HOMEWORK 2 — CS 818A4 — SPRING 2004

Problem 1

When

$$\exists \alpha, \beta.\ (\textbf{lseg}\ \alpha\,(\mathsf{i}, \mathsf{j})\ *\ \textbf{lseg}\ \beta\,(\mathsf{j}, \mathsf{k})) \wedge \gamma = \alpha{\cdot}\beta,$$

we say that j is an *interior pointer* of the list segment described by $\textbf{lseg}\ \gamma\,(\mathsf{i}, \mathsf{k})$.

1. Give an assertion describing a list segment with two interior pointers j_1 and j_2, such that j_1 comes before than, or at the same point as, j_2 in the ordering of the elements of the list segment.

2. Give an assertion describing a list segment with two interior pointers j_1 and j_2, where there is no constraint on the relative positions of j_1 and j_2.

3. Prove that the first assertion implies the second.

Problem 2

A *braced list segment* is a list segment with an interior pointer j to its last element; in the special case where the list segment is empty, j is **nil**. Formally,

$$\textbf{brlseg}\ \epsilon\,(\mathsf{i}, \mathsf{j}, \mathsf{k}) \stackrel{\text{def}}{=} \textbf{emp} \wedge \mathsf{i} = \mathsf{k} \wedge \mathsf{j} = \textbf{nil}$$

$$\textbf{brlseg}\ \alpha{\cdot}a\,(\mathsf{i}, \mathsf{j}, \mathsf{k}) \stackrel{\text{def}}{=} \textbf{lseg}\ \alpha\,(\mathsf{i}, \mathsf{j})\ *\ \mathsf{j} \mapsto \mathsf{a}, \mathsf{k}.$$

1. Prove the assertion

$$\textbf{brlseg}\ \alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k}) \Rightarrow \textbf{lseg}\ \alpha\,(\mathsf{i}, \mathsf{k}).$$

2. Give an annotated specification of a command that changes the final pointer of a braced list segment from k to l. The command should not construct or deallocate any list structure, and it should not change the value of j. It should satisfy

$$\{\textbf{brlseg}\ \alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}\ \cdots\ \{\textbf{brlseg}\ \alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{l})\}.$$

3. Give an annotated specification of a command that concatenates two braced list segments. The command should not construct or deallocate any list structure, and it should not change the value of k_2. It should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i}_1, \mathsf{j}_1, \mathsf{k}_1)\ *\ \mathbf{brlseg}\,\beta\,(\mathsf{i}_2, \mathsf{j}_2, \mathsf{k}_2)\}$$
$$\cdots$$
$$\{\mathbf{brlseg}\,\alpha{\cdot}\beta\,(\mathsf{i}_1, \mathsf{j}, \mathsf{k}_2)\}.$$

4. Give an annotated specification of a command that appends an item a at the beginning of a braced list segment. The command should not change the value of k, and should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\mathsf{a}{\cdot}\alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}.$$

5. Give an annotated specification of a command that appends an item a at the end of a braced list segment. The command should not change the value of k, and should satisfy

$$\{\mathbf{brlseg}\,\alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\alpha{\cdot}\mathsf{a}\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}.$$

6. Give an annotated specification of a command that deletes an item a from the beginning of a nonempty braced list segment. The command should not change the value of k, and should satisfy

$$\{\mathbf{brlseg}\,\mathsf{a}{\cdot}\alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}\ \cdots\ \{\mathbf{brlseg}\,\alpha\,(\mathsf{i}, \mathsf{j}, \mathsf{k})\}.$$