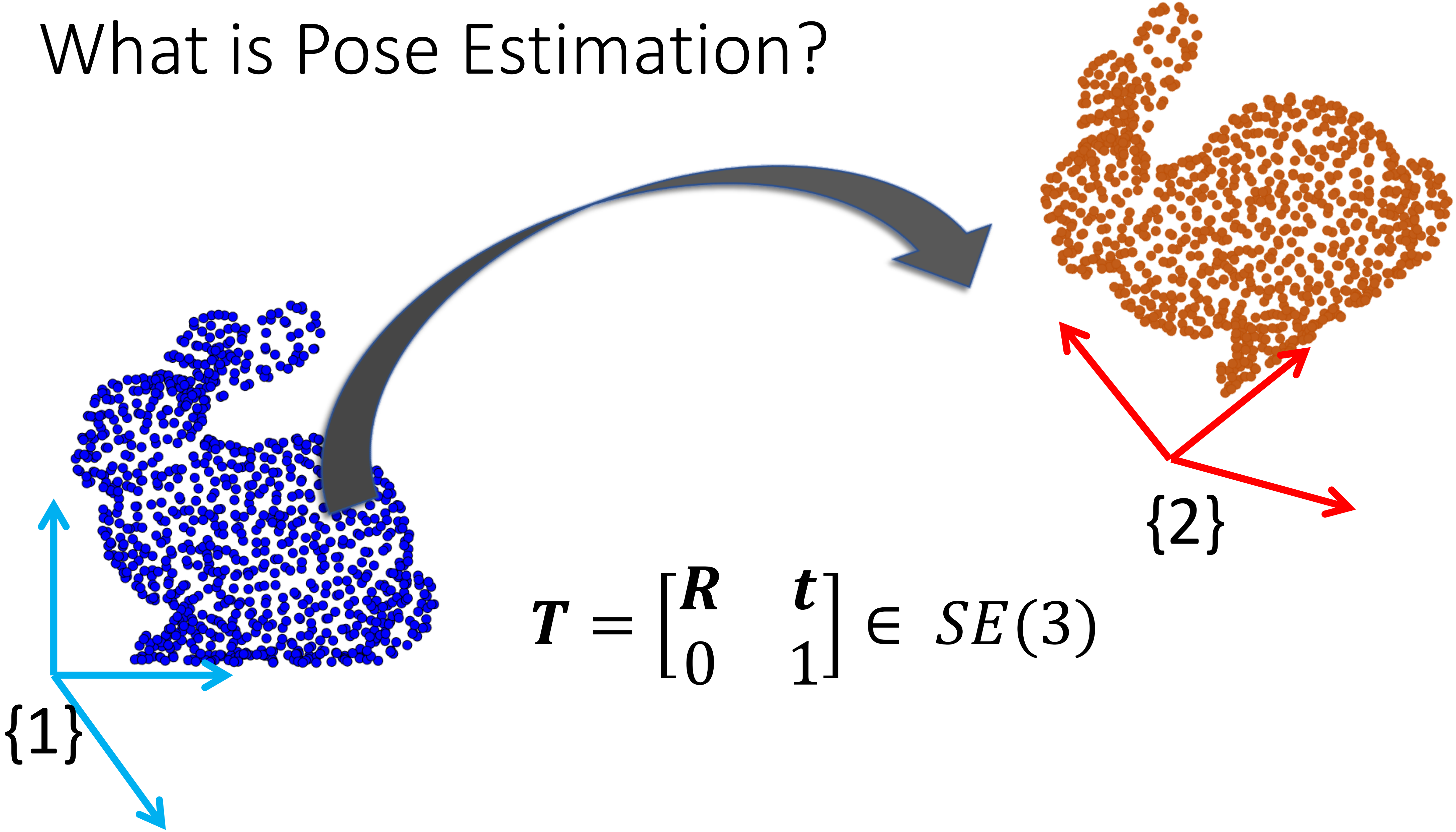
Probabilistic approaches for pose estimation

R. Arun Srivatsan

Thesis Committee

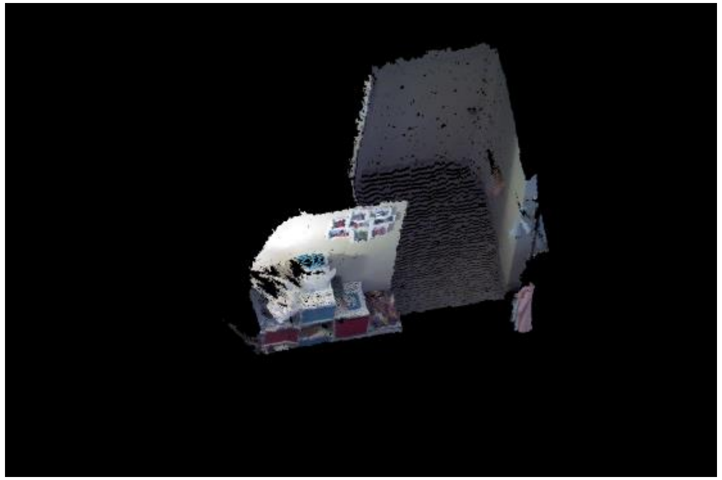
Howie Choset
Michael Kaess
Simon Lucey
Russel Taylor, JHU
Nabil Simaan, VU

What is Pose Estimation?

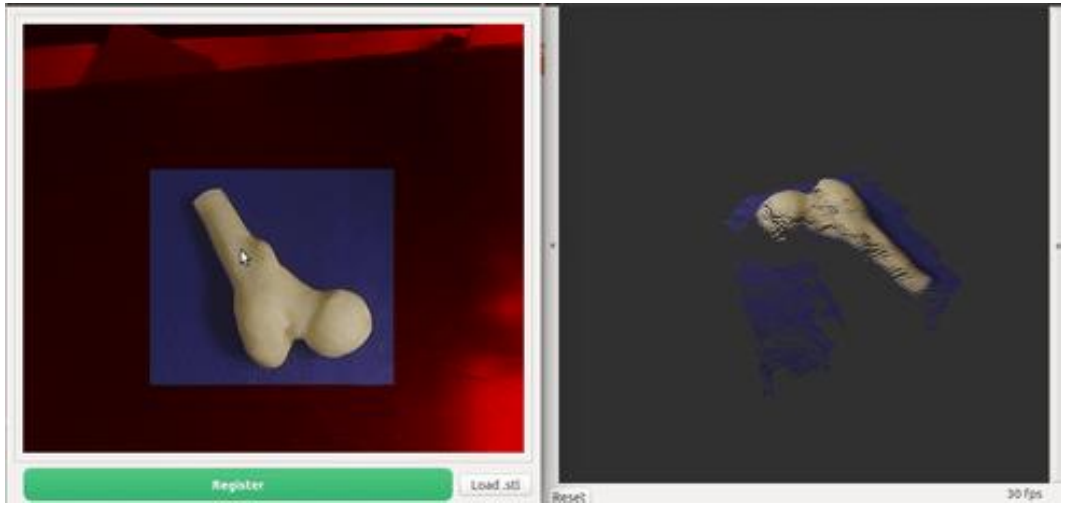


$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \\ 0 & 1 \end{bmatrix} \in SE(3)$$

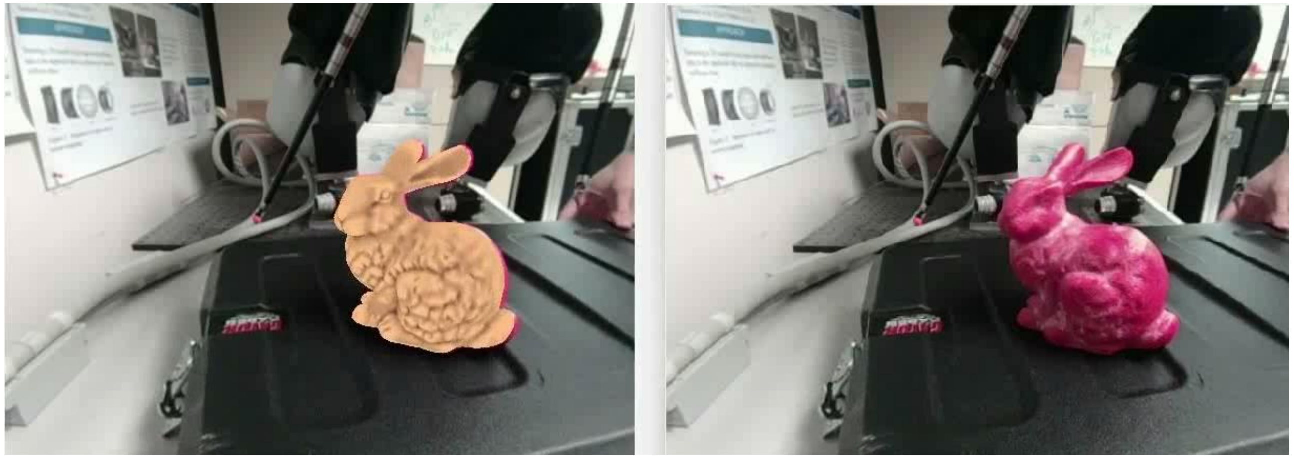
Applications of Pose Estimation



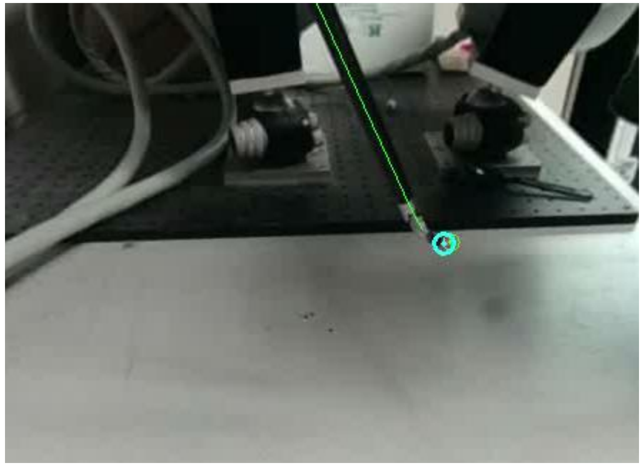
3D reconstruction



Registration

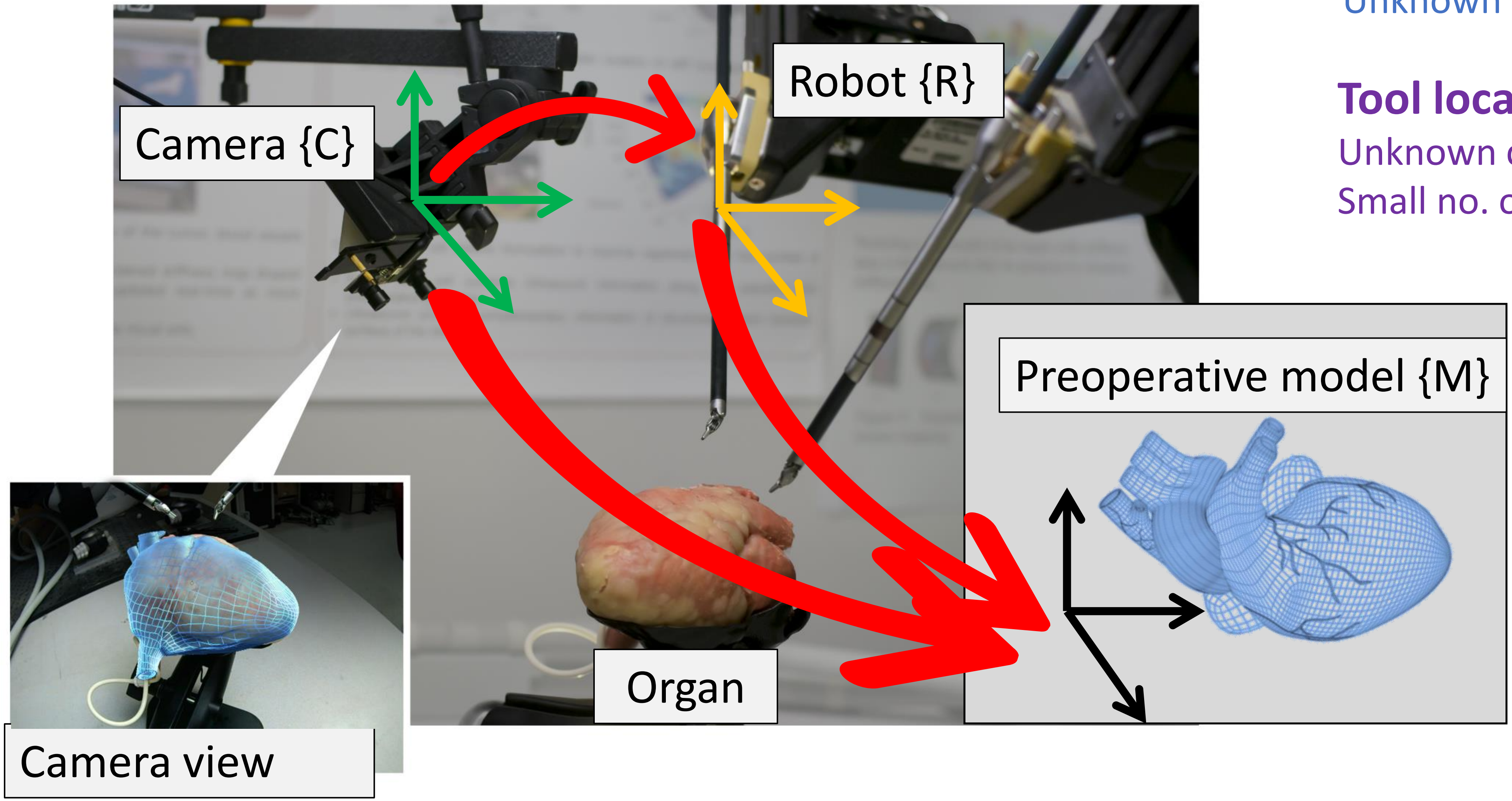


Object tracking



Hand-eye calibration

Motivating example:
Image guided robotic surgery



Hand eye calibration
Known correspondence

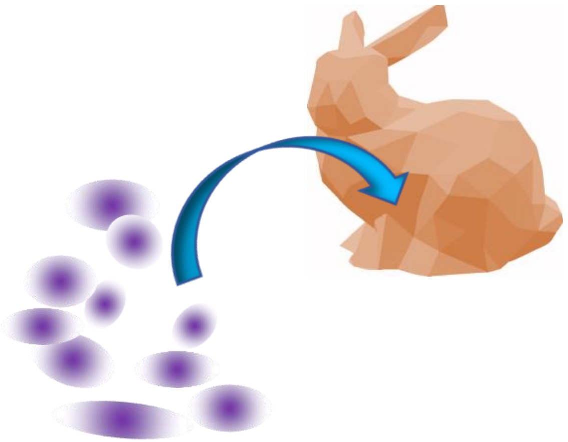
Stereo registration
Unknown correspondence

