Annoucements

- Next Thursday: Chris software overview
- Reading:

  Henrik Wann Jensen, "Global Illumination using Photon Maps," In "Rendering Techniques '96". Eds. X. Pueyo and P. Schrder. Springer-Verlag, pp. 21-30, 1996

  http://graphics.ucsd.edu/~henrik/papers/ewr7/ [web page]

  http://graphics.ucsd.edu/~henrik/papers/ewr7/egwr96.pdf [paper]
- Photon Mapping
  - SIGGRAPH 2002 Course Notes
  - http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15864-s04/www/slides/photonMappingCourse02.pdf

Local Illumination, Reflection, and BRDFs

Adapted from…

Szymon Rusinkiewicz
Princeton University
C0S 526, Fall 2002

Overview

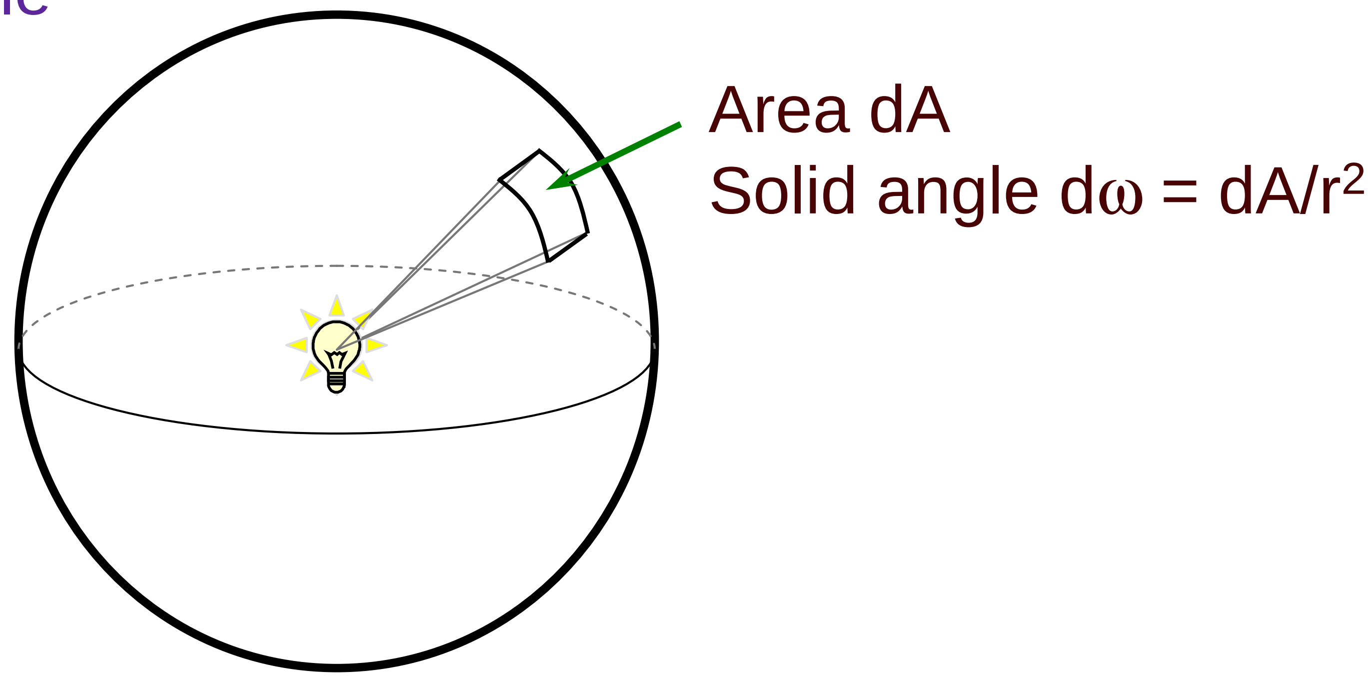
- Radiometry and Photometry
- Definition of BRDF
- BRDF properties and common BRDFs
- Rendering equation

Radiometric Units

- Light is a form of energy – measured in Joules (J)
- Power: energy per unit time
  - Measured in Joules/sec = Watts (W)
  - Also called *Radiant Flux* (Φ)

Point Light Source

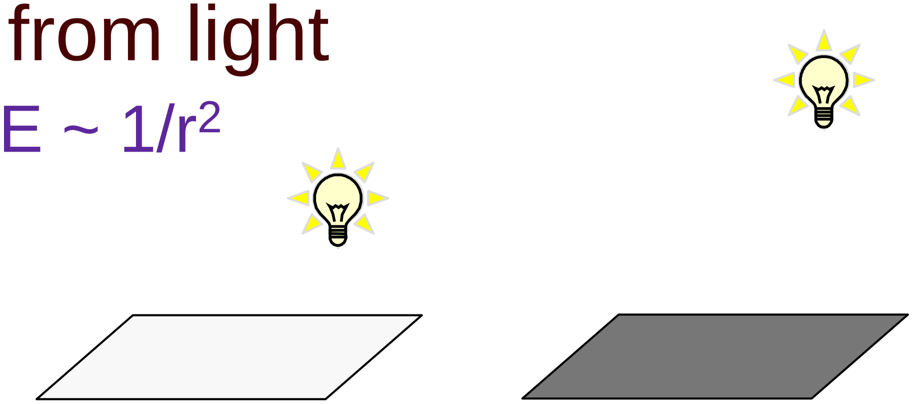
- Total radiant flux in Watts
- How to define angular dependence?
  - Solid angle



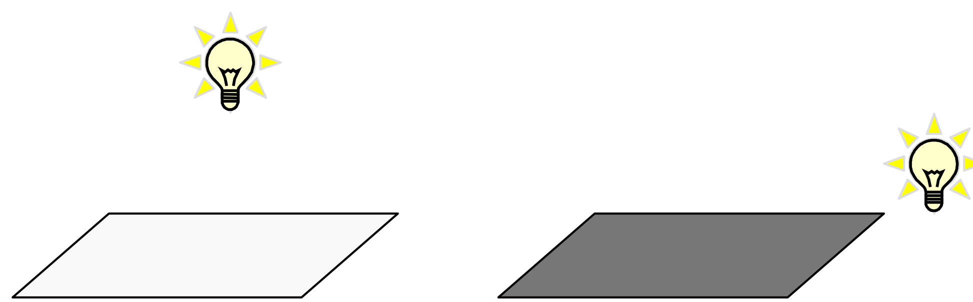
- Power per unit solid angle
  - Measured in Watts per steradian (W/sr)

Light Falling on a Surface

- Power per unit area – *Irradiance* (E)
  - Measured in W/m^2
- Move surface away from light
  - Inverse square law: $E \sim 1/r^2$

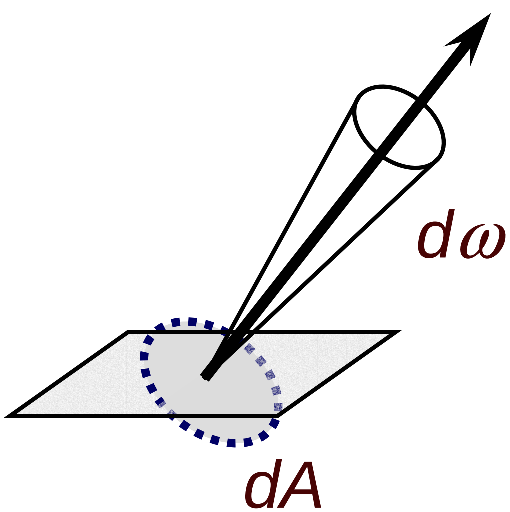


- Tilt surface away from light
  - Cosine law: $E \sim \mathbf{n} \bullet \mathbf{l}$



Light Emitted from a Surface

- Power per unit area per unit solid angle – *Radiance* (L)
  - Measured in $W/m^2/sr$
  - *Projected area* – perpendicular to given direction



$$L = \frac{d\Phi}{dA\,d\omega}$$

Total Light Emitted from a Surface

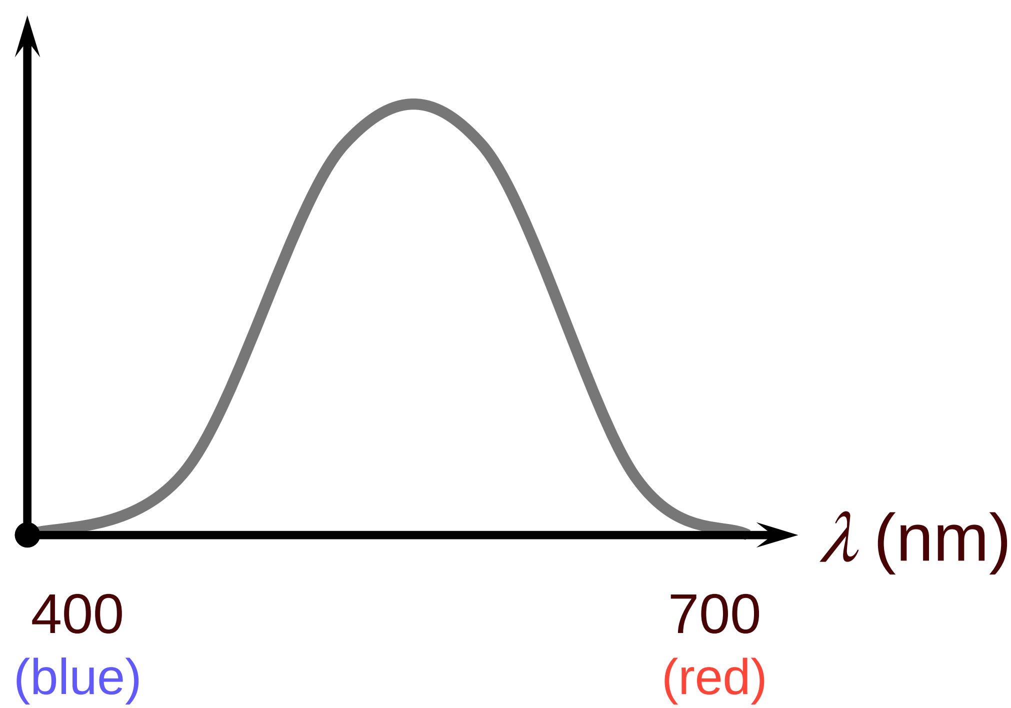
- Radiance integrated over all directions

$$B = \int_{\Omega} L_o(\theta, \phi) \cos\theta \, d\omega$$

- Called *Radiosity* (B)
  - Measured in W/m^2

Radiometry vs. Photometry

- These are all physical (radiometric) units
- Don't take perception into account
- Eye sensitive to different colors



