block Diagram*

Types of stations (base, mobile, portable, transponder, land station, etc.) and number of stations to be used.

Item	Frequency	Type of Station
1	Tx 5.8 GHz Rx 3.475 GHz	Fixed (4-5 locations during the mission)
2	2.4-2.4835 GHz	1 Mobile; 1 Fixed (4-5 locations during the mission)
3	902-928 MHz	1 Mobile; 1 Fixed (4-5 locations during the mission)

Table 7: Type of Stations

Maximum capacity for each kind of equipment.

	Frequency	Max. Capacity
1	5.8 GHz Tx 3.475 GHz Rx	8 Mbps Duplex
2	2.4-2.4835 GHz	4 Mbps Simplex
3	902-928 MHz	115.2 Kbps Duplex

Table 8: Max. capacilty of each Equipments

Appendix D Antenna Pointing- Kinematics

The problem is presented as follows: given the relative position of the rover and the repeater station, and given the angles of the roiver respect to the general frame (earth frame), what are the values of pitch and roll necesaries to point the rover's antenna to the repeater station.

Notation

Robot pitch

Robot yaw

Robot Azimuth

Azimuth motor encoder

Elevation motor encoder

Coordinates of the relay station (GPS)

Coordinates of the robot (GPS)

Location of the pointing frame w.r.t the vehicle frame

Reference Frames

The problem can be expressed in terms of frame transformations. We will say than the rover and the station are located in a general frame {G} fixed with respect to the earth.

This frame has its x-axis pointing towards the East. The y-axis is pointing towards the North. The z-axis is pointing upwards in the local vertical. The origin of this frame is somewhere in the earth, and it is irrelevant because we are interested in the relative position of the rover and the station.

There is another {V} frame attached to the rover (vehicle) in such a way that the origin coincides with the Digital Gyro Compass. The y-axis points forward. The x-axis points to the right side of the vehicle. Finally, the z-axis points up in the local vertical. We know the relative position of {V} with respect to the {G} using the GPS output. The DGC gives the yaw, pitch and roll of the vehicle.

There is a mast on the vehicle, and it has the antenna to be pointed on the top of it. The antenna has been designed to have two axis controlled, that is the yaw and pitch. The frame mounted on the antenna {A} has the y-axis pointing toward the electrical axis of the antenna. So the problem is to control the antenna to intersect the projection of the y-axis of {B} with the station.

There is also an auxiliar frame mounted on the antenna axis intersection, that will be very useful as we will see. That is a frame with its origin coincident with the {A} origin, but with the axis parallel to the {V} axis.

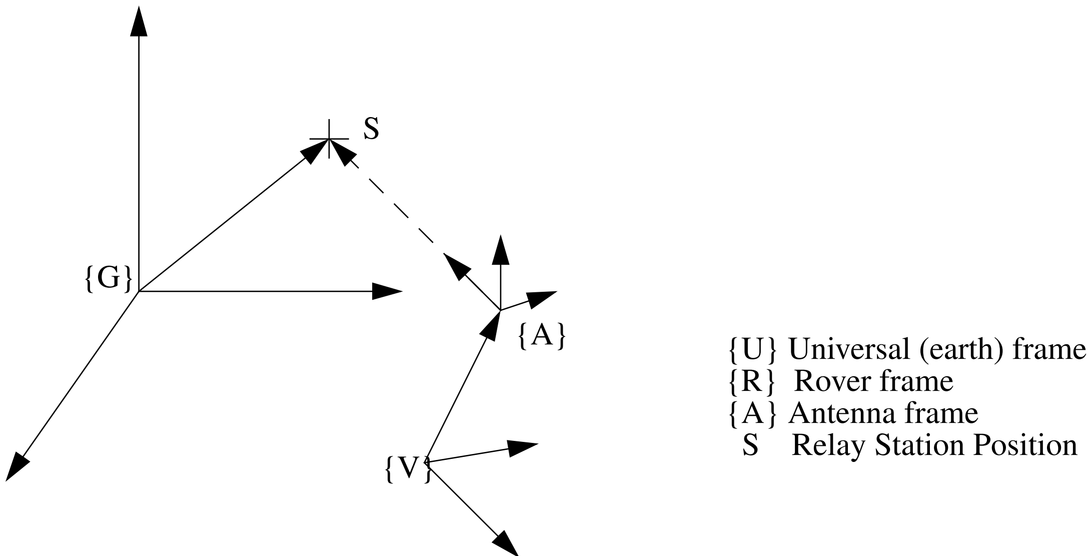

Figure 7 ***Reference frames for Antenna Pointing***

Tha goal or station vector is S, and it is also the result of GPS data.