Tool localization
Unknown correspondence
Small no. of measurements

Outline

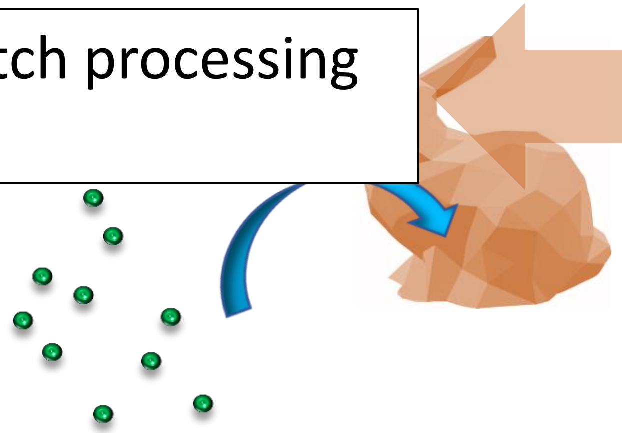
- Prior work
- Motivation
- Key contributions
- Revisiting example
- Future work

Prior work



Deterministic methods

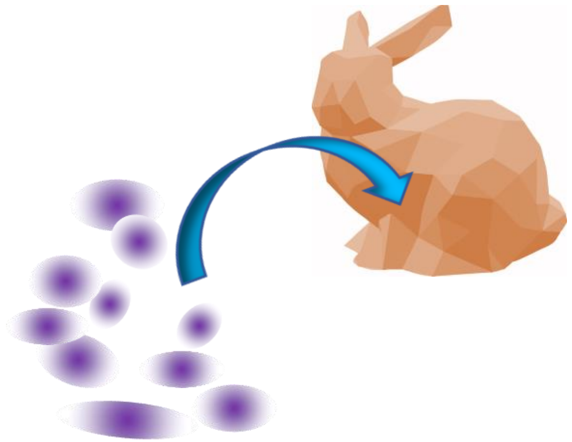
- Horn '87, Arun '87
- ICP, Online processing 2
- Pulli
- Horaud '95
- Go-ICP, Yang '13
- MIP, Batch processing
- Pose



Probabilistic methods

- EKF, Pennec '97
- DQ-IEKF, Goddard '98
- UKF, Moghari '07
- SE-sync, Rosen '16
- EM-ICP, Granger '01
- G-ICP, Segal '09
- GTLS-ICP, Estépar '04
- IMLOP, Billings '15

Prior work



Probabilistic methods

Pose parameter

- Euler angles + Cartesian cords
- Exponential coordinates
- Dual quaternions
- Quaternions + Cartesian coords

Uncertainty distribution

- Gaussian
- Kent
- Bingham

Measurement modalities

- Points
- Surface normals
- Pose
- Color, Stiffness, etc.

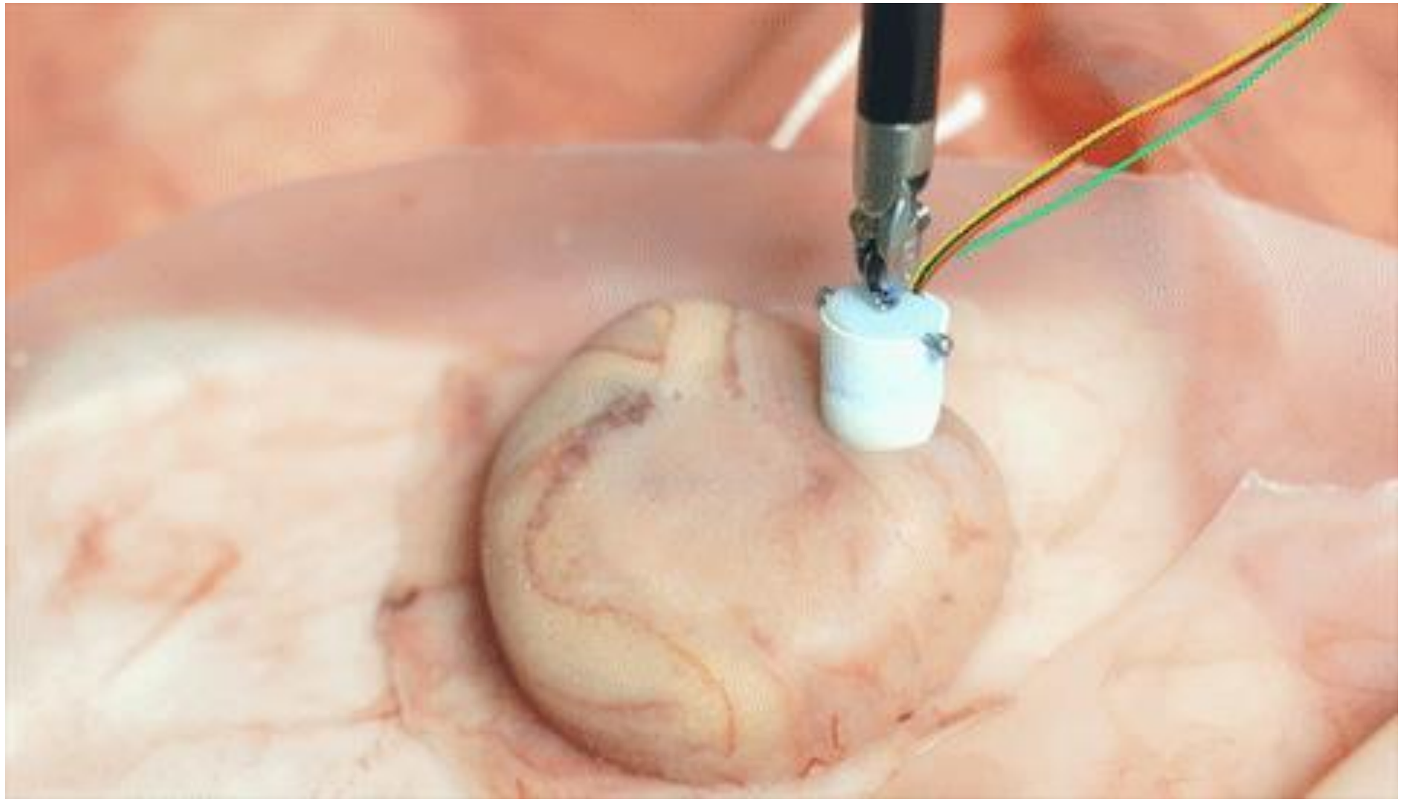
Correspondence

- Known
- Unknown
- 'Some what' known

Motivation and Key Contributions

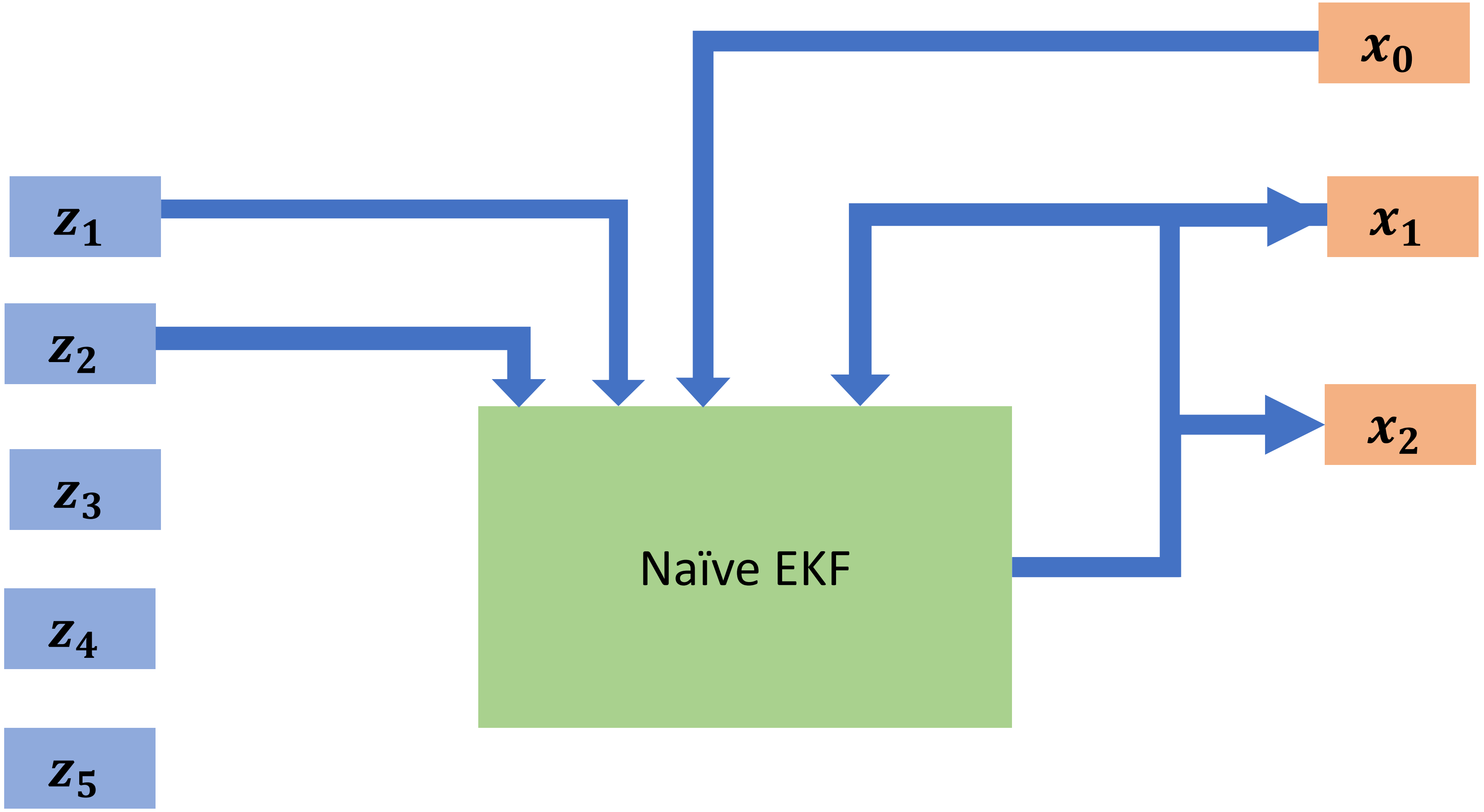
Part 1	Linear update model for pose estimation	State update using 'hash' of measurements
		Finding uncertainty in 'hash' space
Part 2	Using appropriate choice of probability distribution	Novel Bayes filter for pose update
Part 3	Probabilistic nonconvex optimization	Multiple start branch and prune filter
		Sparse point registration
Part 4	Unified framework for probabilistic registration	Unifying prior methods
		Improving existing methods