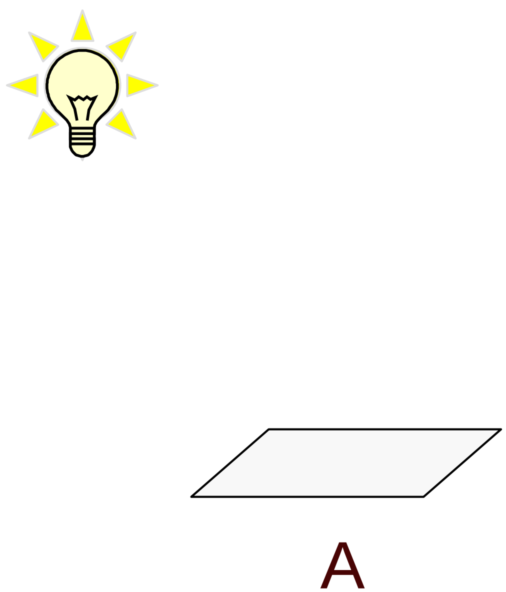
Photometric Units

- Take human perception into account
- Original unit: candle
  - Luminous intensity equal to a “standard candle”
- Today: one of the basic SI units
  - One candela (cd) is the luminous intensity of a source producing 1/683 W at 555 nm (yellow-green).

Radiometric and Photometric Units

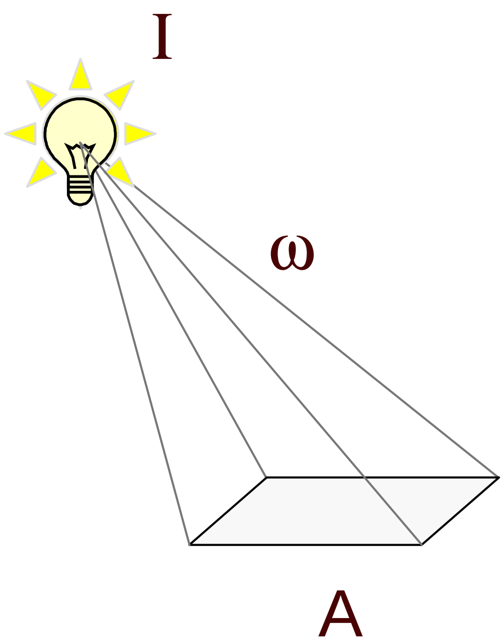
Radiant energy Joule (J)	Luminous energy Talbot
Radiant flux or power (Φ) Watt (W) = J / sec	Luminous power Lumen (lm) = talbots / sec
Radiant intensity (I) W / sr	Luminous intensity Candela (cd)
Irradiance (E) W / m^2	Illuminance Lux = lm / m^2
Radiance (L) $W / m^2 / sr$	Luminance Nit = $lm / m^2 / sr$
Radiosity (B) W / m^2	Luminosity Lux = lm / m^2

Direct Illumination (i.e., Irradiance)



$$E = \frac{\Phi}{A}$$

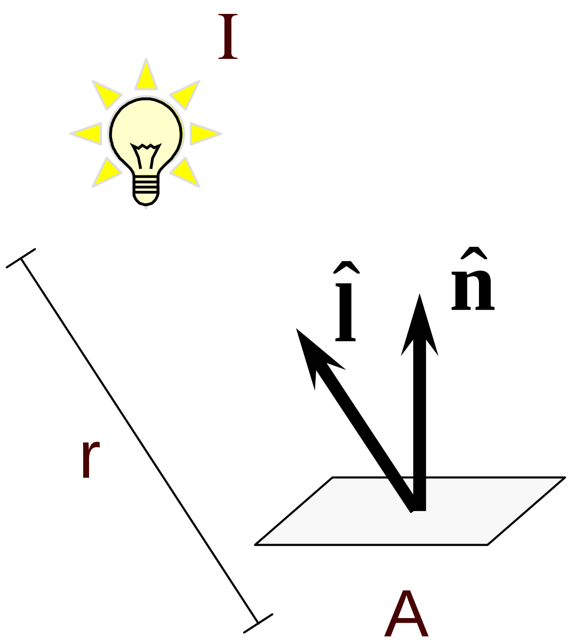
Direct Illumination



$$E = \frac{\Phi}{A}$$

$$\Phi = I\omega$$

Direct Illumination



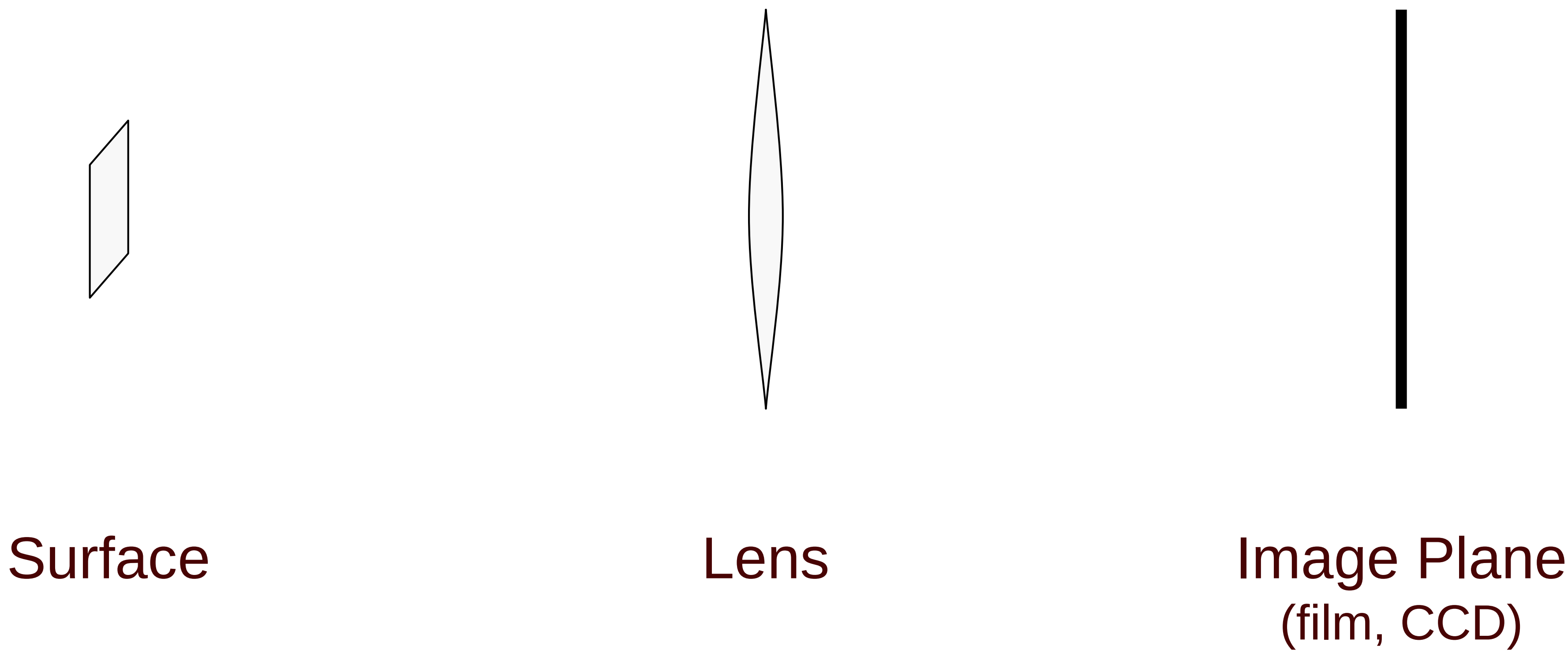
$$E = \frac{\Phi}{A}$$

$$\Phi = I\omega$$

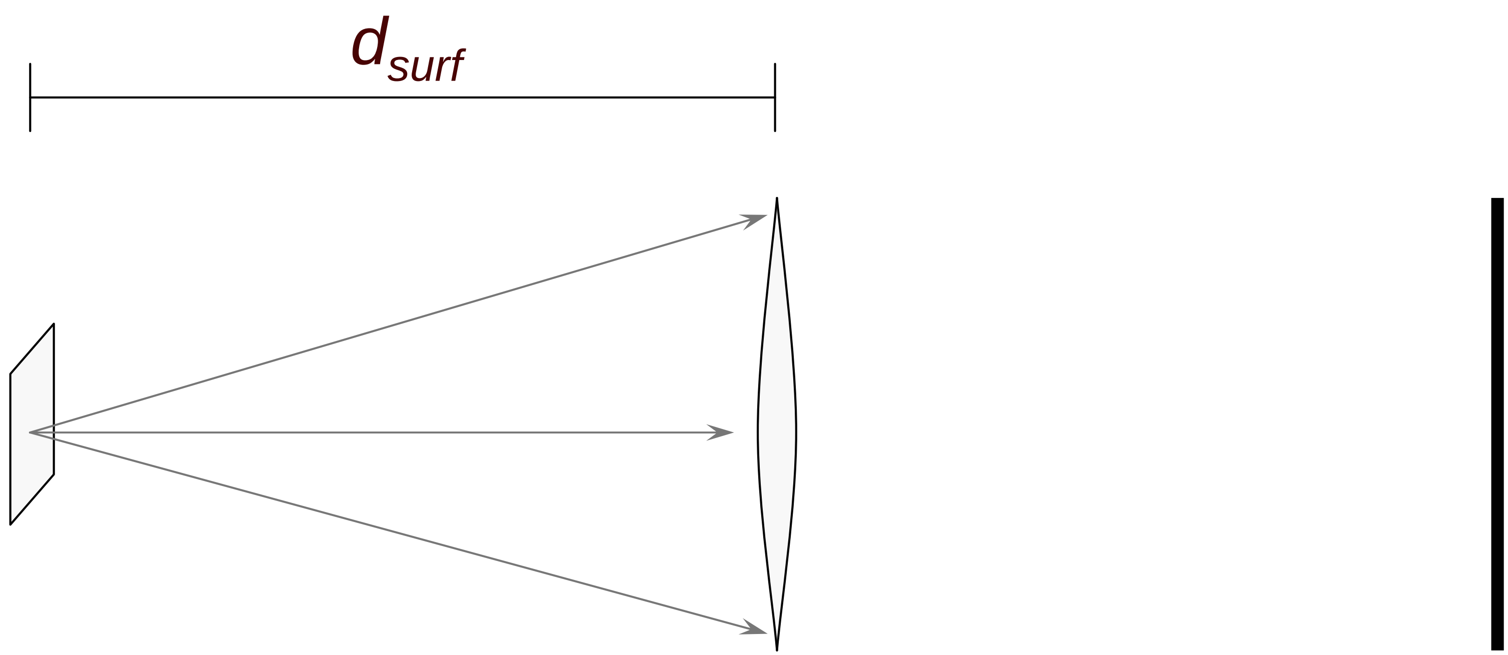
$$\omega = \frac{A(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

