Oddities of Inductive Definitions

$$\mathit{tree}\ t\ z \;=\; t = \mathit{nil} \wedge \mathit{emp} \;\vee\; \exists l, r.\ t \xmapsto{z} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z$$

$\{\ \mathit{tree}\ t\ z\}$
if $t \neq \mathit{nil}$ then
  $\{\exists l, r.\ t \xmapsto{z} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z\}$
  $l := [t]\ ;\ r := [t+1]\ ;$
  $\{t \xmapsto{z} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z\}$
  if $l = r$ then
  ;

$$\mathit{tree}\ t\ z = t = \mathit{nil} \wedge \mathit{emp} \vee \exists l, r.\ t \overset{z}{\mapsto} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z$$

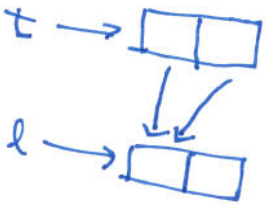
When $z \leq 1/2$

$\{\ \mathit{tree}\ t\ z\}$
if $t \neq \mathit{nil}$ then
$\{\exists l, r.\ t \overset{z}{\mapsto} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z\}$
$l := [t]; r := [t+1];$
$\{t \overset{z}{\mapsto} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z\}$
if $l = r$ then
$\{t \overset{z}{\mapsto} l, l * \mathit{tree}\ l\ z * \mathit{tree}\ l\ z\}$
$\{t \overset{z}{\mapsto} l, l * \mathit{tree}\ l\ (2z)\}$
⋮

$$\mathrm{tree}\ t\ z = t = \mathrm{nil} \wedge \mathrm{emp} \vee \exists l, r.\ t \xmapsto{z} l, r * \mathrm{tree}\ l\ z * \mathrm{tree}\ r\ z$$

When $z = 1/2$

$\{\ \mathrm{tree}\ t\ z\}$
if $t \neq \mathrm{nil}$ then
$\{\exists l, r.\ t \xmapsto{z} l, r * \mathrm{tree}\ l\ z * \mathrm{tree}\ r\ z\}$
$l := [t]; r := [t+1];$
$\{t \xmapsto{z} l, r * \mathrm{tree}\ l\ z * \mathrm{tree}\ r\ z\}$
if $l = r$ then
$\{t \xmapsto{z} l, l * \mathrm{tree}\ l\ z * \mathrm{tree}\ l\ z\}$
$\{t \xmapsto{z} l, l * \mathrm{tree}\ l\ (z/2)\}$
if $l \neq \mathrm{nil}$
$\{t \xmapsto{z} l, l * \exists l', r'.\ l \xmapsto{1} l', r' * \mathrm{tree}\ l'\ 1 * \mathrm{tree}\ r'\ 1\}$
$[l] := \mathrm{nil}$
⋮



Creating the Dag

$$\mathit{tree}\ t\ z \;=\; t = \mathit{nil} \wedge \mathit{emp} \;\vee\; \exists l, r.\ t \xmapsto{z} l, r * \mathit{tree}\ l\ z * \mathit{tree}\ r\ z$$

$\{emp\}$

$t' := cons(nil, nil);$

$\{t' \xmapsto{1} nil, nil\}$

$t := cons(t', t');$

$\{t \xmapsto{1} t', t' * t' \xmapsto{1} nil, nil\}$

$\{t \xmapsto{1/2} t', t' * t \xmapsto{1/2} t', t' * t' \xmapsto{1/2} nil, nil * t' \xmapsto{1/2} nil, nil\}$

$\{t \xmapsto{1/2} t', t' * \exists l, r.\ t \xmapsto{1/2} l, r * l \xmapsto{1/2} nil, nil * r \xmapsto{1/2} nil, nil\}$

$\{t \xmapsto{1/2} t', t' * \exists l, r\ t \xmapsto{1/2} l, r * \mathit{tree}\ l\ 1/2 * \mathit{tree}\ r\ 1/2\}$

$\{t \xmapsto{1/2} t', t' * \mathit{tree}\ t\ 1/2\}$

$\{t \xmapsto{1/2} t', t'\}$ $\qquad$ $\{\mathit{tree}\ t\ 1/2\}$

⋮ $\qquad$ $=$ $\qquad$ ⋮