Simultaneous Compliance and Registration Estimation

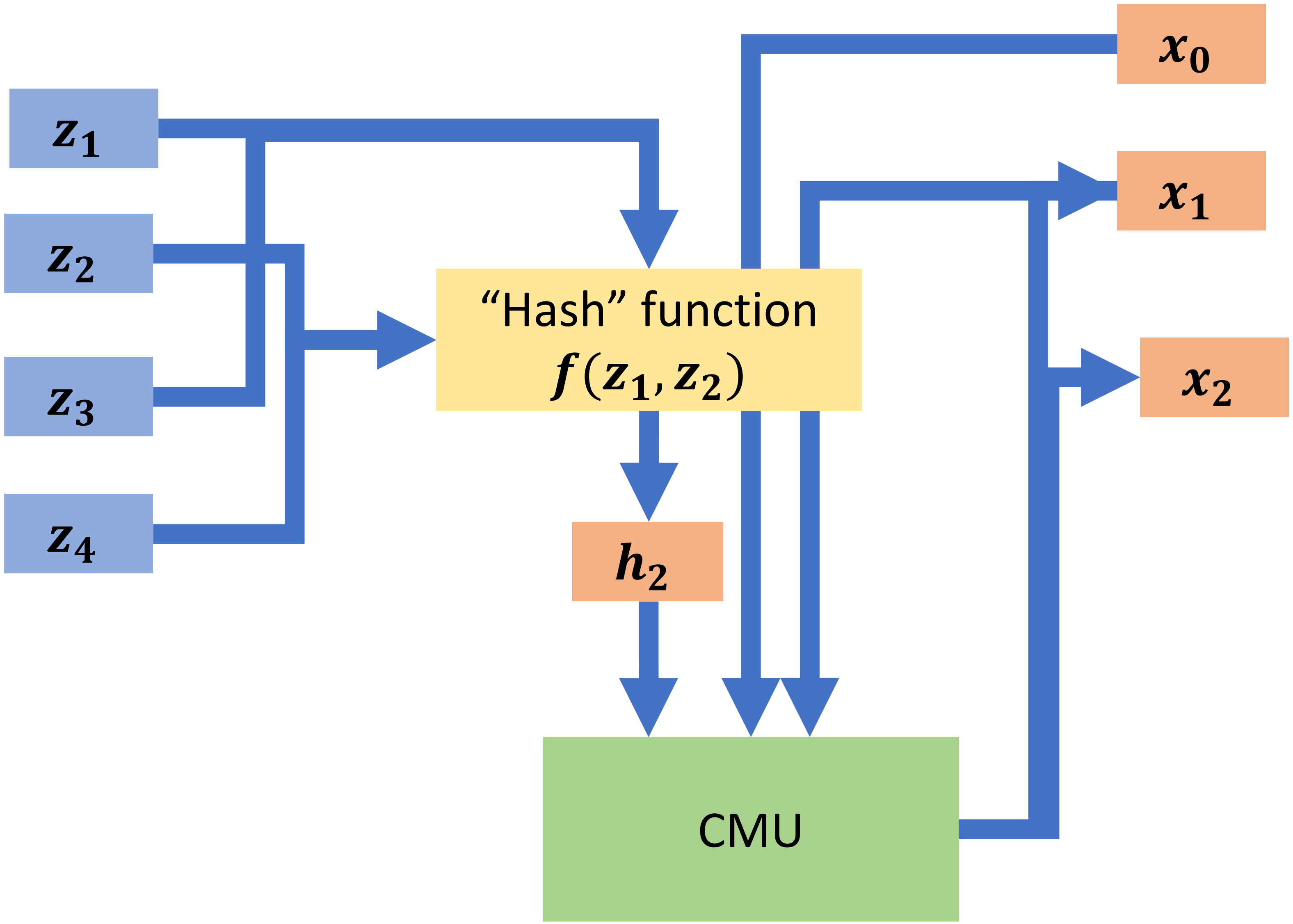


Complementary Model Update (CMU)*

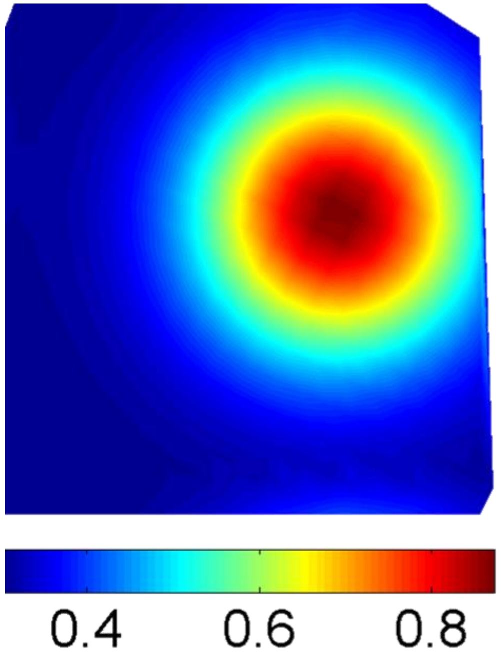
*R. A. Srivatsan et al. “Complementary model update: A method for simultaneous registration and stiffness mapping in flexible environments”, In ICRA, 2016
R. A. Srivatsan et al. “Simultaneous Registration and Stiffness mapping of a Flexible Environment using Stiffness and Geometric Prior”, Hamlyn symposium, 2016


z_1
z_2
z_3
z_4
z_5
Naïve EKF
x_0
x_1
x_2

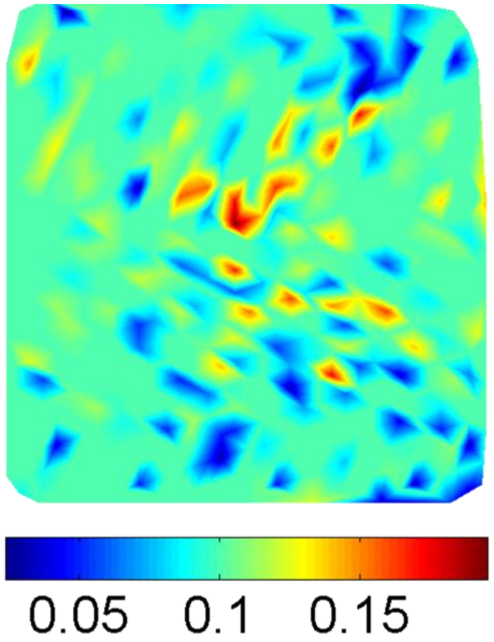
State update using 'hash' of measurements



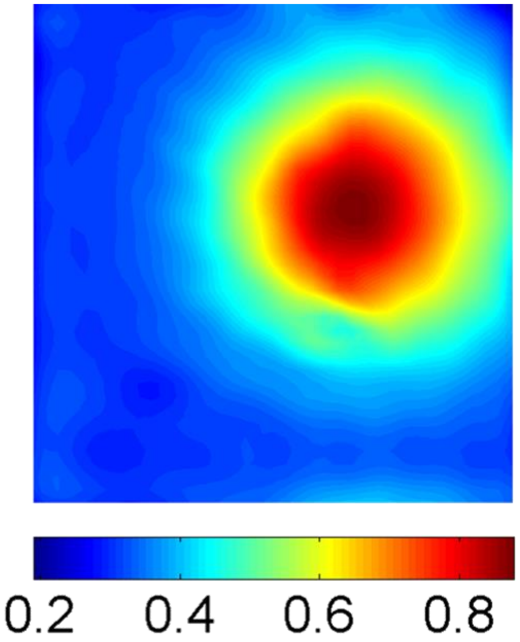
Results: Stiffness estimate using CMU



Actual



Naïve EKF

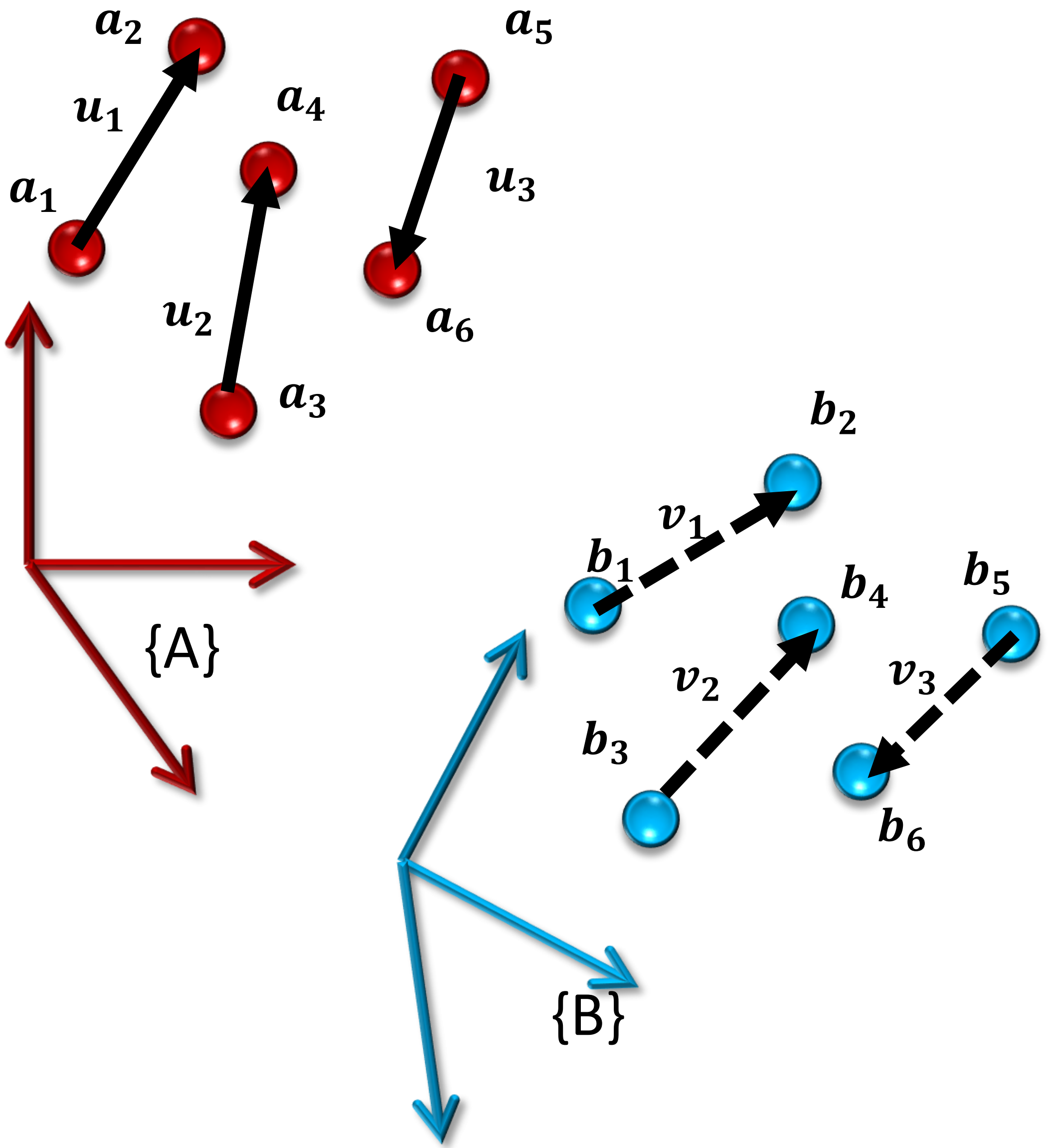


CMU

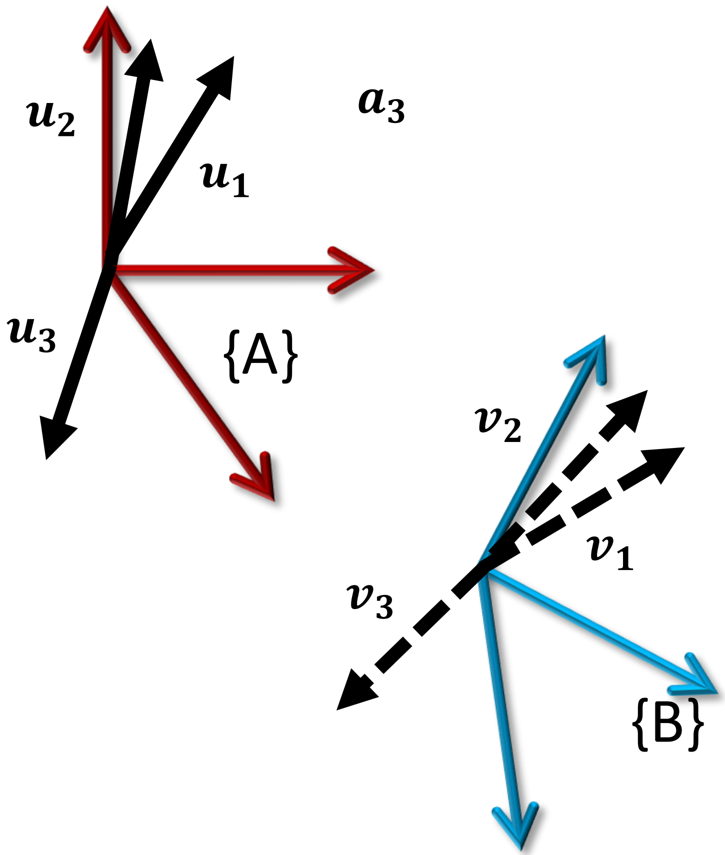
Motivation and Key Contributions

Linear update model for pose estimation	State update using ‘hash’ of measurements
	Finding uncertainty in ‘hash’ space
Part 2	Novel Bayes filter for pose update
Part 3	Multiple start branch and prune filter
	Sparse point registration
Part 4	Unifying prior methods
	Improving existing methods

