Physically Based Modeling
CS 15-863 Spring 1997
Take Home Midterm
Due March 10

Question 1

Consider the following planar physical system. There are three mass points with position (p_{0_x}, p_{0_y}), (p_{1_x}, p_{1_y}), and (p_{2_x}, p_{2_y}) and masses m_0, m_1 and m_2, respectively.

The mass points bound a certain amount of an incompressible fluid—that is, if we draw lines between each pair of points, to form a triangle in the plane, the fluid must remain in that triangle (imagine a very light, stretchable membrane that fits snugly around the three mass points, containing the fluid). Because the fluid is incompressable, the area that the fluid occupies must remain constant.

Assuming that the mass of the fluid is negligible, derive equations for the acceleration of the three mass points, for a given instant of time. That is, assume that the three points currently have coordinates (p_{0_x}, p_{0_y}), (p_{1_x}, p_{1_y}) and (p_{2_x}, p_{2_y}). The velocity of the three points is arbitrary, but is consistent with the constant area assumption. External forces of (F_{0_x}, F_{0_y}), (F_{1_x}, F_{1_y}), and (F_{2_x}, F_{2_y}) act on the three particles respectively. Give an equation for the resulting acceleration of each mass point.

Question 2

The *kinetic energy* T of a particle p with velocity $v = \dot{p}$ and mass m is defined to be

$$T = \frac{1}{2} m v \cdot v. \tag{1}$$

(a) Suppose that the particle is subject to a single constraint $C(p) = 0$. Show that if the external force on the particle is zero, the kinetic energy of the particle remains constant. Do *NOT* assume that the particle is initially motionless, but do assume that any initial velocity v is consistent with the constraint $C(p) = 0$.

(b) Generalize (a) to show that if the particle is subject to multiple constraints

$$C_1(p) = C_2(p) = \cdots = C_k(p) = 0 \tag{2}$$

the kinetic energy is constant. Assume that the dimension of the particle (i.e. the number of coordinates of p) is greater than k.

Question 3

A particle with coordinates (x, y) is constrained to the line $y = mx + b$ using the implicit constraint $C(x, y) = (mx + b - y)^2$ and the method of Lagrange multipliers. Assuming a feedback term of the form $\frac{1}{\tau^2}C - \frac{2}{\tau}\dot{C}$, and assuming that $C \neq 0$ initially, how will the particle behave in the absence of external forces both in theory and practice?