$$\Rightarrow E = \frac{I\,(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{r^2}$$

Imaging



Imaging



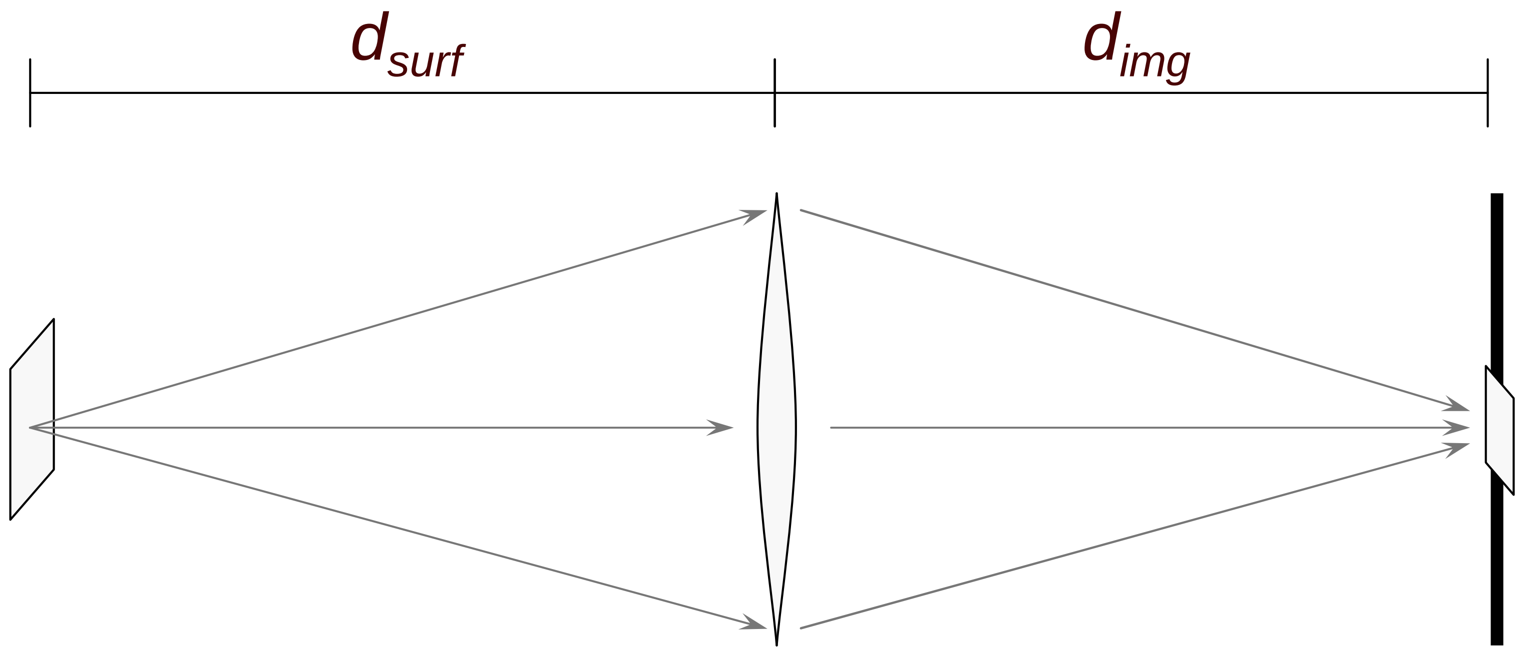
Area A_{surf}
Radiance L

$$\Rightarrow I = L\,A_{surf}$$

Area $A_{aperture}$

$$\Rightarrow \Phi = L\,A_{surf}\,\frac{A_{aperture}}{d_{surf}^{2}}$$

Imaging



Area A_{surf}
Radiance L

$$\Rightarrow I = L\,A_{surf}$$

Area $A_{aperture}$

$$\Rightarrow \Phi = L\,A_{surf}\,\frac{A_{aperture}}{d_{surf}^{2}}$$

Area A_{img}

$$\Rightarrow E = \frac{\Phi}{A_{img}}$$

Imaging

$$I = L\, A_{surf}$$

$$\Phi = L\, A_{surf} \frac{A_{aperture}}{d_{surf}^2}$$

$$E = \frac{\Phi}{A_{img}}$$

$$E = L \frac{A_{aperture}\, A_{surf}}{d_{surf}^2\, A_{img}}$$

$$\frac{A_{surf}}{A_{img}} = \left(\frac{d_{surf}}{d_{img}} \right)^2$$

$$E = L \frac{A_{aperture}}{d_{img}^2}$$

← Depends only on camera

- Punch line: cameras "see" radiance

Surface Reflectance – BRDF

- Bidirectional Reflectance Distribution Function

$$f_r(\omega_i \rightarrow \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$$

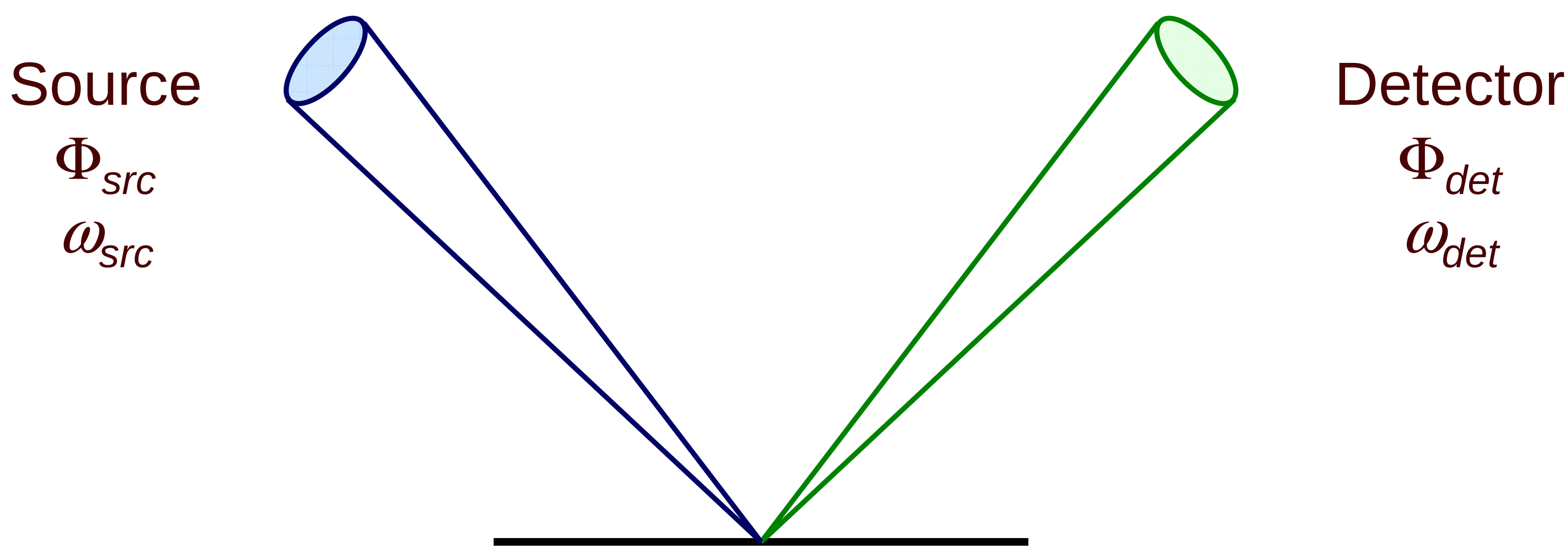
- 4-dimensional function: also written as

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) = \frac{dL_o(\theta_o, \varphi_o)}{dE_i(\theta_i, \varphi_i)}$$

(the symbol ρ is also used sometimes)

