Physically Based Modeling

CS 15-863 Spring 1997

Assignment 3: A Double Pendulum

Due February 27

In this assignment, you'll implement a "double pendulum." A double pendulum is a system consisting of two masses, which we'll assume to be particles. The first mass is constrained to lie on the parabola $y = ax^2$, where a is the constant of your choice (other than zero). The second mass must be at a constant distance r from the first mass.

For this assignment, we'd like you derive and implement a double pendulum two different ways: first, use Lagrange multipliers with an implicit constraint of the form $\mathbf{C}(\mathbf{p}) = 0$, where $\mathbf{p}$ is the vector of both particles positions. Second, use reduced coordinates, with a parametric function $\mathbf{p}(\mathbf{u})$. (What dimension should $\mathbf{u}$ have?) Also, we'd like you to hand in both derivations.

What to Implement

For both the reduced coordinates case and the Lagrange multiplier case, you should simulate the system with a constant gravity force acting on both particles. As in previous assignment, you should be able to turn a mouse spring on and off, and pull on either of the two particles. You will want to add some damping to the system. The damping force in R^2 on each particle should be of the form $F_{damp} = -kv$ where v is the particle's velocity. For the Lagrange multiplier case, you should add "feedback" terms onto the right-hand-side of the equation as described in the handout so that small errors in the constraint or the constraints' first derivative are cancelled.

For the Lagrange multiplier case, draw a vector from each particle to indicate the constraint force acting on the particle. (Choose an appropriate scaling factor so that the constraint force vectors are neither too big nor too small to view during the simulation.) Draw both the parabola (whatever part fits on the screen), and draw a line between the two particles.

What to Hand In

In the reduced coordinates approach, specify what the generalized coordinates you have chosen to describe the system are, and write down the particle positions in terms of those coordinates. Derive expressions for the particles' velocity in terms of your generalized coordinates, and derive the matrices $\partial\mathbf{p}/\partial\mathbf{u}$ and $\frac{d}{dt}(\partial\mathbf{p}/\partial\mathbf{u})$. Derive the generalized force when an external force F_1 in R^2 acts on the first particle at the same time as a force F_2 acts on the second particle.

In the Lagrange multiplier approach, write out the constraint equations. Derive the matrices $\mathbf{J} = \partial\mathbf{C}/\partial\mathbf{p}$ and $\dot{\mathbf{J}}$.

Helpful Hint

You'll probably make a mistake somewhere in your derivation, or in coding this up. To easily test if your system is working correctly, start the double pendulum so that the first mass has position $(0, 0)$ and the second has position $(r, 0)$ i.e. the pendulum is stretched out horizontally. Turn gravity and damping off. Choose an arbitrary initial velocity for the system that is consistent with the constraints. If you've gotten everything right, then the kinetic energy of the system $\frac{1}{2}(m_1 v_1 \cdot v_1 + m_2 v_2 \cdot v_2)$ should remain constant as the system moves around. If the kinetic energy varies (or your objects fly apart!) you've made a mistake.