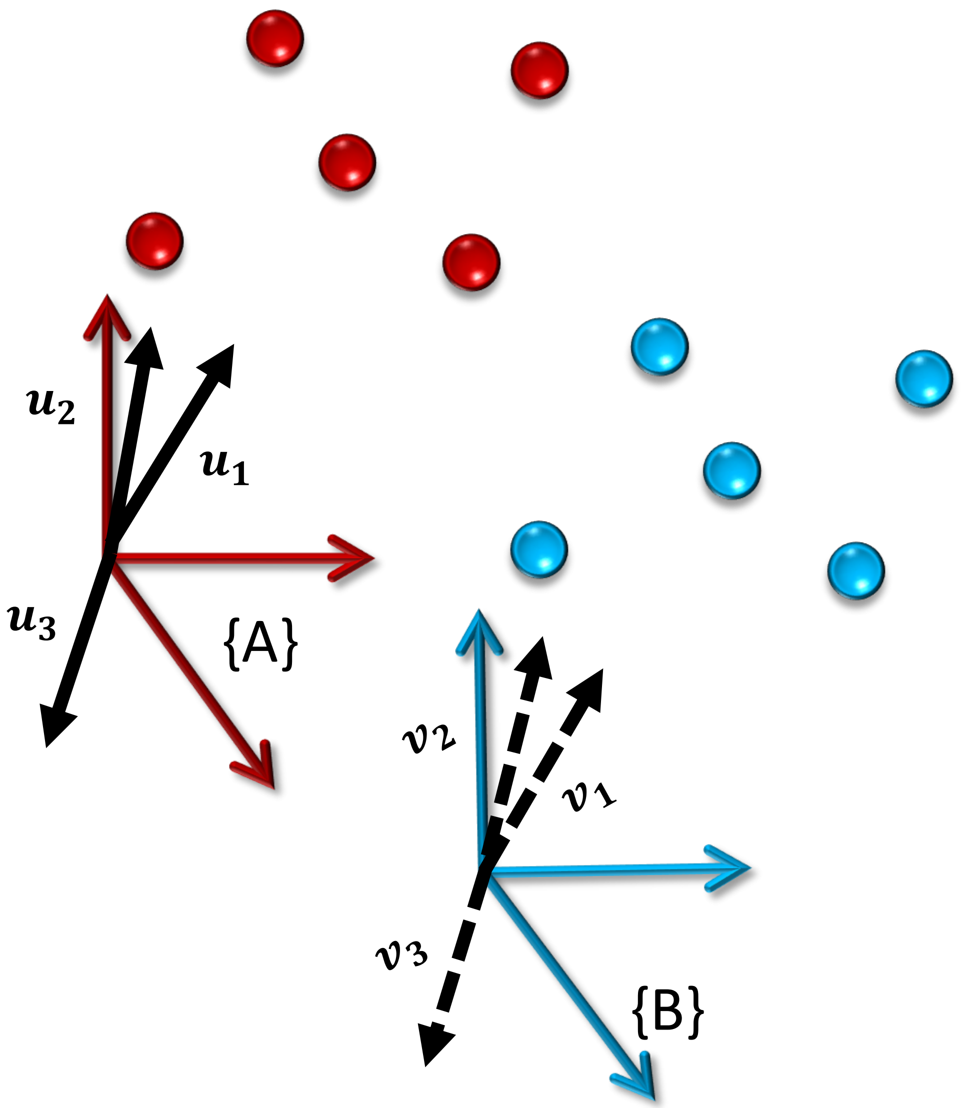
Register frames {A} and {B}



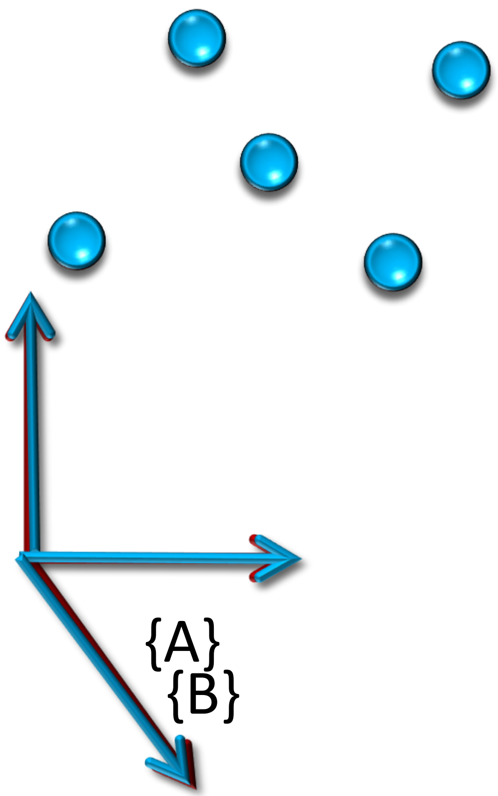
Align vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$



After orienting, estimate translation

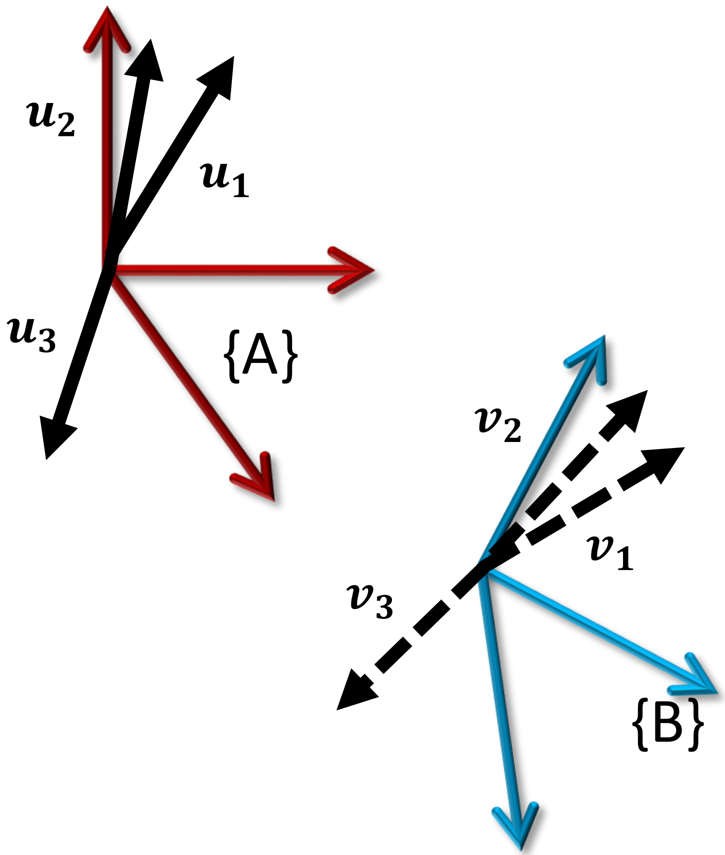


After orienting, estimate translation



Aligning vectors $\boldsymbol{u_i}$ and $\boldsymbol{v_i}$

$\boxed{\text{Unit quaternion: } \boldsymbol{q} \in R^4,\ \boldsymbol{q}^T\boldsymbol{q} = 1}$



$$\boldsymbol{R v_i - u_i = 0}$$

$$\boldsymbol{H(u_i, v_i) q = 0}$$

$$\underbrace{\boldsymbol{H(u_i^s, v_i^s) q}} + \boldsymbol{\Delta s = 0}$$

Uncertainty

$\boldsymbol{h_k}$ = “Hashed” measurement

Finding uncertainty in ‘hash’ space

$\boldsymbol{Q}$ is obtained is closed form using a result from Stochastic Filtering Theory*

$$\Delta s \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q})$$

$$\boldsymbol{H}(\boldsymbol{u}_i, \boldsymbol{v}_i)\boldsymbol{q} = \mathbf{0}$$

$$\underbrace{\boldsymbol{H}(\boldsymbol{u}_i^s, \boldsymbol{v}_i^s)\boldsymbol{q}}_{\boldsymbol{h}_k = \text{“Hashed” measurement}} + \underbrace{\Delta\mathbf{s}}_{\text{Uncertainty}} = \mathbf{0}$$

*Andrew H Jazwinski. Stochastic processes and filtering theory. Courier Corporation, 2007

Linear update model for pose estimation*

$$P(\boldsymbol{z}_k|\boldsymbol{q}_k) = \frac{1}{N_1}\exp\left(-(\boldsymbol{h}_k - \boldsymbol{0})^T \boldsymbol{Q}_k^{-1}(\boldsymbol{h}_k - \boldsymbol{0})\right)$$

$$= \frac{1}{N_1}\exp\left(-(\boldsymbol{H}\boldsymbol{q}_k)^T \boldsymbol{Q}_k^{-1}(\boldsymbol{H}\boldsymbol{q}_k)\right)$$

$$\boldsymbol{z}_k = \left(\boldsymbol{a}_i^T, \boldsymbol{b}_i^T\right)^T$$

*R. A. Srivatsan, G. Rosen, F. Naina, H. Choset, "Estimating SE (3) elements using a dual quaternion based linear Kalman filter", In Robotics: Science and Systems 2016.

Motivation and Key Contributions

Part 1	State update using ‘hash’ of measurements
	Finding uncertainty in ‘hash’ space
Part 2	Novel Bayes filter for pose update
Part 3	Multiple start branch and prune filter
	Sparse point registration
Part 4	Unifying prior methods
	Improving existing methods

Bayes Rule

$$p(\boldsymbol{q}|\boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_{1:k}|\boldsymbol{q})\; p(\boldsymbol{q})$$

Likelihood Prior

$$\boldsymbol{q_k} = \underset{\boldsymbol{q}}{argmax}\; \boldsymbol{p}(\boldsymbol{q}|\boldsymbol{z}_{\boldsymbol{1:k}})$$

Bayesian filter for pose estimation

$$p(\boldsymbol{q}|\boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_{1:k}|\boldsymbol{q})\ p(\boldsymbol{q})$$

$$\boxed{\frac{1}{N_1}\exp\left(-(\boldsymbol{Hq})^T\boldsymbol{Q}_k^{-1}(\boldsymbol{Hq})\right)}$$

?

Bayesian filter for pose estimation

$$p(\boldsymbol{q}|\boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_{1:k}|\boldsymbol{q})\ p(\boldsymbol{q})$$

- $\boldsymbol{q}$ and $-\boldsymbol{q}$ are equivalent
- $||\boldsymbol{q}|| = 1$

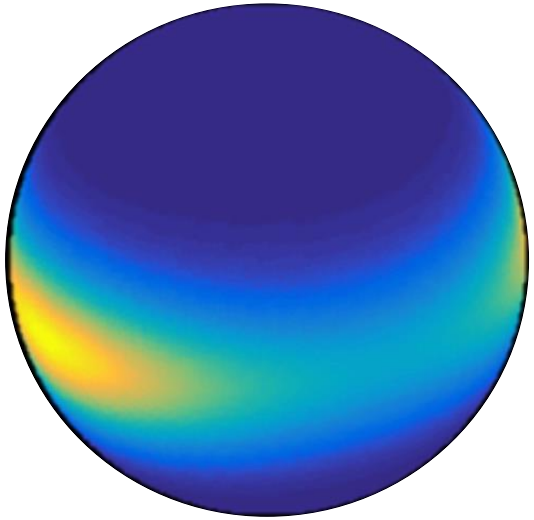
Gaussian Distribution

Bayesian filter for pose estimation

$$p(\boldsymbol{q}|\boldsymbol{z}_{1:k}) \propto \; p(\,\boldsymbol{z}_{1:k}|\boldsymbol{q})\; p(\boldsymbol{q})$$

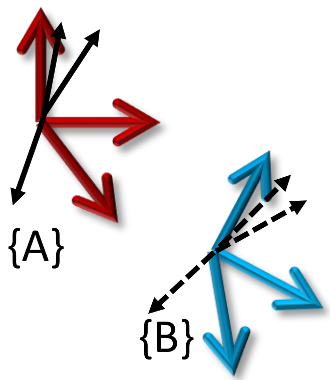
- $\boldsymbol{q}$ and $-\boldsymbol{q}$ are equivalent
- $||\boldsymbol{q}|| = 1$

Bingham Distribution*



*Christopher Bingham. "An antipodally symmetric distribution on the sphere", The Annals of Statistics, pages 1201-1225, 1974.