Defining Surface Reflectance

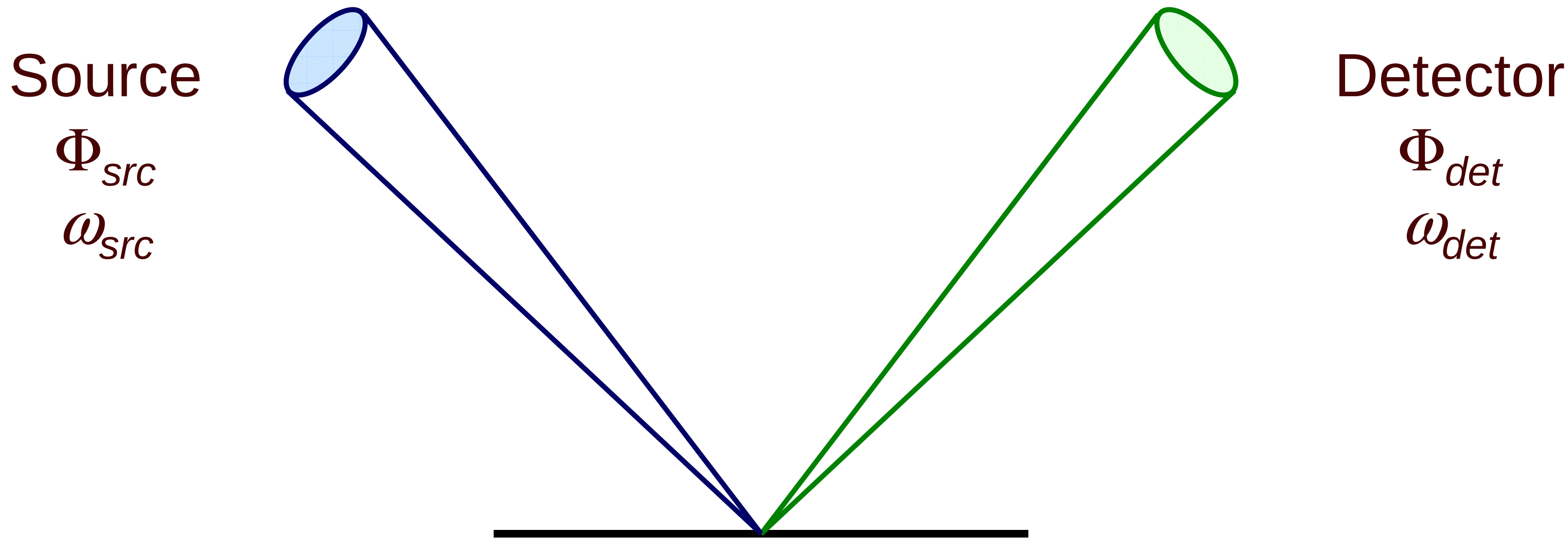
- Why is BRDF defined in this way?
- Key point: BRDF is a differential quantity, so limit must exist



Definition of BRDF

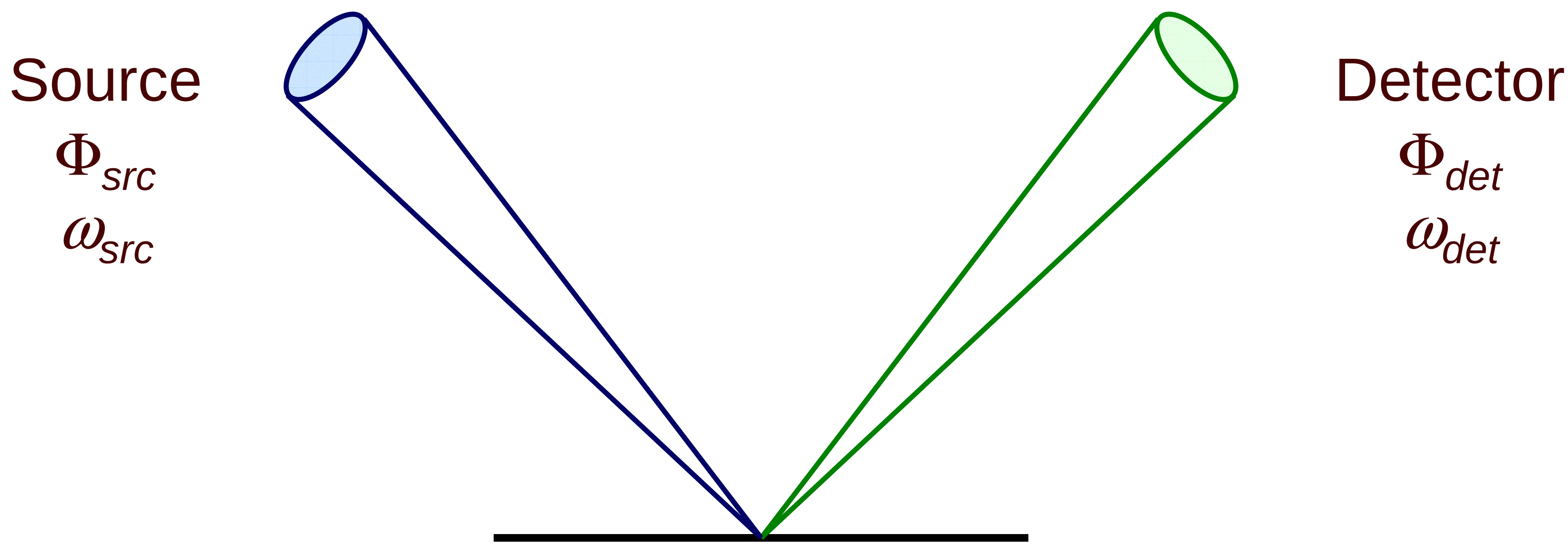
- First attempt:

$$f_r = \frac{\Phi_{det}}{\Phi_{src}}$$



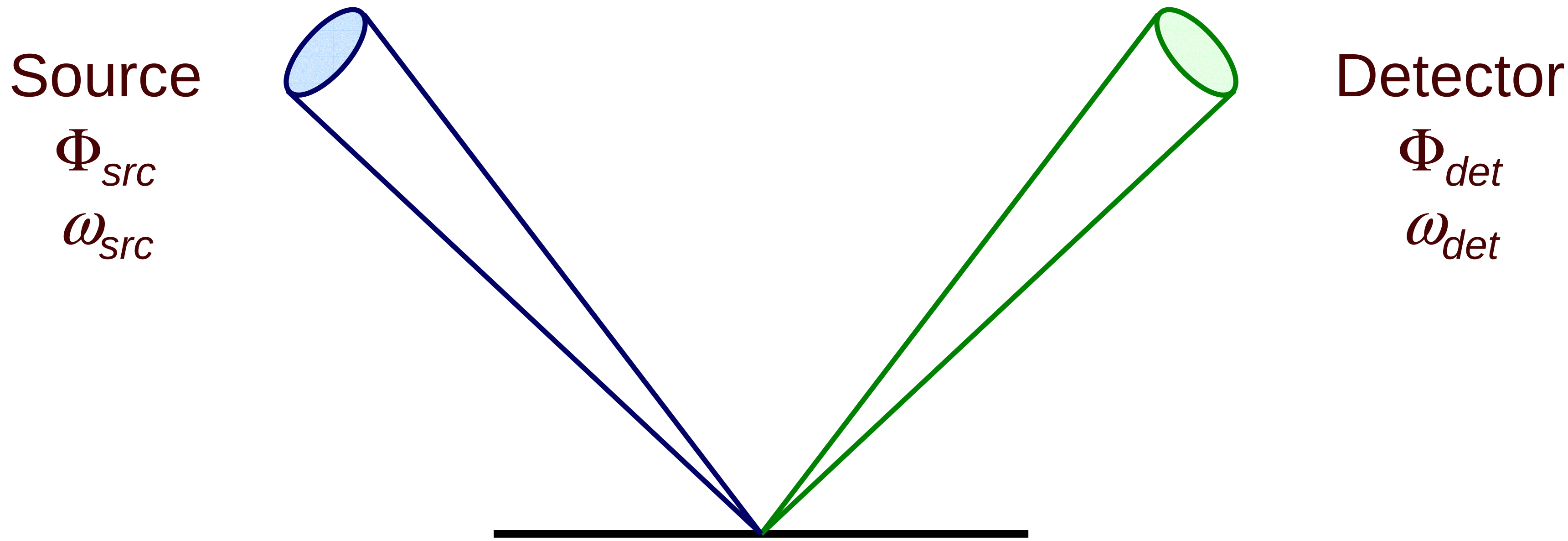
Definition of BRDF

- Should f_r vary with ω_{src}? No.



Definition of BRDF

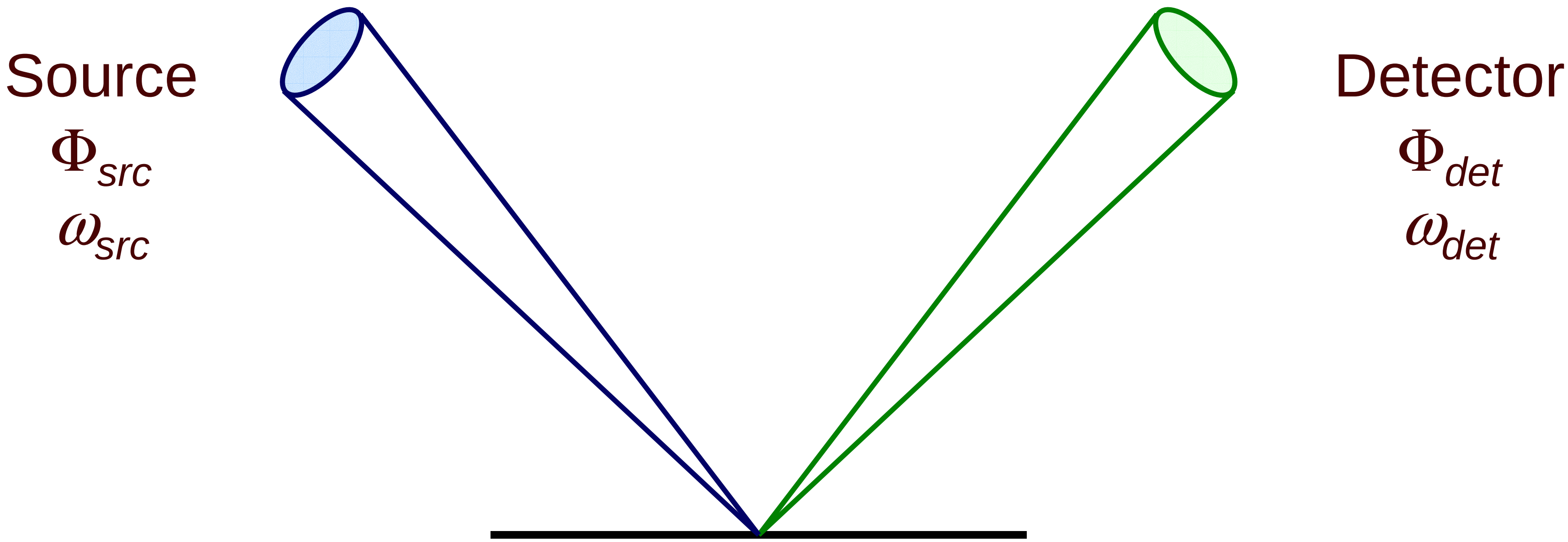
- Should f_r vary with ω_{det}? Yes.



