Homework 3 — CS 819A4 — Spring 2003
assigned April 22, due April 29

Problem 1

Derive the axiom schemata

$$m \leq j \leq n \Rightarrow \Big(\big(\bigodot_{i=m}^{n} p(i)\big) \Rightarrow (p(j) \ * \ \mathbf{true})\Big)$$

from the axiom schemata for iterating separating conjunction given earlier at the beginning of the fifth section of the class notes.

Problem 2

The recursive procedure $\mathsf{subst1}(\mathsf{a}, \mathsf{j}\,; \mathsf{i})$ defined in the sixth section of the class notes will perform a mutation operation on every field of the tree structure whose root is i. Except when a field contains the atom a, however, the mutation operation will merely overwrite the current value with the same value.

This inefficiency can be avoided by a recursive procedure $\mathsf{substinloc}$ that accepts, instead of i, a location k containing i, and mutates this location only if i changes.) This procedure should satisfy

$$\begin{array}{l} \{\exists \mathsf{i}.\ \mathsf{k} \mapsto \mathsf{i} \ * \ \mathsf{tree}\,\tau\,(\mathsf{i}) \ * \ (\mathsf{p} \wedge \mathsf{dag}\,\tau'\,(\mathsf{j}))\} \\ \mathsf{substinloc}(\mathsf{a}, \mathsf{j}, \mathsf{k}) \\ \{\exists \mathsf{i}.\ \mathsf{k} \mapsto \mathsf{i} \ * \ \mathsf{tree}\,(\tau/\mathsf{a} \to \tau')\,(\mathsf{i}) \ * \ \mathsf{p}\}, \end{array}$$

Write the procedure $\mathsf{substinloc}$, and give an annotated specification proving that it meets the above specification.

Problem 3

Show, by an informal metaproof, that the following laws hold, whenever p is an arbitrary assertion and i and i' are intuitionistic assertions:

$$i \ * \ p \Rightarrow i \qquad i \Rightarrow p \mathrel{-\!\!*} i \qquad i \ * \ i' \Rightarrow i \wedge i'.$$