Bayesian filter for pose estimation



$$p(\boldsymbol{q}|\boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_{1:k}|\boldsymbol{q})\ p(\boldsymbol{q})$$

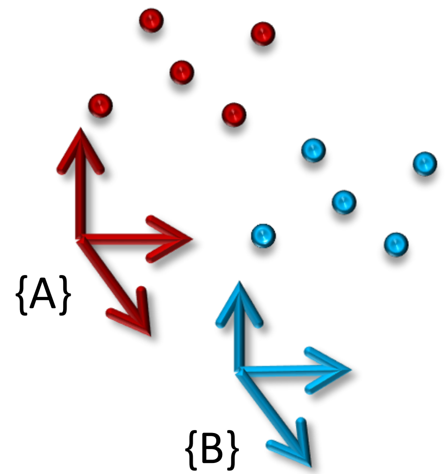
Bingham distribution

Bingham distribution

Bingham distribution

Maximum Likelihood Estimate available in analytical form

After orienting, estimate translation



$$p(\boldsymbol{t}|\boldsymbol{q}_k, \boldsymbol{z}_k) \propto p(\boldsymbol{q}_k, \boldsymbol{z}_k|\boldsymbol{t})\ p(\boldsymbol{t})$$

$$\boldsymbol{t_k} = \underset{\boldsymbol{t}}{argmax}\ \boldsymbol{p}(\boldsymbol{t}|\boldsymbol{q}_k, \boldsymbol{z}_k)$$ Linear Kalman filter

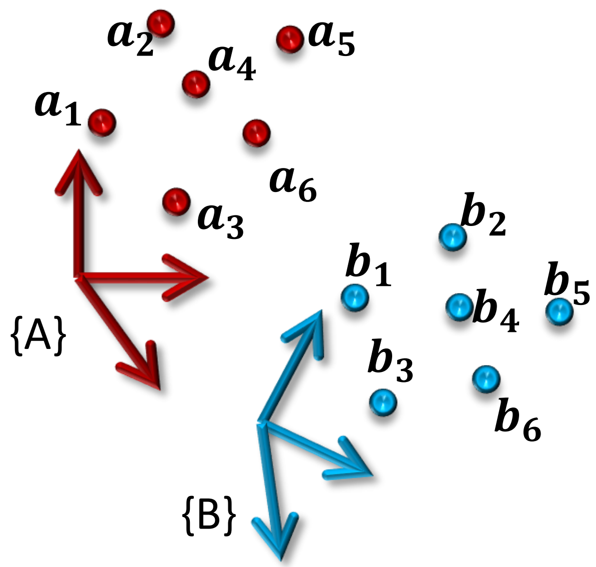
Gaussian distribution

Gaussian distribution

R. A. Srivatsan et al. “Probabilistic Pose Estimation Using a Bingham Distribution-Based Linear Filter”, IJRR, 2018
R. A. Srivatsan et al. "Bingham distribution-based linear filter for online pose estimation." In *RSS*, 2017.

Results for known correspondence

- 1000 experiments
- 100 points with noise
- Large initial pose error

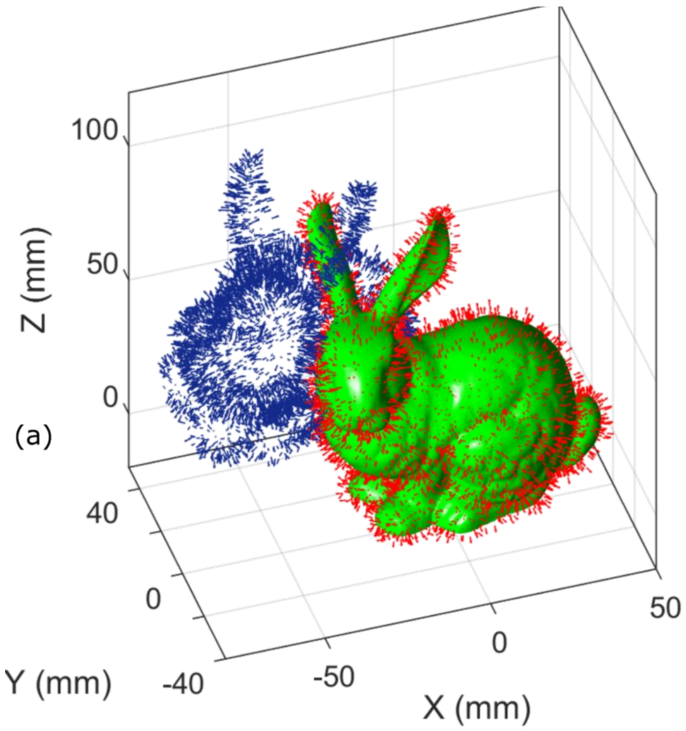


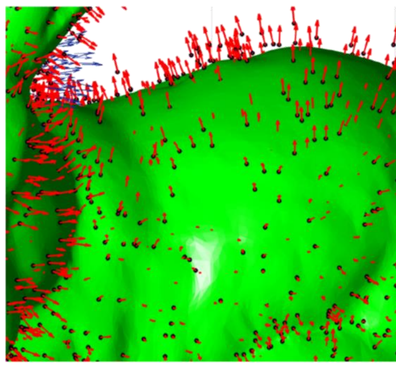
	RMS error (in mm)	Success rate (%)
Bingham filter	**2.06**	**100**
Linear Kalman filter	2.72	99.70
Moghari-UKF	18.84	99.30
Pennec-EKF	83.41	83.90
Incremental Horn's method	4.96	97.30

Results for unknown correspondence

- 5000 sensed points
- 86,632 model points
- Noisy Point and normal measurements

	RMS error (in mm)	Time (s)
Bingham filter	**0.31**	**0.27**
ICP	2.04	0.55
Pulli-ICP	0.90	5.38
Go-ICP	0.72	0.81 (C++)
IMLOP	0.34	63.32
Inc- IMLOP	0.84	2.45





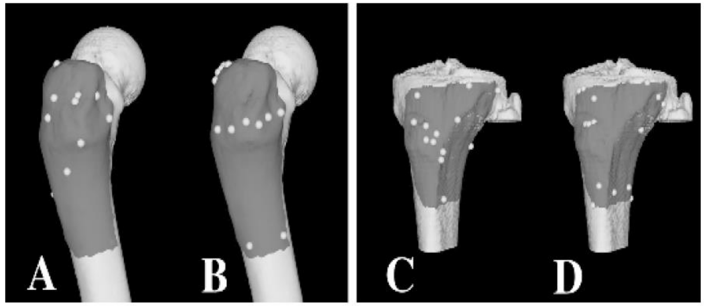
Motivation and Key Contributions

Part 1	State update using ‘hash’ of measurements
	Finding uncertainty in ‘hash’ space
Part 2	Novel Bayes filter for pose update
Probabilistic nonconvex optimization	Multiple start branch and prune filter
	Sparse point registration
Part 4	Unifying prior methods
	Improving existing methods

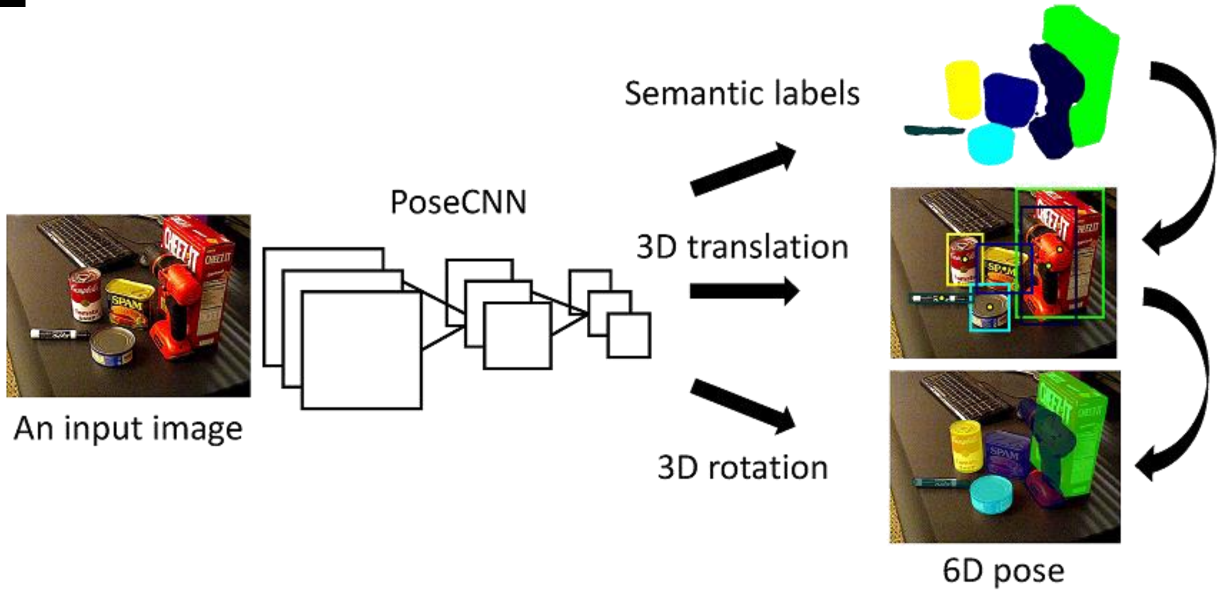
How to deal with unknown correspondence?

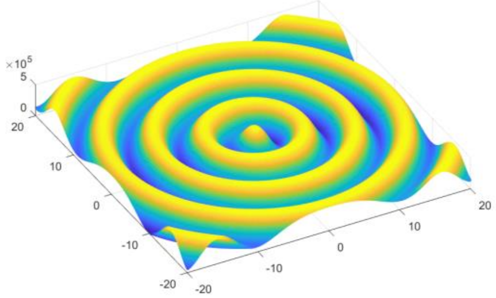
Point cloud form Microsoft kinect

- Find the correct correspondence
- Start from a 'good' initial pose
- Use a global optimizer