Definition of BRDF

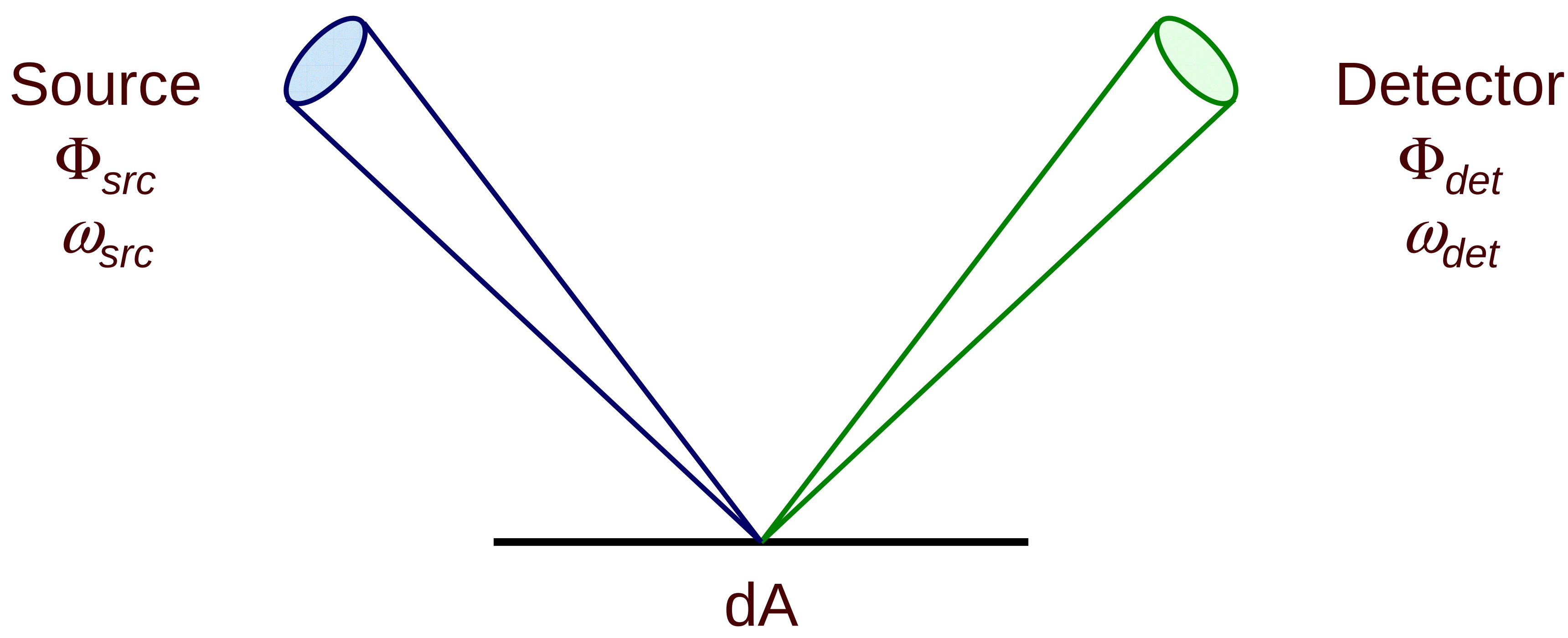
- Thus,

$$f_r = \frac{\Phi_{det} / \omega_{det}}{\Phi_{src}}$$



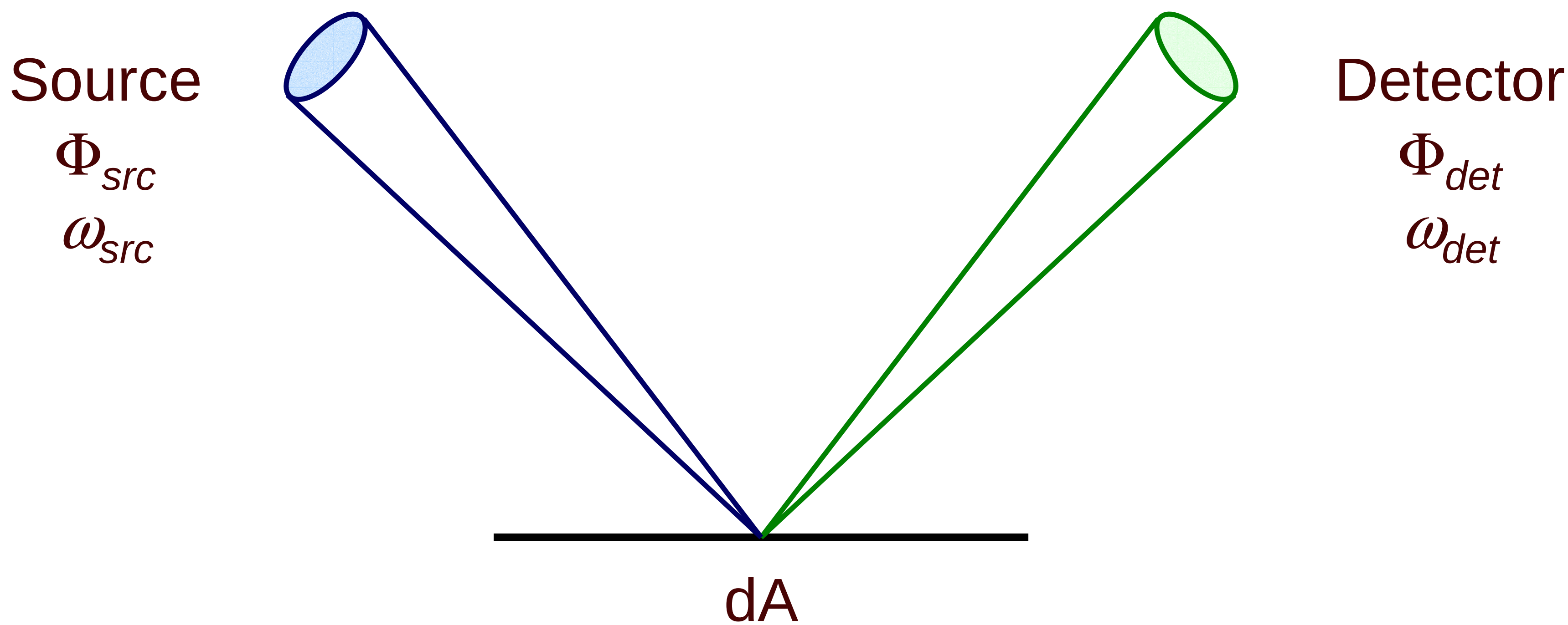
Definition of BRDF

- What about surface area?
  f_r must be independent of surface area



Definition of BRDF

$$f_r = \frac{\Phi_{det} / (\omega_{det} \cdot dA)}{\Phi_{src} / dA} = \frac{L}{E}$$



Properties of the BRDF

- Energy conservation:

$$\int_{\Omega} f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) \cos\theta_o d\omega_o \leq 1$$

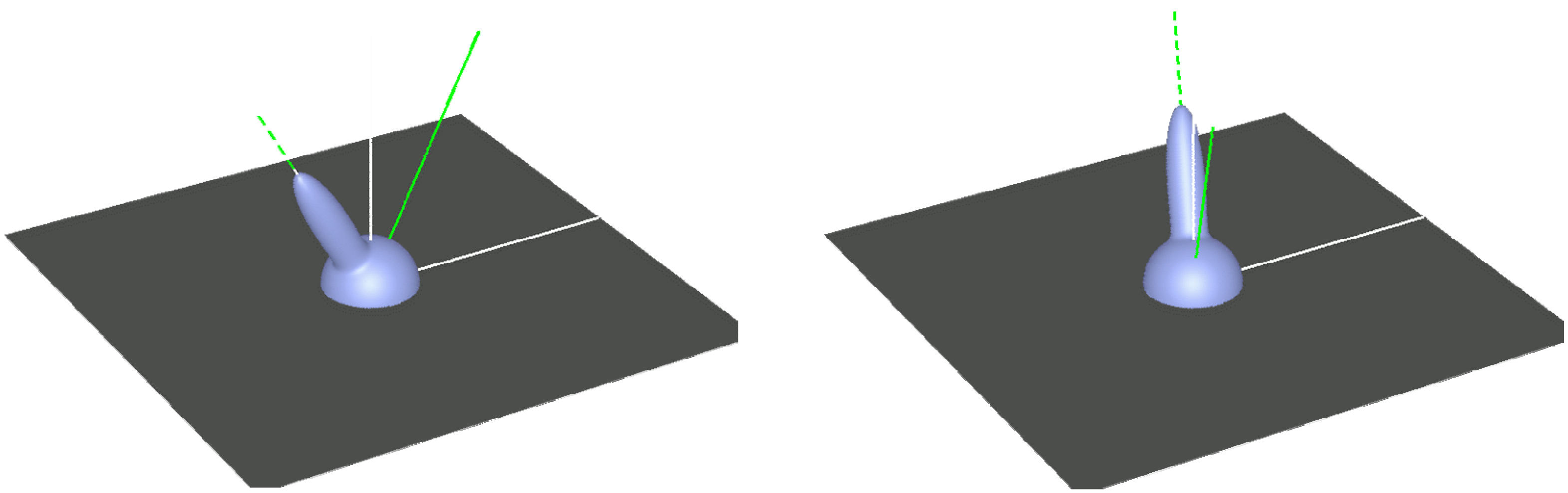
- Helmholtz reciprocity:

$$f_r(\omega_i \rightarrow \omega_o) = f_r(\omega_o \rightarrow \omega_i)$$

(not always obeyed by "BRDFs" used in graphics)

