# Multimodal Optimization

- Intro to optimization
- Genetic Algorithms
- Other techniques: hillclimbing, simulated annealing
- Paper: Genetic Algorithms and Technical Analysis in fx data

## Almost Everything is Optimization

#### Basic optmization framework:

- Some set of parameters that define a parameter space
- A fitness function that maps every point in the parameter space onto a fitness value
- Optmization is simply the process of adjusting these parameters until fitness is maximized (or some other criteria is reached).

## Almost Everything is Optimization, Continued

Almost all machine learning algorithms contain have some element of optimization at their core. Examples:

- Simple: maximize  $-((x-10)^2)$
- Neural networks: parameters are weights, fitness function is error on learning task
- Programs: parameters are various primitives in programming languae, fitness function is performance on desired task

# Appropriate Techniques

## **Optimization**

- Smooth functions with one local optimum can use calculus based methods (Lagrange, derivatves)
- With neural networks, all we have is a local gradient; pick the optimal direction ango
- When the landscape is not smooth, we must use other methods (like genetic algorithms)

# How to Optimize with No Derivatives

#### Hillclimbing:

- Look one step away in random direction in parameter space
- Did we improve in fitness?
  - Yes: Move, and look again
  - No: Stay, and look again

# Multiple Optima

- Sometimes, we have multiple optima
- Hillclimbing gets stuck in 'basin of attraction'.
- Solution: Multiple Restarts
- Or: Genetic Algorithms

# Genetic Algorithms

\*Slides from book go here\*

## Simulated Annealing

Examine fitness of neighboring point in parameter space

- If better than current point, accept move
- If worse then than current point, accept with the following probability (called the 'Metropolis Criterion'):  $e^{(f(y)-f(x)/T)}$

#### Where

- -f(y) is the fitness of the current point
- -f(x) is the fitness of the neighboring point
- -T is an annealing schedule parameter that decreases with time

Essentially, this is hillclimbing, but with a small probability of accepting a downhill move. This probability decreases with time.

## Simulated Annealing Continued

- This is based on very well worked out physics.
- Theoretically, if the annealing is 'slow' enough, the algorithm is guaranteed to converge
- Works very well in practice usually better than GAs.

#### Which Method?

Unfortunately, much more of an art than a science

- GA-like methods are good for parameters spaces of unknown size (like genetic programming trees)
- Hillclimbing (and multi-restart hillclimbing) are simple to implement with few parameters.
- Simulated Annealing is also simple, and in general the most effective.

Bottom Line: Genetic Algorithms have their uses, but don't fall prey to STD (sexy terminology disease). The evolutionary metaphor means nothing – pay attention to the underlying computation.

If somebody is using GAs, ask them why hillclimbing or simulated annealing aren't appropriate.

## GAs and Forex Trading Rules

"Is Technical Analysis in the Foreign Exchange Market Profitable?" Neely, Weller, Ditmar

#### Basic idea:

Evolve technical analysis style trading rules using genetic programming.

Authors are economists and treat genetic algorithms as a black box. Much more interested in statistical validation of results than in tweaking the algorithm

#### Overview

- Parameter Space: Program trees that map past price data onto long/short signals in fx trading
- Fitness function: excess returns provided by those strategies

#### Structure of GP Tree

- Operations on price time series moving averages, max, min, etc
- Arithmetic operators
- Boolean operations
- if-then, if-then else
- numerical constants
- Boolean constants: 'true', 'false'

#### Fitness function

Tree is a function: maps past price data onto an 'long' (borrow dollars to buy marks, say) or 'short' (borrow marks to buy dollars) signal. Metric of success: excess returns over cost of capital.

#### Daily returns

$$r_t = lnS_{t+1} - lnS_t + ln(1 + i_t^*) - ln(1 + i_t)$$

- $r_t$  is daily log return
- $S_t$  is exchange rate at time t
- $\bullet$   $i_t^*$  foreign risk-free rate at time t
- $i_t$  US risk-free rate at time t

## Fitness function, continued

#### Total returns:

$$r = \sum_{t=0}^{T-1} z_t r_t + n \cdot \ln((1-c)/(1+c))$$

- $\bullet$  c is transaction cost
- $\bullet$  *n* is number of transactions
- $r_t$  is daily log returns
- $z_t$  is long/short signal (provided by tree)  $z_t = 1$  for long,  $z_t = -1$  for short,

#### Data

Data is daily forex closes, for dollar, mark, swiss frank, and yen, with accompanying risk-free rates.

Split data into 3 chunks:

- Training: 1975-77; Train initial population of rules
- Validation: 1978-80; Pick best rule over validation set
- Test: 1981-1995; Report results on test set

(This terminology differs from the paper, which confusingly refers to the 'validation' set as a 'selection' set, and the 'test' set as the 'validation' set).

## Why Validation?

- Optimization problems often 'noiseless'; see block stacking problem in book.
- But, this data is extremely noisy (efficient market hypothesis, anyone?)
- Thus, need to prevent overfitting.
- After training on training set, pick only the best rule on validation set

### Results

Excess profits across all currencies

Annualized returns on test set:

- $\bullet$  6.05% for dollar/mark
- $\bullet$  2.34% for dollar/yen
- 2.27% for dollar/pound
- 4.1% for mark/yen

### Benchmark?

The real question: How to interpret the results?

What benchmark?

- Raw excess returns?
- Buy-and-hold?
- Sharpe Ratios?
- Simple Moving Average Rule?

## Statistical Tests: The Bootstrap

#### Statistical Tests:

- Normal hypothesis testing? No
- Bootstrapping? Yes, allows our null-models to have arbitrary distributions
  - random walk/sampling with replacement
  - -ARMA(2,2) model
  - -ARMA(2,2) + GARCH(1,1)
- Examine out-of-sample profits of generated rules on bootstrapped data
- Result: Bootstrapped data sets do *not* produce excess returns
- Bootstrapping is the gold standard for this sort of work

# Questions?

- Only tested one set of rules (dollar/DM) with bootsrap
- Is this really better than a simple 150 day moving average rule?
- Is validation really helpful?

# Implications and Future Directions

- If ARMA + GARCH doesn't explain everything, what does this say about pricing options?
- What about other markets?
- Can we tweak the algorithm esp, better guards against overfitting?

# What do you make of the results?