Ellis et al. '01





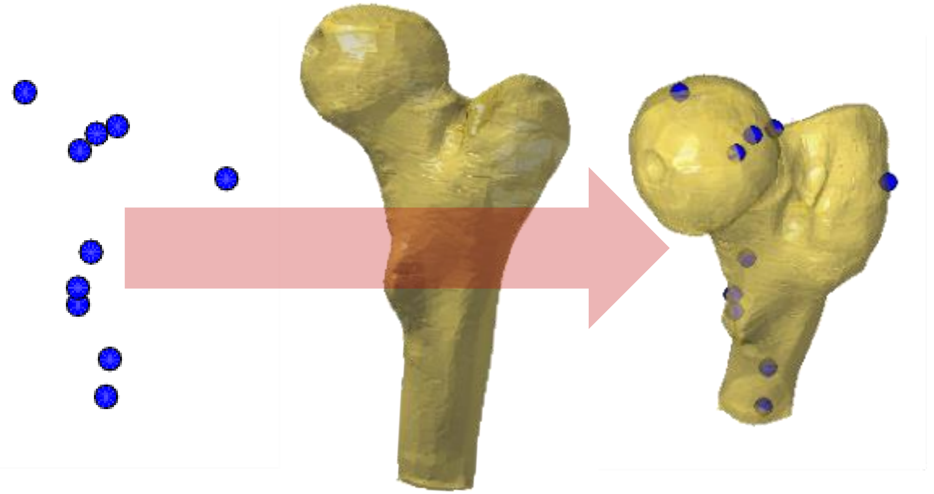
Pose CNN, Xiang et al. '18

Using uncertainty to search for optimal pose

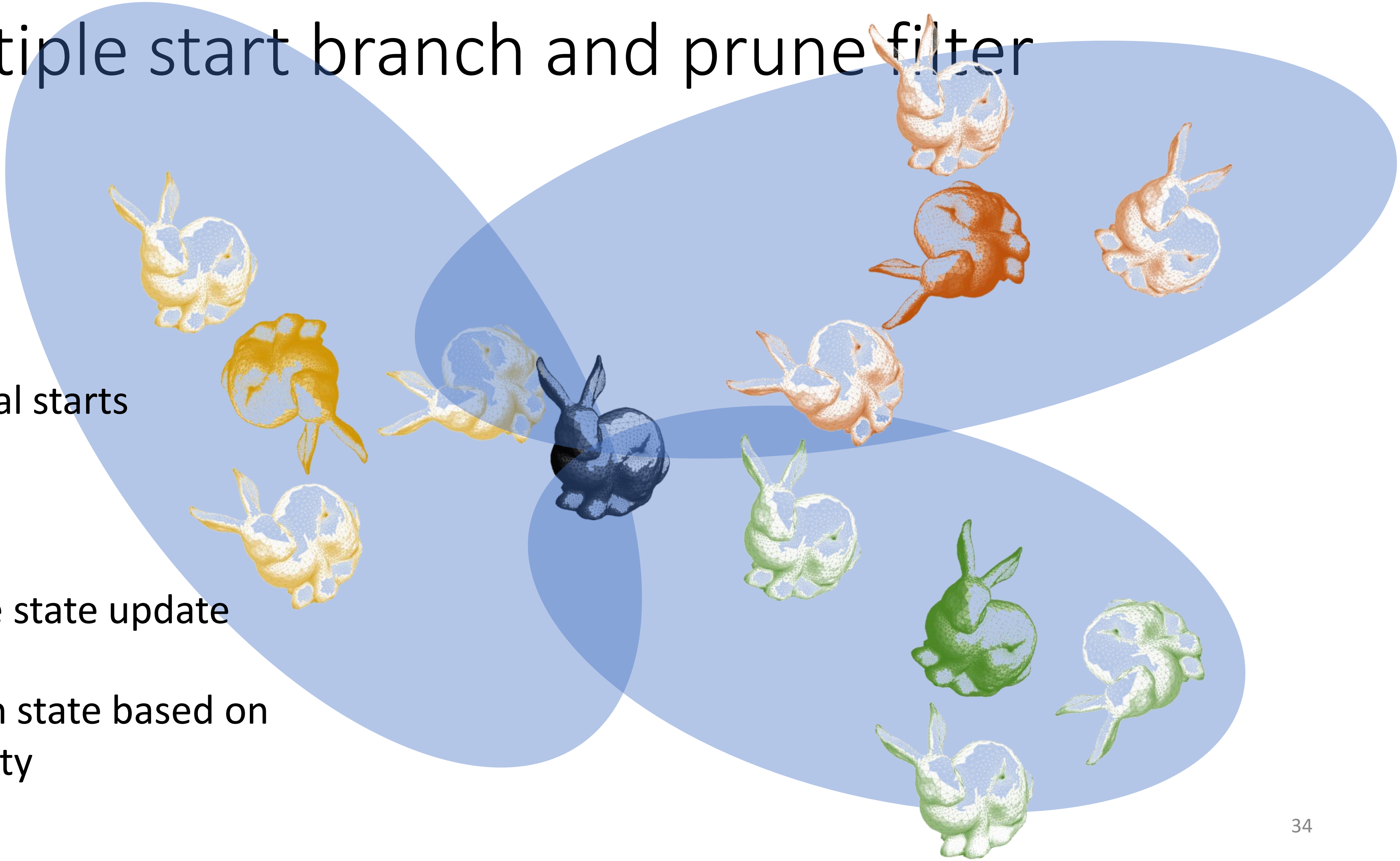
Two probabilistic approaches

- Online estimation
  - Multiple start branch and prune filter

- Batch estimation
  - Sparse point registration



Multiple start branch and prune filter

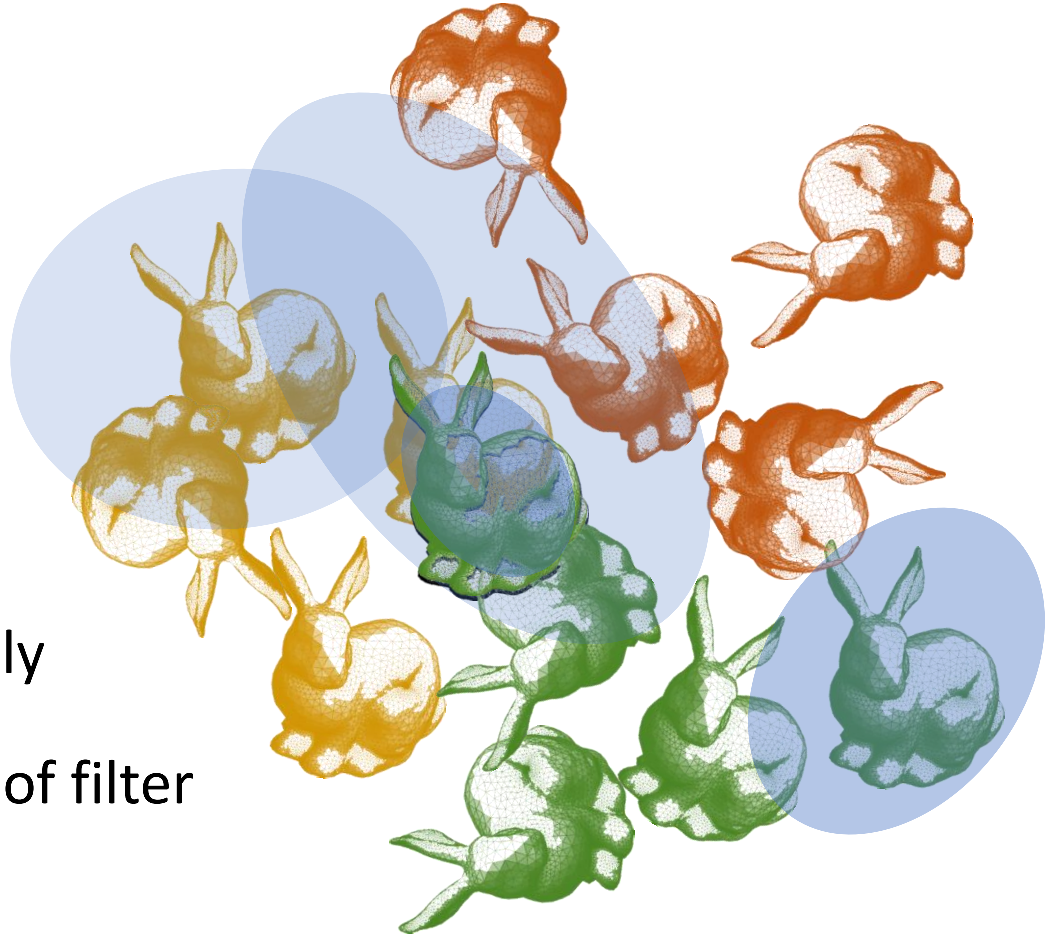


Multiple initial starts

Perform one state update

Perturb each state based on its uncertainty

Multiple start branch and prune filter



Repeat this process iteratively

Prune based on 'innovation' of filter

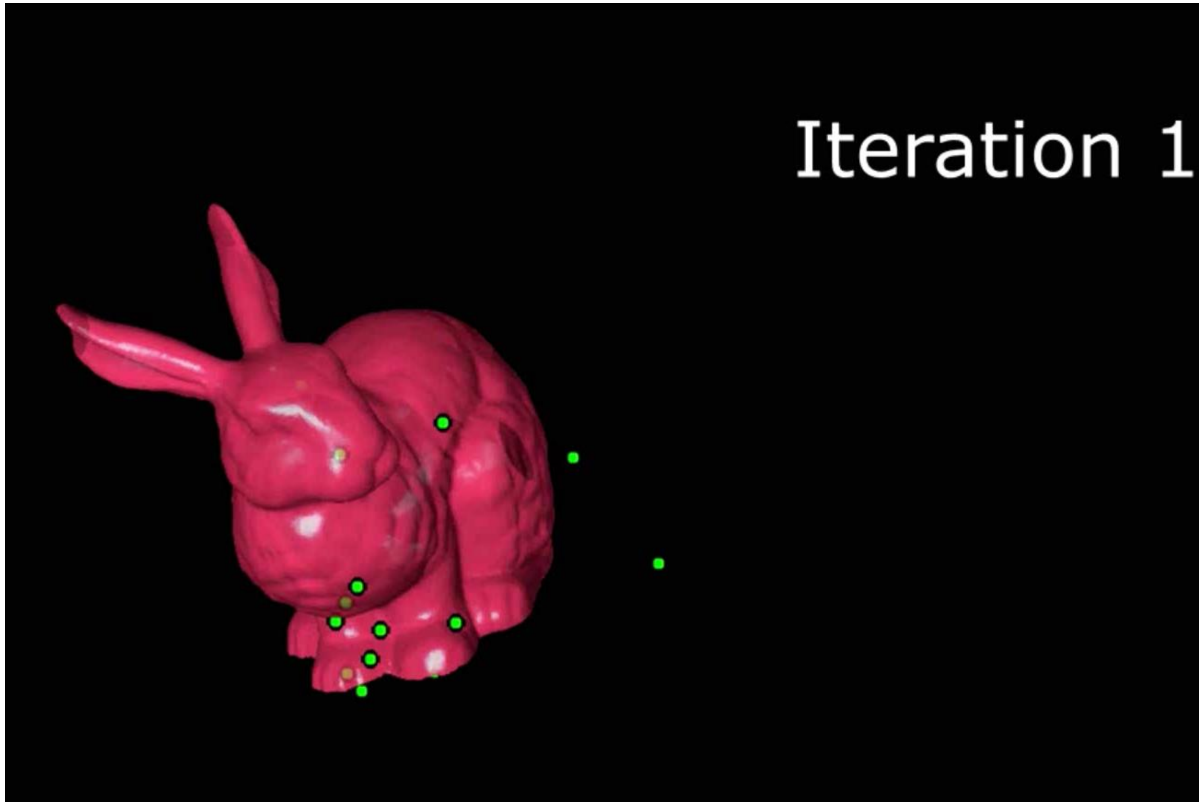
Results for online registration

Initial error in
Translation: $[30, -40, 15]$mm
Rotation: $[-55, 80, -20]^{\circ}$

	RMS (mm)	Time (s)
Our approach (MSBP*)	0.48	28.00
ICP	35.06	**0.58**
Simulated Annealing + ICP (Luck '00)	2.36	353.67
Genetic Algorithm + ICP (Silva '05)	**0.08**	1051.00
Go- ICP (Yang '13)	0.80	0.72 (C++)

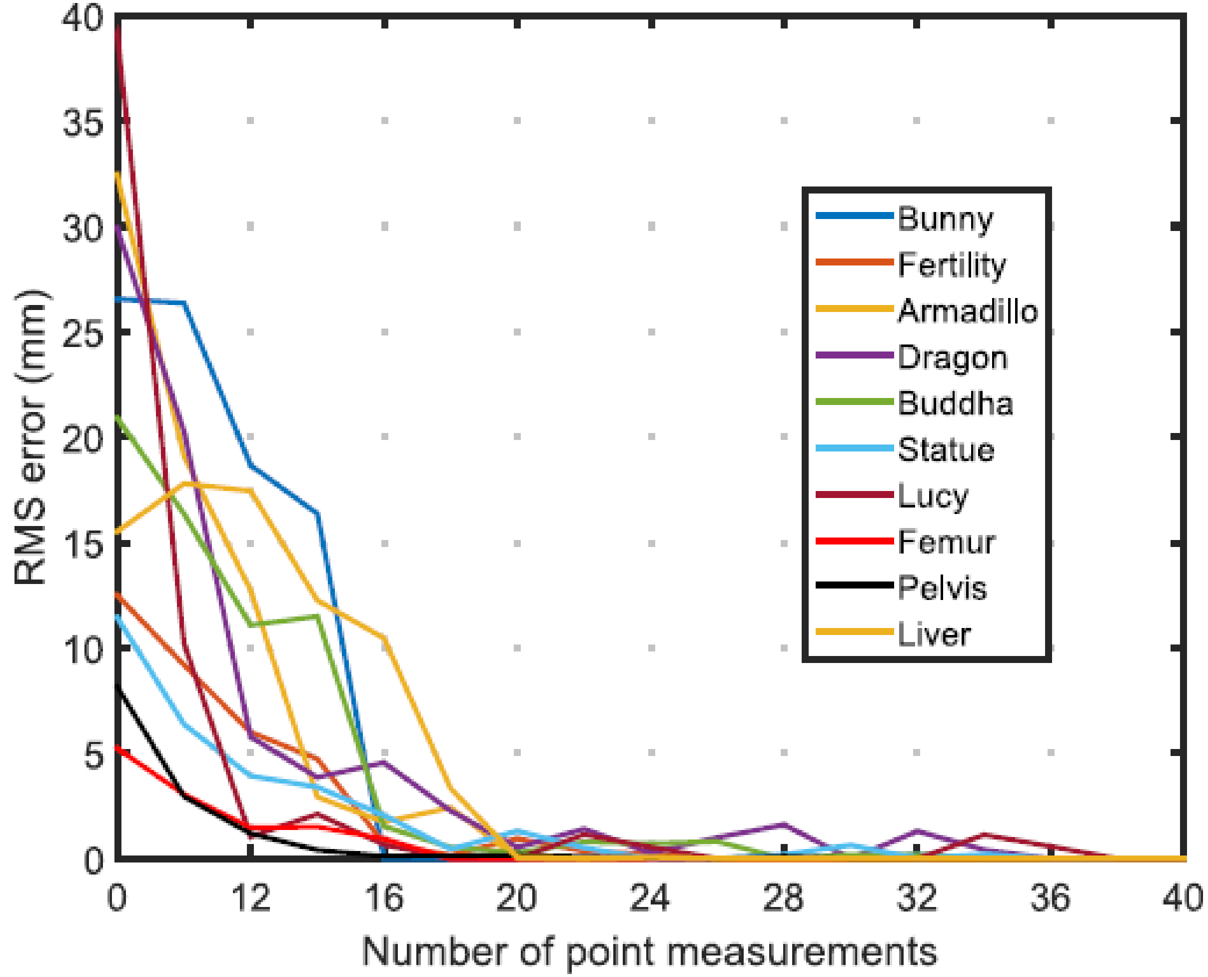
* R. A. Srivatsan and H. Choset, "Multiple start branch and prune filtering algorithm for nonconvex optimization", In WAFR, 2016