Isotropy

- A BRDF is isotropic if it stays the same when surface is rotated around normal

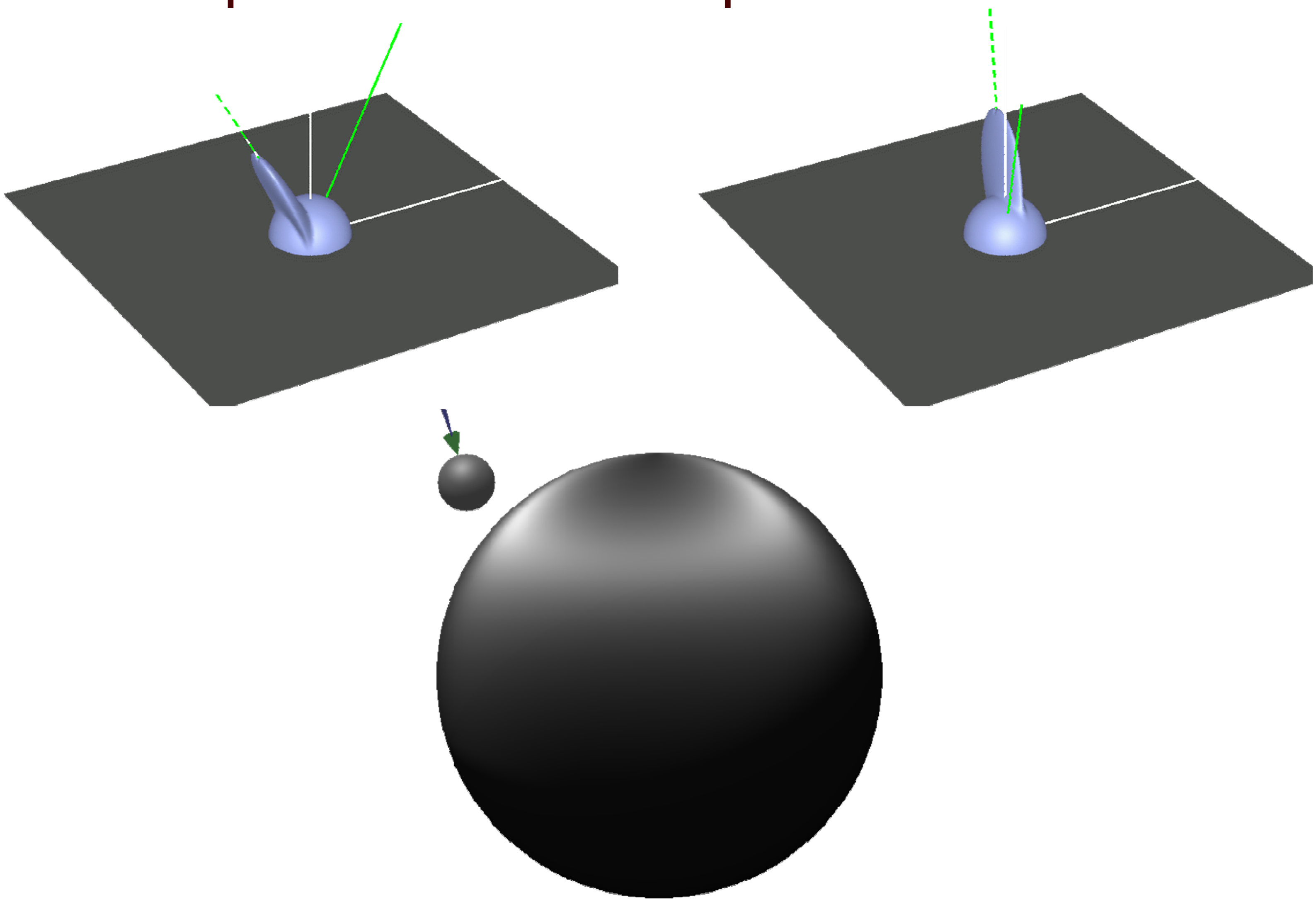


- Isotropic BRDFs are 3-dimensional functions:

$$f_r(\theta_i, \theta_o, \varphi_i - \varphi_o)$$

Anisotropy

- Anisotropic BRDFs **do** depend on surface rotation



Diffuse

- The simplest BRDF is "ideal diffuse" or *Lambertian*: just a constant

$$f_r(\omega_i \rightarrow \omega_o) = k_d$$

- Note: does *not* include $\cos(\theta_i)$
  - Remember definition of irradiance

Diffuse BRDF

- Assume BRDF reflects a fraction ρ of light

$$\int_{\Omega} f_{r,Lambertian}(\omega_i \to \omega_o) \cos\theta_o \, d\omega_o = \rho$$

$$\int_{\substack{\theta \in [0..\pi/2] \\ \varphi \in [0..2\pi]}} k_d \cos\theta_o \sin\theta_o \, d\theta_o d\varphi_o = \rho$$

$$2\pi k_d \int_{\theta \in [0..\pi/2]} \sin\theta_o \cos\theta_o \, d\theta_o = \rho$$

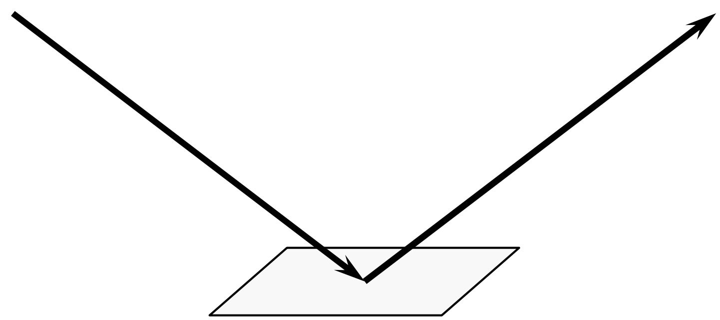
$$\pi k_d = \rho$$

$$\therefore f_{r,Lambertian} = \frac{\rho}{\pi}$$

- The quantity ρ is called the albedo

Ideal Mirror

- All light incident from one direction is reflected into another



- BRDF is zero everywhere except where

$$\theta_o = \theta_i$$

$$\varphi_o = \varphi_i + \pi$$

Ideal Mirror

- To conserve energy,

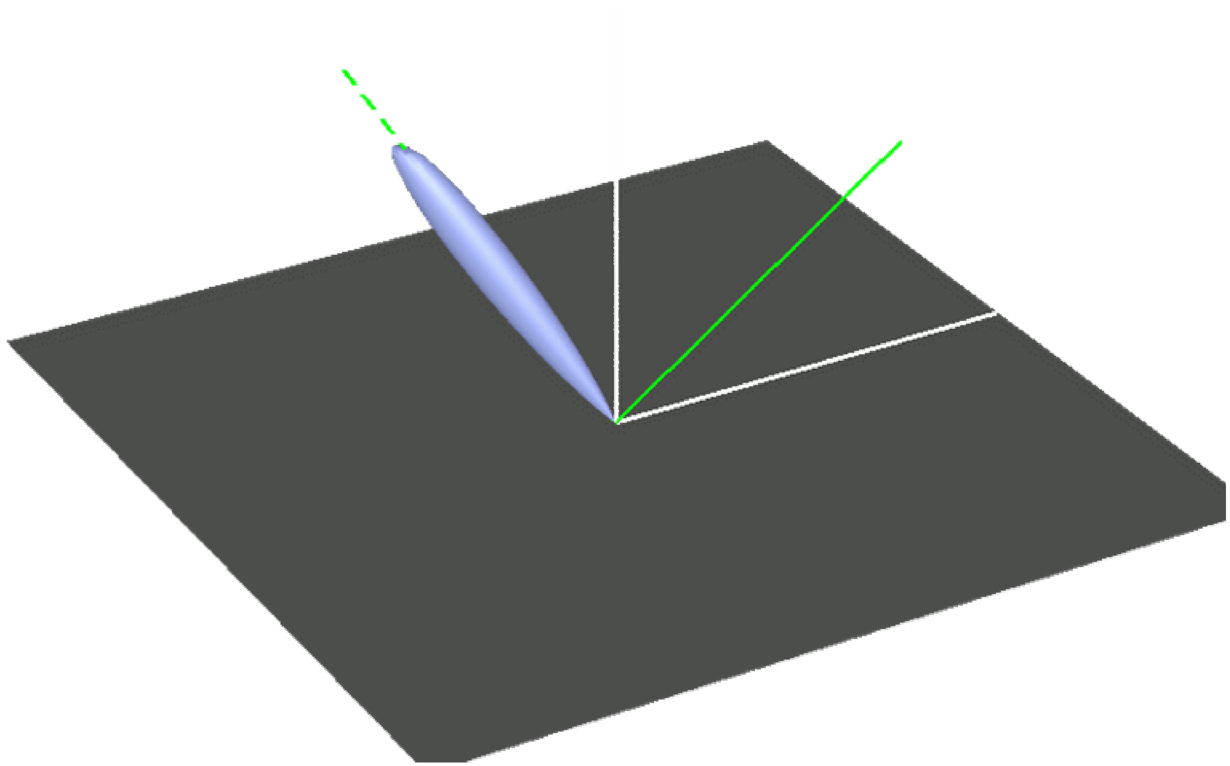
$$\int_{\Omega} f_{r,Mirror}(\omega_i \rightarrow \omega_o)\cos\theta_o \, d\omega_o = 1$$

- So, BRDF is a delta function at direction of ideal mirror reflection

$$f_{r,Mirror} = \frac{\delta(\theta_i - \theta_o)\,\delta(\varphi_i - \varphi_o)}{\cos(\theta_i)}$$

Glossy Reflection

- Non-ideal specular reflection
- Most light reflected *near* ideal mirror direction