Kinemetic Transformations

The solution to the problem is a combination of direct and inverse kinematics. Since we know the angles and position of the origin of the {V} frame, we can obtain the transformation matrix to express S in {V} coordinates. After that, it is easy to translate that point to have it in {B} coordinates.

Now, once S is expressed in {B} cartesian coordinates, it happens that the antenna moves in spherical coordinates. Then, if we use the cartesian to spherical coordinates, we will get the angles we need to point rthe antenna to. This is the inverse kinematics part because given the point, we need the joint angles.

First Transformation {G} to {V}

The transformation matrix, given the angles yaw, pitch and roll is (from mobile robots notes and handouts) the product of the translation and rotation matrices in homogeneous coordinates:

$$ {}^{V}_{G}T = \begin{bmatrix} c_\alpha c_\beta & s_\alpha c_\beta & -s_\gamma & 0 \\ c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & s_\beta s_\gamma & 0 \\ c_\alpha s_\beta c_\gamma + s_\alpha c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma & c_\beta c_\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -{}^{G}v_x \\ 0 & 1 & 0 & -{}^{G}v_y \\ 0 & 0 & 1 & -{}^{G}v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Once in {V} coordinates, the translation to the top of the mast is accomplished by multiplyig the vector by the next transformation matrix:

$$
{}^{B}_{V}T = \begin{bmatrix} 1 & 0 & 0 & -{}^{V}a_x \\ 0 & 1 & 0 & -{}^{V}a_y \\ 0 & 0 & 1 & -{}^{V}a_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Now we have that the full transformation matrix from the {G} frame to the {B} frame is the product of:

$$
{}^{B}_{G}T = {}^{B}_{V}T \cdot {}^{V}_{G}T
$$

Multiplying this matrix and the station vector, we obtain the vector in {B} coordinates. Now, to calculate the yaw angle of the antenna, we note that that angle is the angle of the proyection of the station vector on the ${}^{B}X$-${}^{B}Y$ plane, with respect to the ${}^{B}Y$ axis.

Similarly, the pitch is the angle between the S vector and the ${}^{B}X$-${}^{B}Y$ plane. The equation's to solve for those angles are:

$$
\theta = \operatorname{atan}({}^{B}s_x, {}^{B}s_y)
$$
$$
\lambda = \operatorname{atan}\left({}^{B}s_z, \sqrt{(({}^{B}s_x)^2 + ({}^{B}s_y)^2})\right)
$$

It is possible to obtain equations for the coordinates of the station in the {B} frame. After multiplying the matrices and the vector, we get the following equations:

$$ {}^{B}s_x = ({}^{G}s_x - {}^{G}v_x) \cdot c_\alpha c_\beta + ({}^{G}s_y - {}^{G}v_y) \cdot s_\alpha c_\beta - ({}^{G}s_z - {}^{G}v_z) \cdot s_\gamma - {}^{V}a_x $$

$$ {}^{B}s_y = ({}^{G}s_x - {}^{G}v_x) \cdot (c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma) + ({}^{G}s_y - {}^{G}v_y) \cdot (s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma) - ({}^{G}s_z - {}^{G}v_z) \cdot c_\beta s_\gamma - {}^{V}a_y $$

$$ {}^{B}s_z = ({}^{G}s_x - {}^{G}v_x) \cdot (c_\alpha s_\beta c_\gamma + s_\alpha c_\gamma) + ({}^{G}s_y - {}^{G}v_y) \cdot (s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma) - ({}^{G}s_z - {}^{G}v_z) \cdot c_\beta c_\gamma - {}^{V}a_z $$