Sparse point registration *



R. A. Srivatsan et al. “Sparse Point Registration." in ISRR 2017, special invitation to IJRR 2018

Results for batch registration

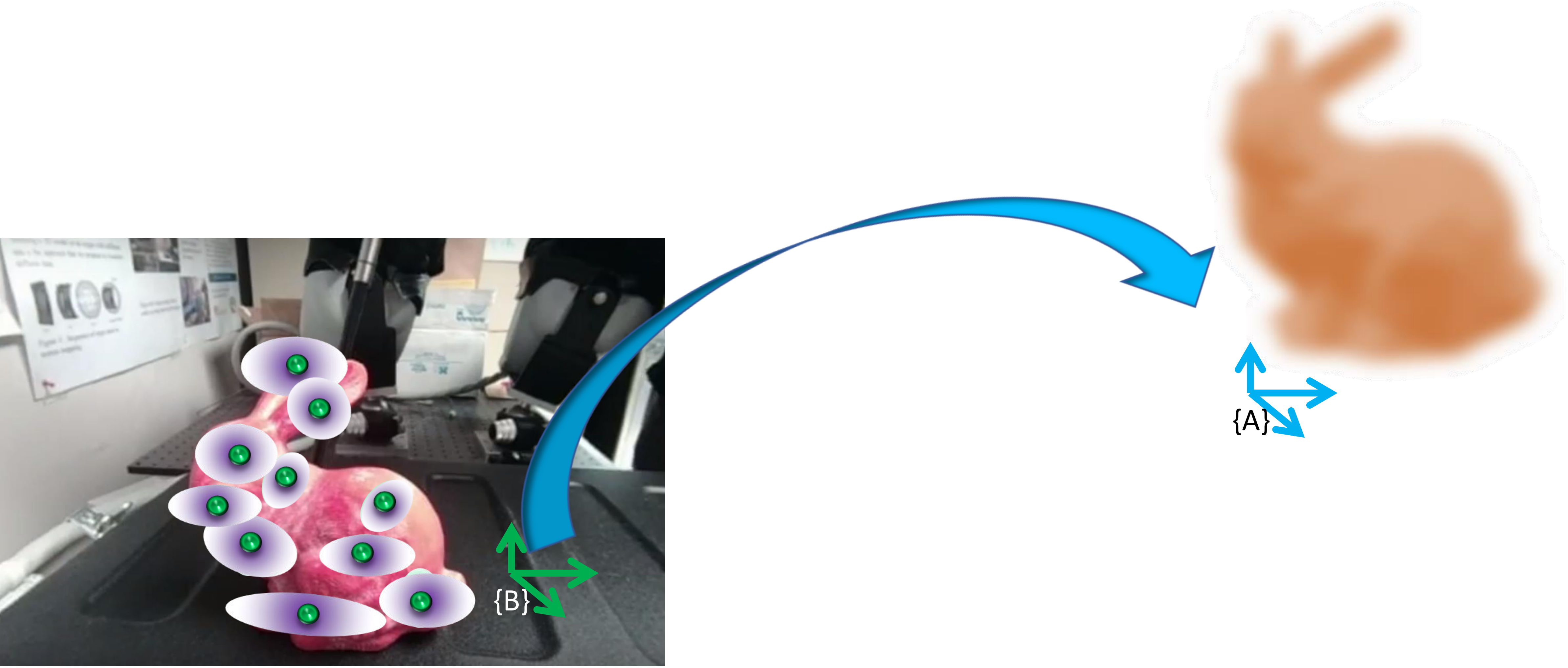


Results using standard datasets

Motivation and Key Contributions

Part 1	State update using ‘hash’ of measurements
	Finding uncertainty in ‘hash’ space
Part 2	Novel Bayes filter for pose update
Part 3	Multiple start branch and prune filter
	Sparse point registration
Unified framework for probabilistic registration	Unifying prior methods
	Improving existing methods

A generalized view of registration



A generalized view of registration

	Points	Sensor noise	Pose prior
ICP, Go-ICP	✓	✗	✗
IMLP, G-ICP	✓	✓	✗
Moghari-UKF, Pennec-EKF, Bingham filter	✓	✓	✓

Revisiting Bayes rule

$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\operatorname{argmax}}\, P(\boldsymbol{x}|\boldsymbol{S},\boldsymbol{M})$$

$\boldsymbol{x}$ $\rightarrow$ pose parameters

$\boldsymbol{S}$ $\rightarrow$ set of sensed points

$\boldsymbol{M}$ $\rightarrow$ set of model points

Objective function for globally optimal pose

$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\operatorname{argmax}} \left\{ \log\big(P(\boldsymbol{x})\big) + \sum_i \log\left(\sum_j \frac{1}{n_{\boldsymbol{M}}} P\big(\boldsymbol{s}_{\boldsymbol{i}} \big| \boldsymbol{m}_{\boldsymbol{j}}, \boldsymbol{x}\big)\right) \right\}$$

	RMS error (mm)
Our objective + PSO	0.04
Go-ICP	0.90

Go-ICP (Yang et al ‘13) and MIP (Izatt et al ‘17) are both special cases of our approach

Locally optimal variant for pose estimation

$$w_{ij} = \frac{P(\boldsymbol{s_i} | \boldsymbol{m_j}, \boldsymbol{x_k})}{\sum_k P(\boldsymbol{s_i} | \boldsymbol{m_k}, \boldsymbol{x_k})}$$

Iterate

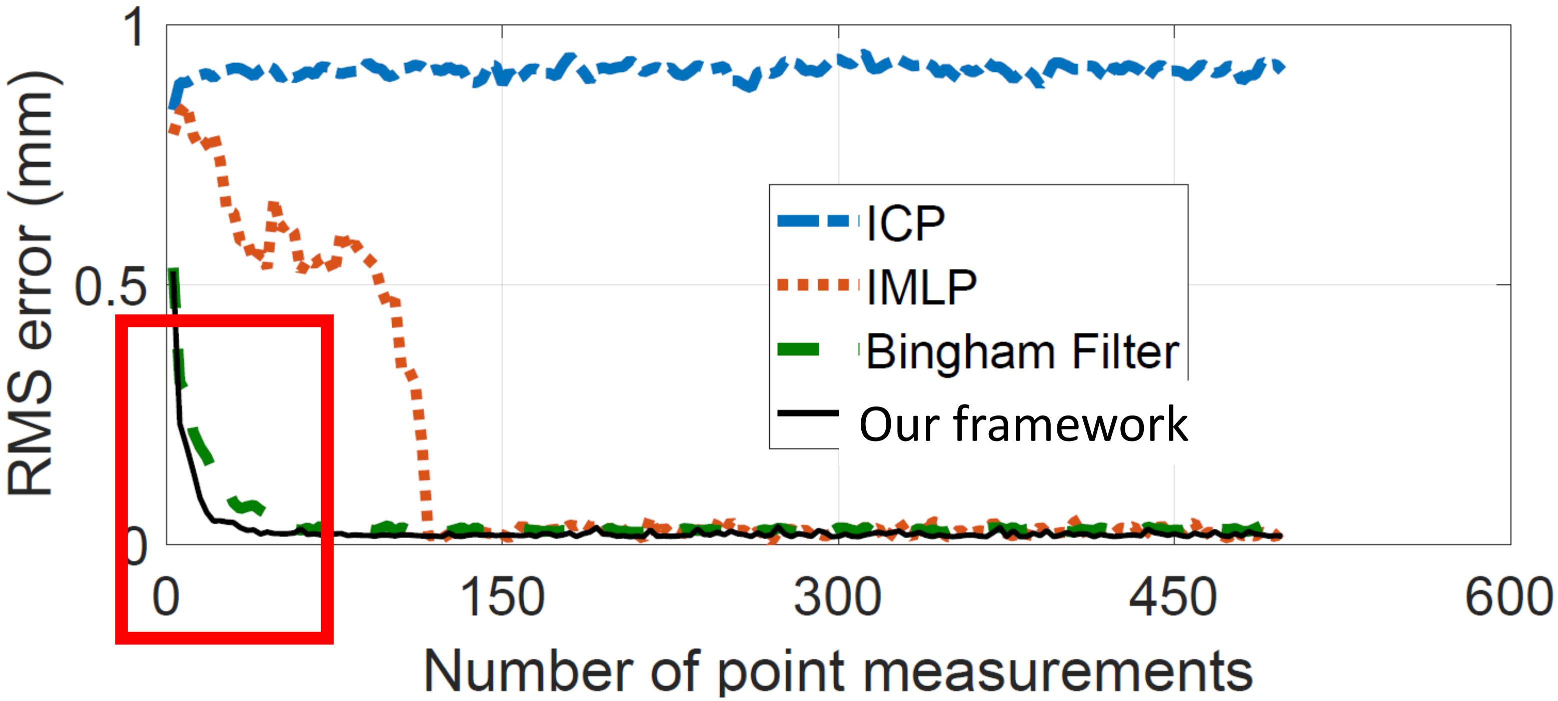
$$\boldsymbol{x_{k+1}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \left\{ -\log(P(\boldsymbol{x})) - \sum_{ij} w_{ij} \log\left(P(\boldsymbol{s_i} | \boldsymbol{m_j}, \boldsymbol{x})\right) \right\}$$

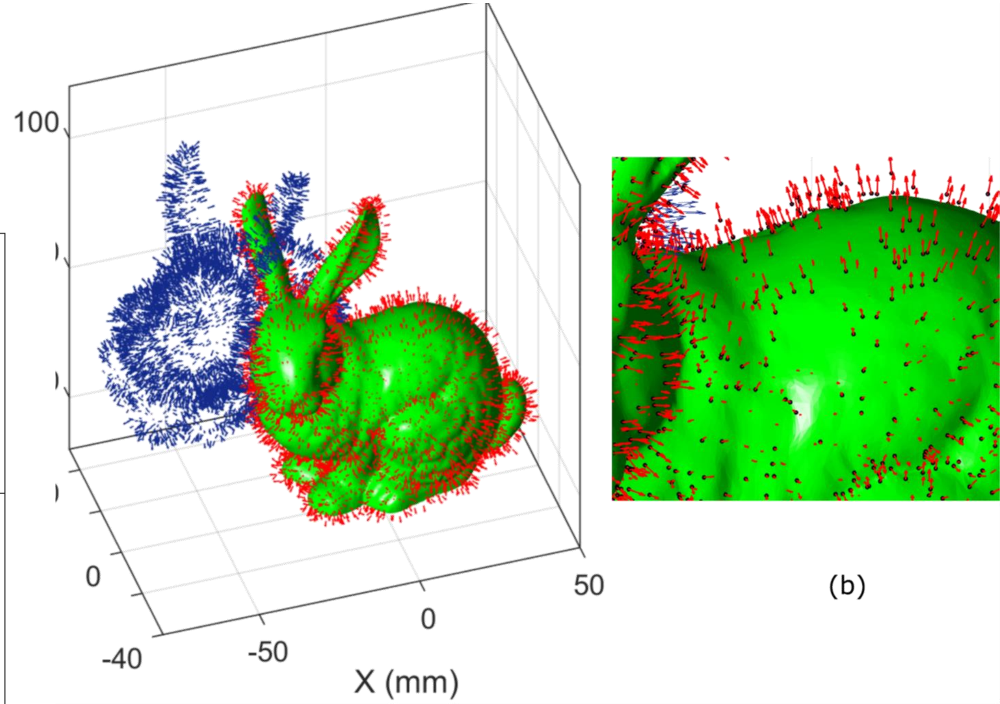
Special cases of our locally optimal framework

1. ICP, Besl and McKay ‘92
2. EM-ICP, Granger *et al.* ‘01
3. GTLS-ICP, Estepar *et al.* ‘04
4. G-ICP, Segal *et al.* ‘09
5. A-ICP, Hein *et al.* ‘12
6. IMLP, Billings *et al.* ‘15
7. Moghari-UKF, Moghari *et al.* ‘07
8. Pennec-EKF, Pennec *et al.* ‘01
9. Bingham filter, Srivatsan *et al.* ‘17

Improvements to existing methods

- Bingham distribution-based Bayes filter
  - Soft probabilistic correspondence





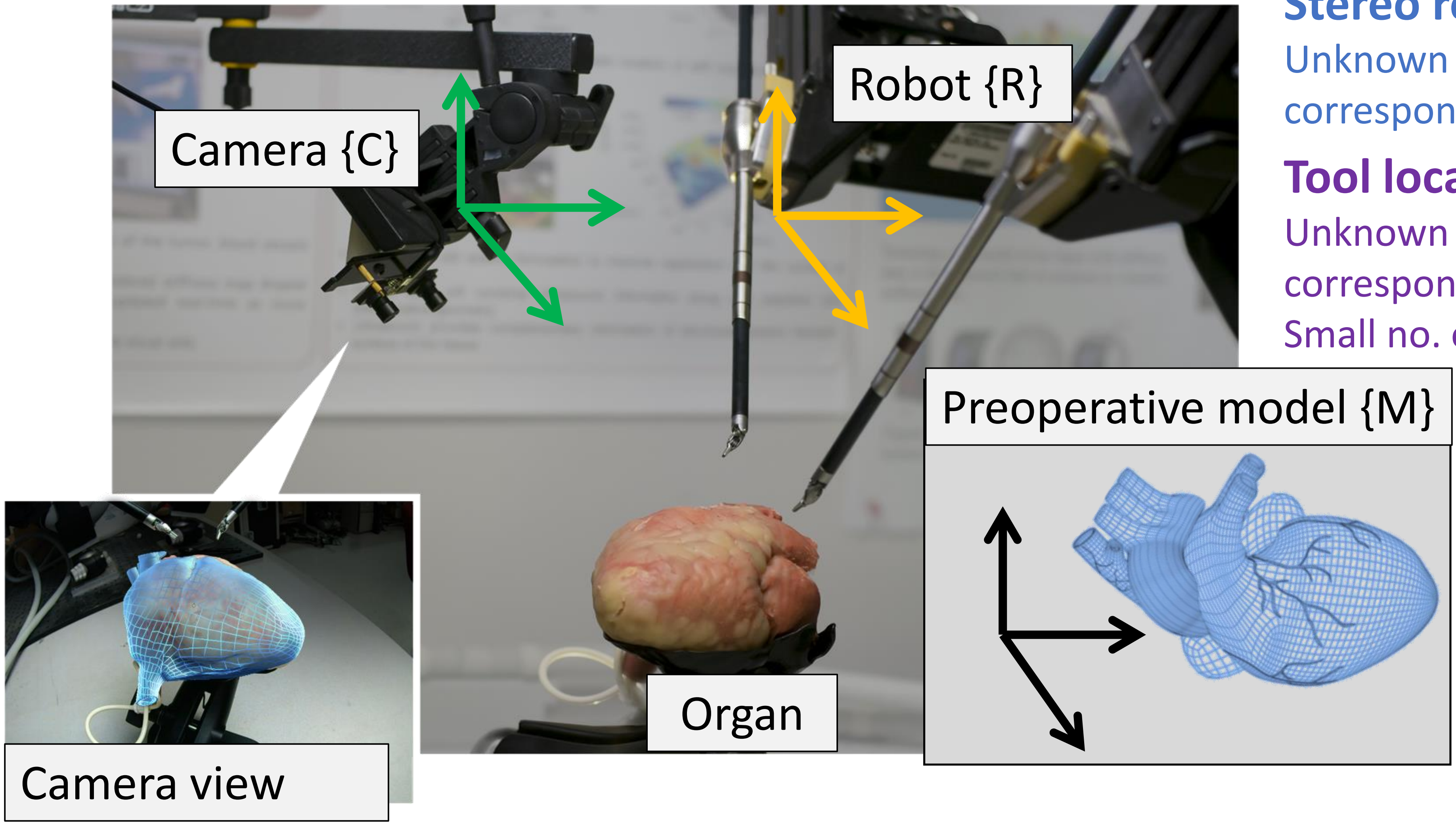
Motivation and Key Contributions

Part 1	State update using 'hash' of measurements
	Finding uncertainty in 'hash' space
Part 2	Novel Bayes filter for pose update
Part 3	Multiple start branch and prune filter
	Sparse point registration
Part 4	Unifying prior methods
	Improving existing methods