Phong BRDF

- Phenomenological model for glossy reflection

$$f_{r,Phong} = k_s(\hat{l} \cdot \hat{r})^n$$

l is a vector to the light source
r is the direction of mirror reflection

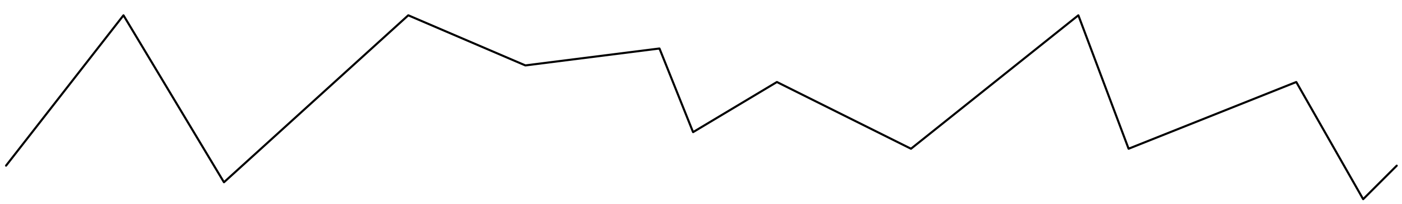
- Exponent *n* determines width of specular lobe
- Constant k_s determines size of lobe

Torrance-Sparrow BRDF

- Physically-based BRDF model
  - Originally used in the physics community
  - Adapted by Cook & Torrance and Blinn for graphics

$$f_{r,T-S} = \frac{DGF}{\pi \cos\theta_i \cos\theta_o}$$

- Assume surface consists of tiny "microfacets" with mirror reflection off each

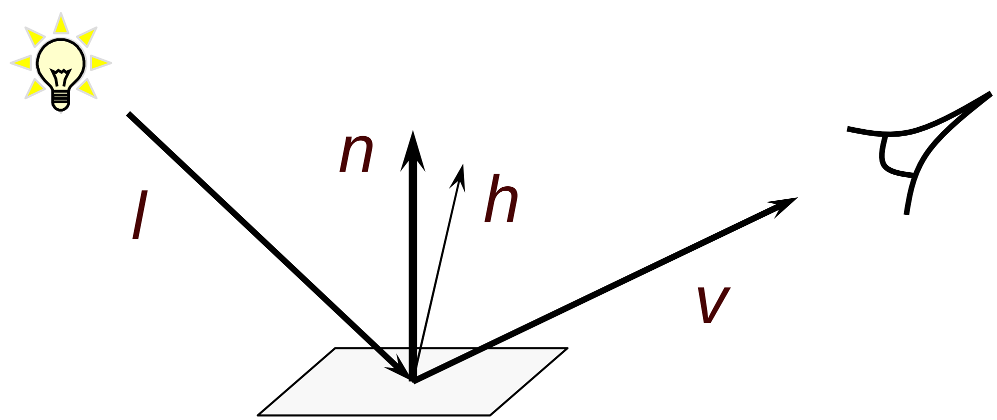


Torrance-Sparrow BRDF

- *D* term is distribution of microfacets (i.e., how many are pointing in each direction)
- Beckmann distribution

$$D = \frac{e^{-[(\tan\beta)/m]^2}}{4m^2 \cos^4 \beta}$$

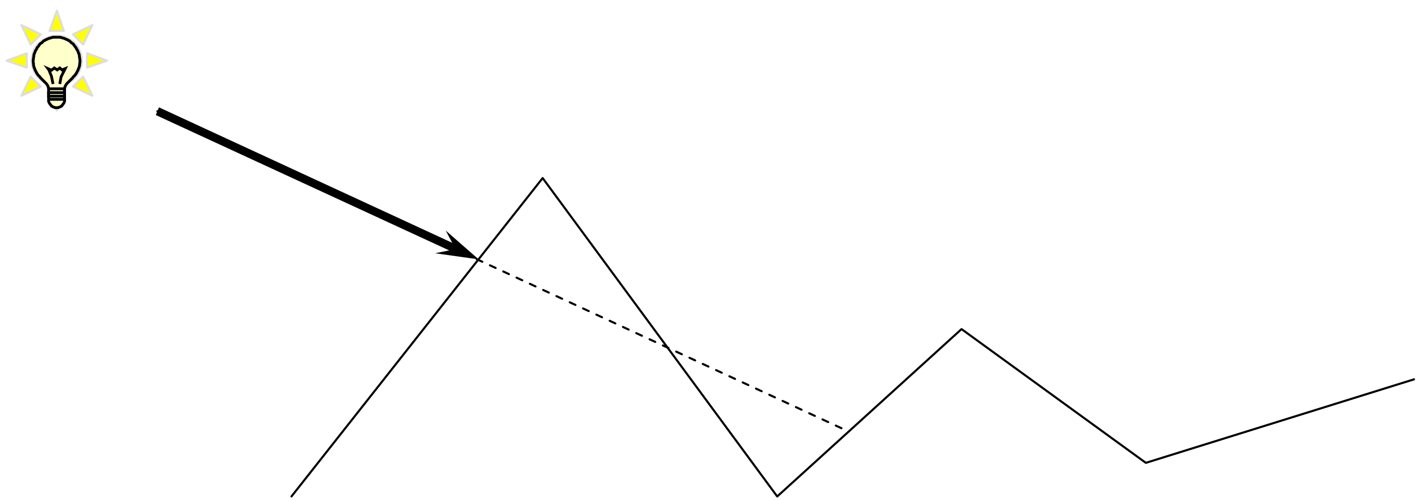
β is angle between *n* and *h*
h is halfway between *l* and *v*
m is “roughness” parameter



Torrance-Sparrow BRDF

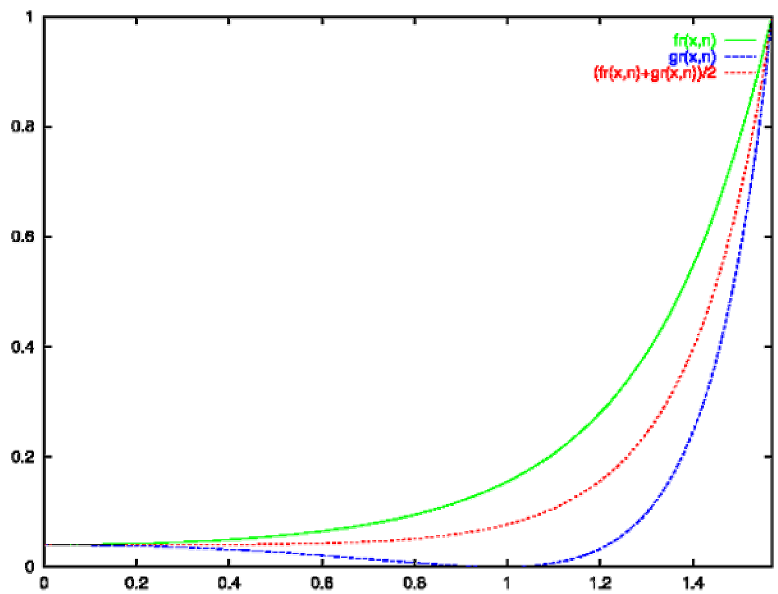
- *G* term accounts for self-shadowing

$$G = \min\left\{1, \frac{2(n \cdot h)(n \cdot v)}{(v \cdot h)}, \frac{2(n \cdot h)(n \cdot l)}{(v \cdot h)}\right\}$$

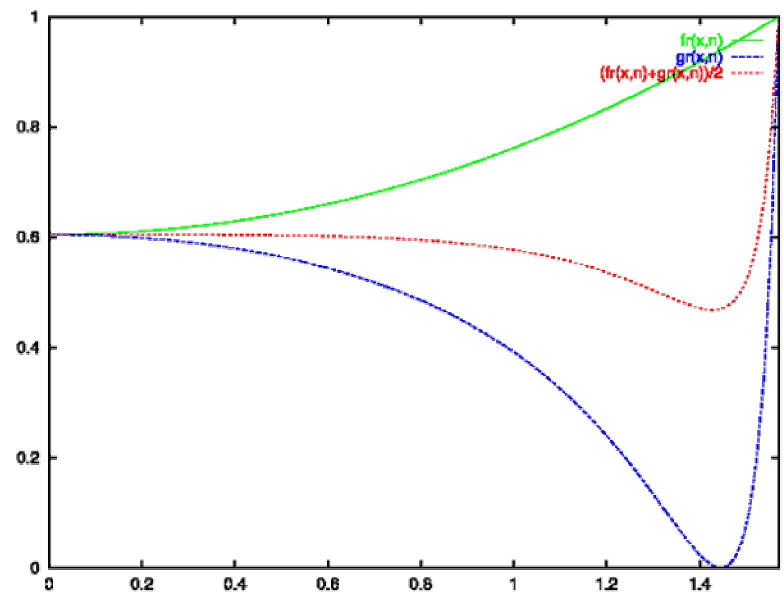


Torrance-Sparrow BRDF

- *F* term is Fresnel term – reflection from an ideal smooth surface (solution of Maxwell's equations)
- Consequence: most surfaces reflect (much) more strongly near grazing angles



Dielectric



Metal

(note behavior at Brewster's angle)

