Physically Based Modeling

CS 15-863 Spring 1997

Take Home Final

Due May 1

Question 1

Consider the following planar physical system. There are three mass points, p_1, p_2 and p_3. The initial positions of these points in the plane are $p_1 = (-2, 0)$, $p_2 = (2, 0)$ and $p_3 = (-1, 2)$. No gravity or damping acts on the particles, and initially, each point is motionless. The points are connected together with springs to form a triangle. Each point has unit mass. The stiffness coefficient k_s of each spring is $k_s = 1/10$, and the drag coefficient of the spring is $k_d = 1/10$ also. The rest length r of each spring is 2.

Recall that the energy stored in the spring between particles 1 and 2 is

$$1/2 k_s (|p_1 - p_2| - r)^2 \tag{1}$$

and similarly for the energy in the spring between particles 2 and 3, and between particle 1 and 3. The kinetic energy of particle i is

$$\frac{1}{2} \dot{p}_i \cdot \dot{p}_i \tag{2}$$

The *total energy* of the system is the sum of the three spring energies, and the kinetic energies of the particles.

(a) Calculate the total energy of the system initially.

(b) After approximately how long will the system's total energy have decayed to below 1% of its initial value? (It is not necessary to show your work; merely explain how you arrived at your answer.)

Question 2

Consider a particle attached to a glass wire. Specifically, imagine a glass wire, whose shape is the parabola $y = x^2$, and a unit mass particle which slides down the parabola, accelerated by a gravity force $(0, -1)$ (i.e. unit gravity, straight down). The glass wire is fragile; a force of 25 units or more on the wire, at any point, will shatter the wire. Compute the minimum height from which you could start the particle, initially at rest, so that the wire will shatter. Assume the interaction of the particle and the glass is frictionless, and there is no damping on the particle.

Question 3

Consider a standard 12-hour analog clock with hour, minute, and second hands. Someone stole the motor, but the hands are still coupled by some kind of (hidden) mechanism so that if you move one of the hands, the other two always move with appropriate relative speeds. We label the end-points of the hour, minute, and second hands as **H**, **M**, and **S**, respectively. We assume that all the mass of the system is concentrated at these three points, denoting these three masses by m_h, m_m, and m_s. We label the lengths of the three hands as r_h, r_m, and r_s. The center of the clock face, where the three hands meet, is fixed in place.

(a) Derive *equations of motion* for the clock, **for a first-order (** $f = mv$ **) world.** Specifically, given forces $\mathbf{f_h}$,$\mathbf{f_m}$ and $\mathbf{f_s}$ applied to points **H**, **M**, and **S**, how would we compute the hands' velocities. Use the notation introduced above for radii, masses, and forces. Your derivation should work for arbitrary radii and masses (i.e. don't assume particular numerical values for the r's and m's.) Clearly define any additional symbols you introduce. Feel free to use labels or diagrams as appropriate.

(b) Suppose we want to make this clock actually run, keeping correct time, by lashing a controllable rocket engine onto point **M** on the minute hand. Give a formula for the *minimal force, as a function of time,* required to keep the clock accurate, assuming it's been set correctly initially. It might be convenient to measure time in minutes (or seconds) since midnight.