Outline

- Prior work
- Motivation
- Key contributions
- Revisiting example
- Future work

Revisiting example: Image guided robotic surgery



Hand eye calibration
Known correspondence

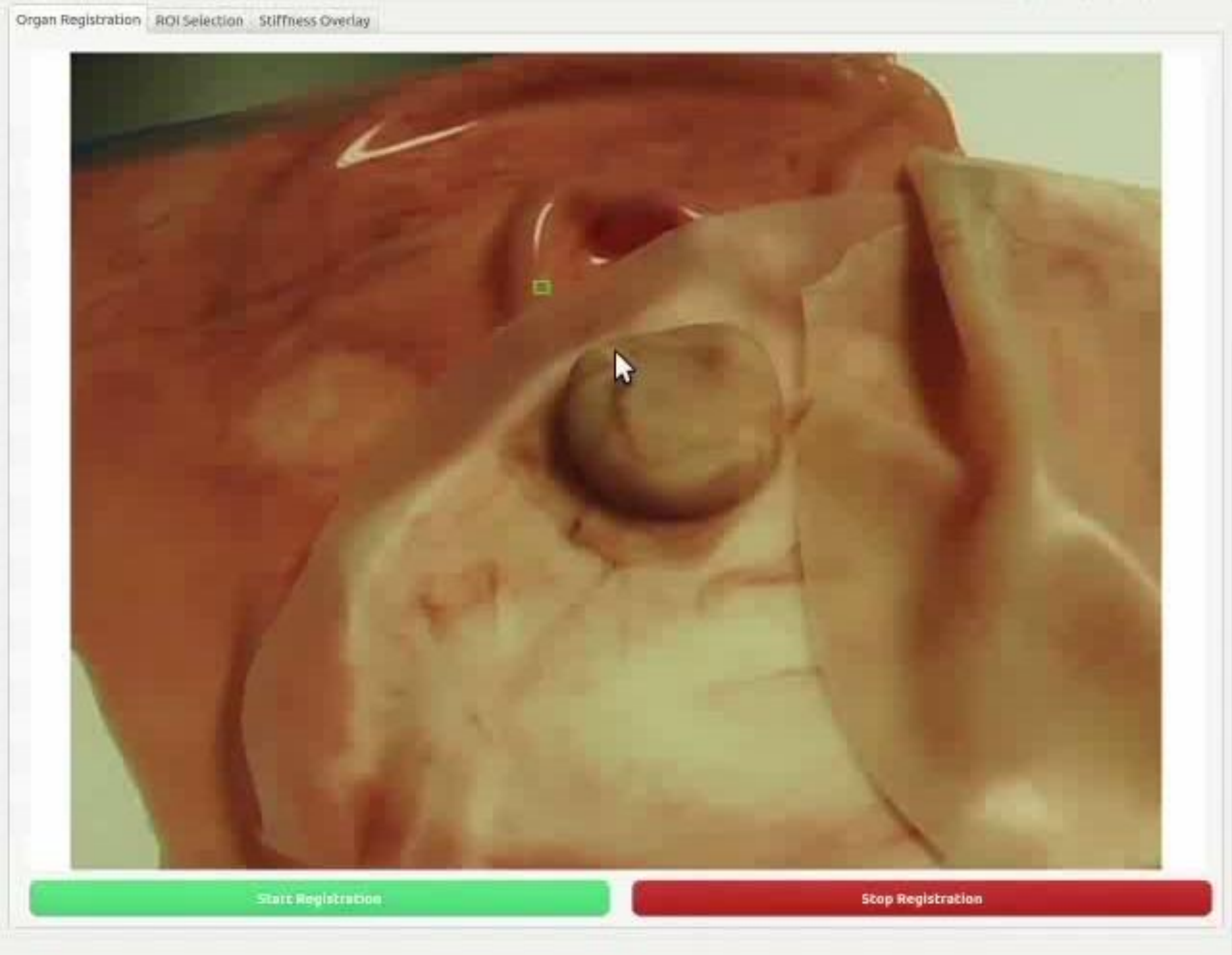
Bingham filter

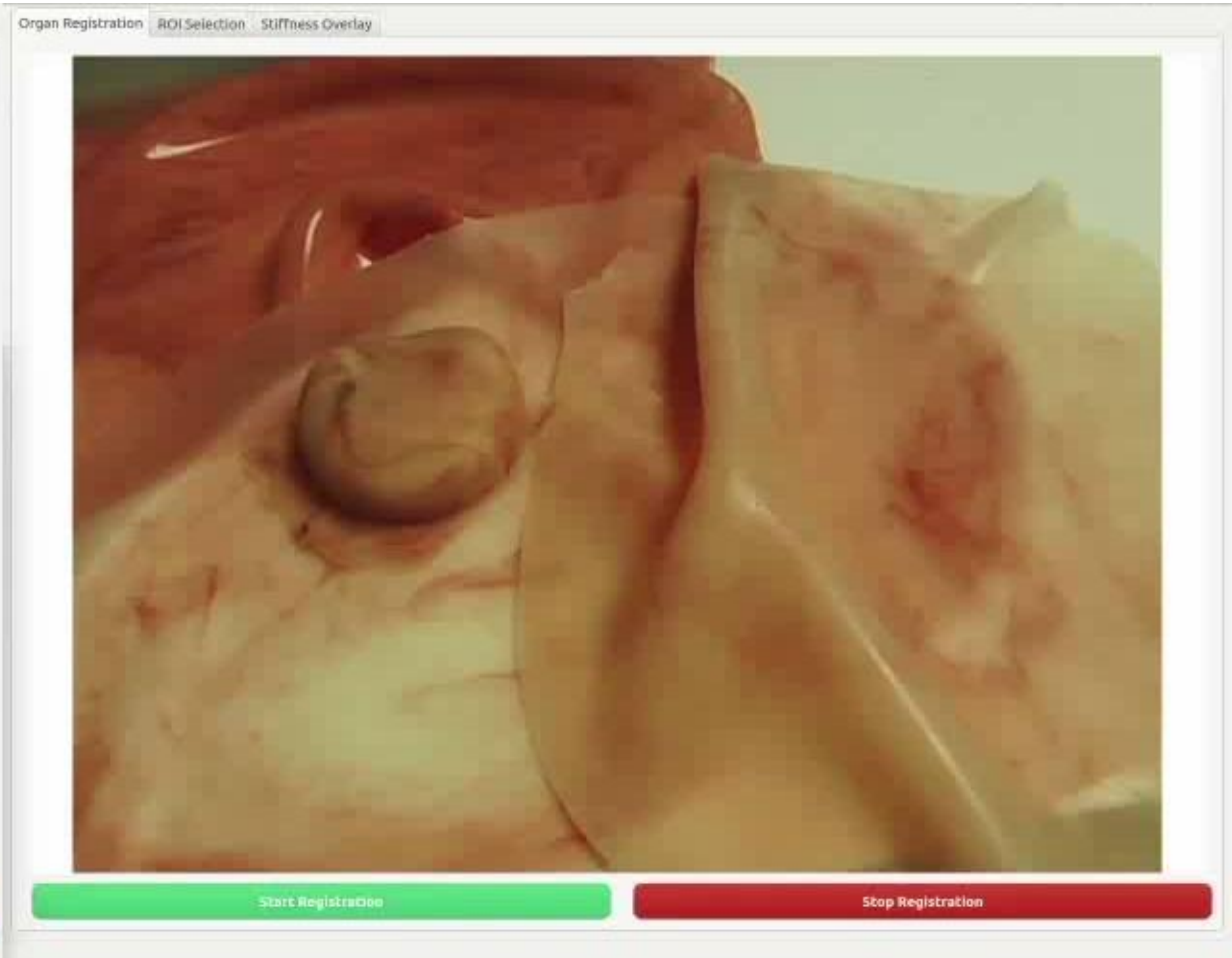
Stereo registration
Unknown correspondence

Multi start branch and prune filter

Tool localization
Unknown correspondence
Small no. of measurements

Sparse point registration





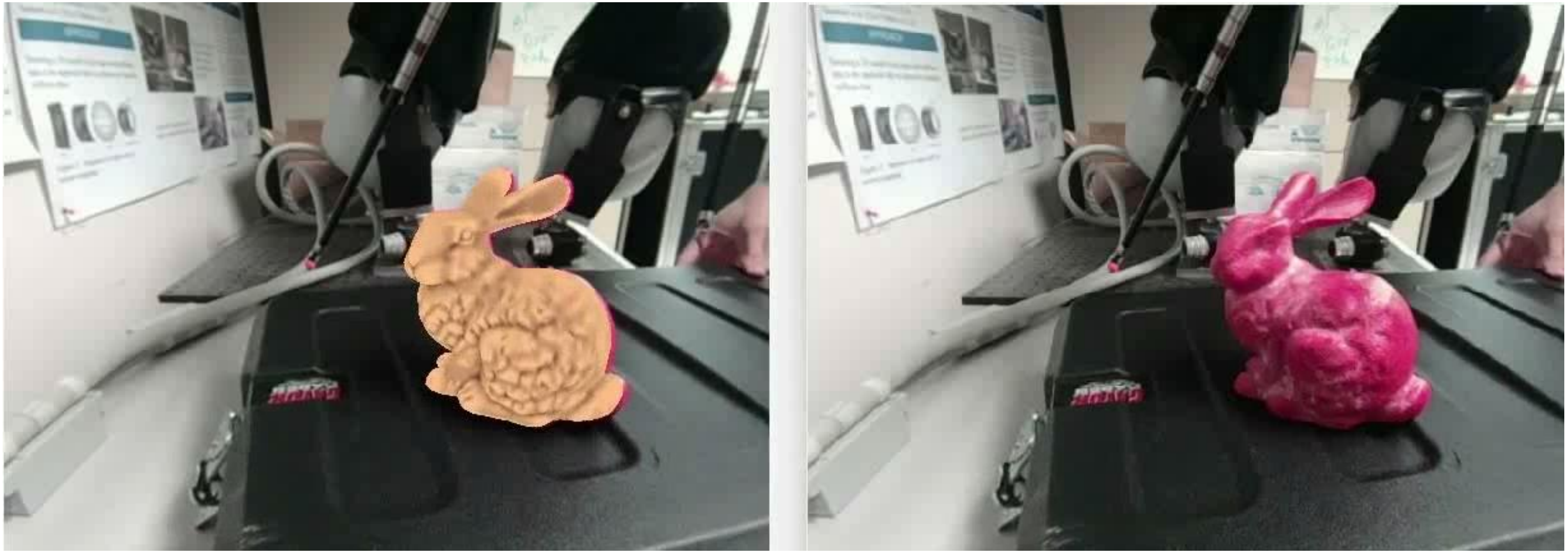
N. Zevallos et al. “A Real-time Augmented Reality Surgical System for Overlaying Stiffness Information”, conditionally accepted for RSS 2018

N. Zevallos et al. “A surgical system for automatic registration, stiffness mapping and dynamic image overlay”, in ISMR, 2018 (Nominated for best paper award)

Outline

- Prior work
- Motivation
- Key contributions
- Revisiting example
- Future work

Future work: Dynamic pose estimation



Future work: Estimate multiple constrained poses

Human pose estimation for movies and VR
Bogo et al. ‘16

Robot manipulation
Pauwels et al. ‘14

Reassembling fractured objects