Aside: Brewster's Angle

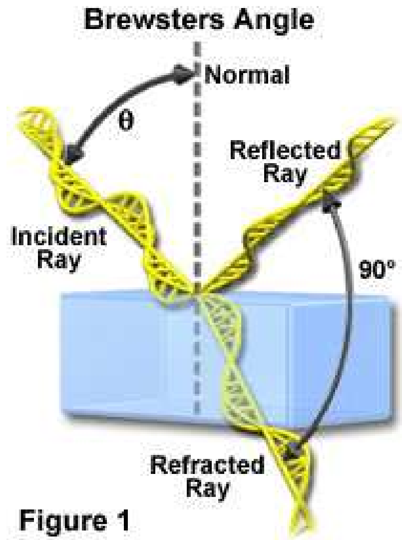
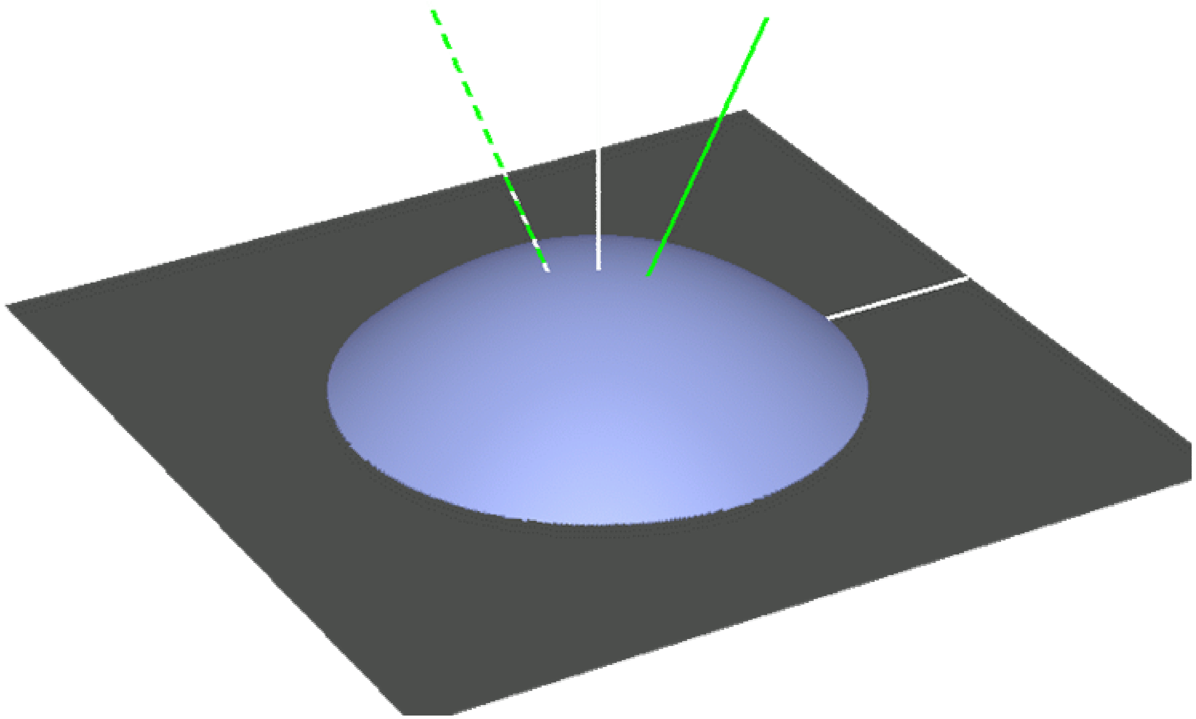


Figure 1

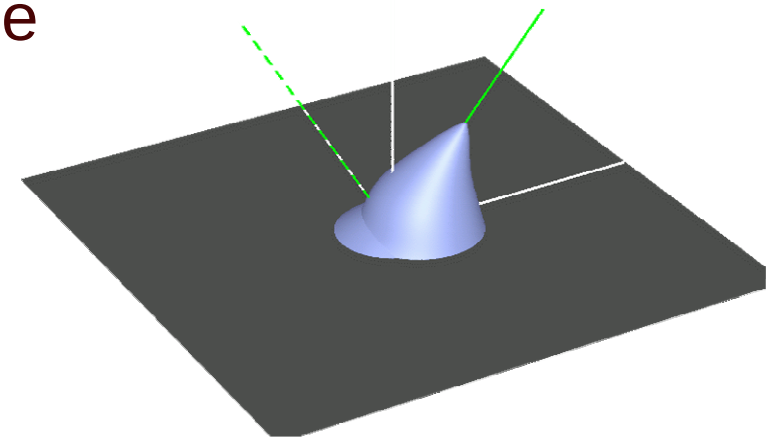
Other BRDF Features

- BRDFs for dusty surfaces scatter light towards grazing angles

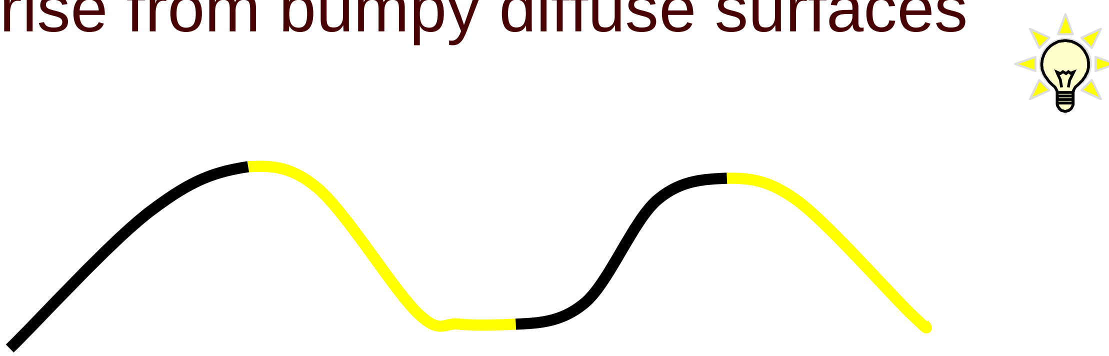


Other BRDF Features

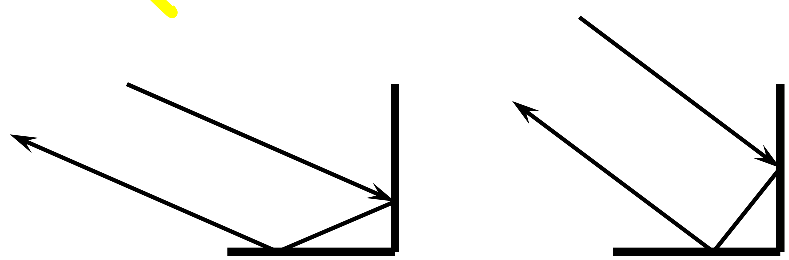
- Retroreflection: strong reflection back towards the light source



- Can arise from bumpy diffuse surfaces



- … or from corner reflectors



BRDF Representations

- Physically-based vs. phenomenological models
- Measured data
- Desired characteristics:
  - Fast to evaluate
  - Maintain reciprocity, energy conservation
  - For global illumination: easy to importance sample

Beyond BRDFs

- So far, have assumed 4D BRDF
- Function of wavelength: 5D
- Fluorescence (absorb at one wavelength, emit at another): 6D
- Phosphorescence (absorb now, emit later): 7D
- Temporal dependence: 8D
- Spatial dependence: 10D
- Subsurface scattering: 12D
- Polarization
- Wave optics effects (diffraction, interference)
- ...

Rendering Equation

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega} f_r(x,\omega_i \to \omega_o)\, L_i(x,\omega_i) \cos\theta_i \, d\omega$$

Outgoing radiance

Emitted radiance

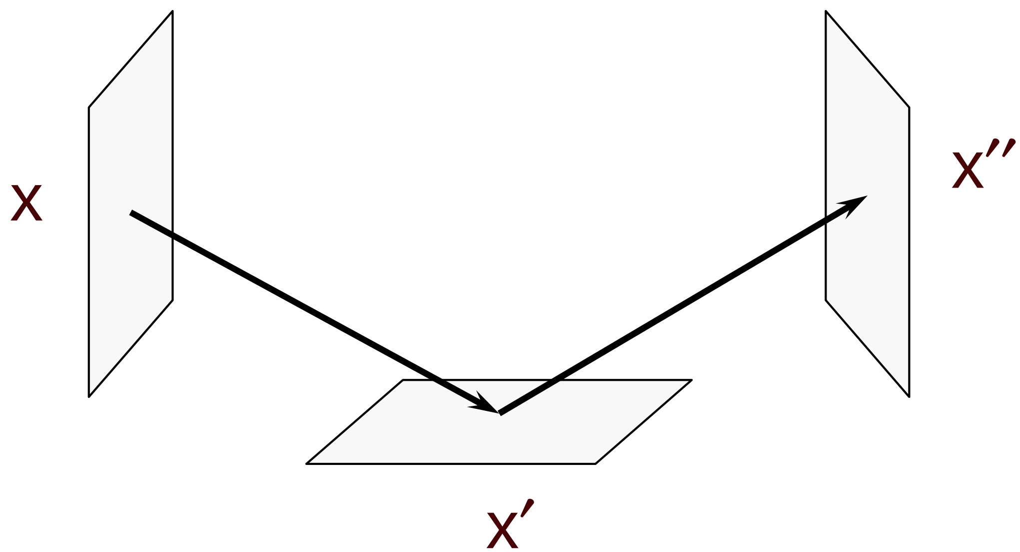
BRDF

Irradiance

Rendering Equation

- Originally expressed by [Kajiya 1986] as

$$I(x' \to x'') = I_e(x' \to x'') + G(x', x'') \int_S f_r(x \to x' \to x'') \, I(x \to x') \, V(x, x') \, dA$$



Rendering Equation

- Originally expressed by [Kajiya 1986] as

$$I(x' \to x'') = I_e(x' \to x'') + G(x', x'') \int_S f_r(x \to x' \to x'') I(x \to x') V(x, x') \, dA$$

- Integral is over all points in the scene
- *G*(*x*,*x'*) is a geometry term:

$$G(x, x') = \frac{\cos \theta_i' \cos \theta_o}{\left\| x - x' \right\|^2}$$

Rendering Equation

- Originally expressed by [Kajiya 1986] as

$$I(x' \to x'') = I_e(x' \to x'') + G(x', x'') \int_S f_r(x \to x' \to x'') I(x \to x') V(x, x') \, dA$$

- Integral is over all points in the scene
- $V(x,x')$ is a visibility term and is either 0 or 1

Rendering Equation

- Originally expressed by [Kajiya 1986] as

$$I(x' \to x'') = I_e(x' \to x'') + G(x', x'') \int_S f_r(x \to x' \to x'')\, I(x \to x')\, V(x, x')\, dA$$

- Integral is over all points in the scene
- *I*(*x*→*x'*) is the *two-point transport intensity:*

$$I(x \to x') = L(x, \omega)\, G(x, x')\, dA\, dA'$$

(note: this is not the same *I* we've seen before…)

Rendering Equation

- Next 3-4 weeks in the course: ways to solve the rendering equation