Photon Mapping
(Helper Slides)

Sampling Patterns

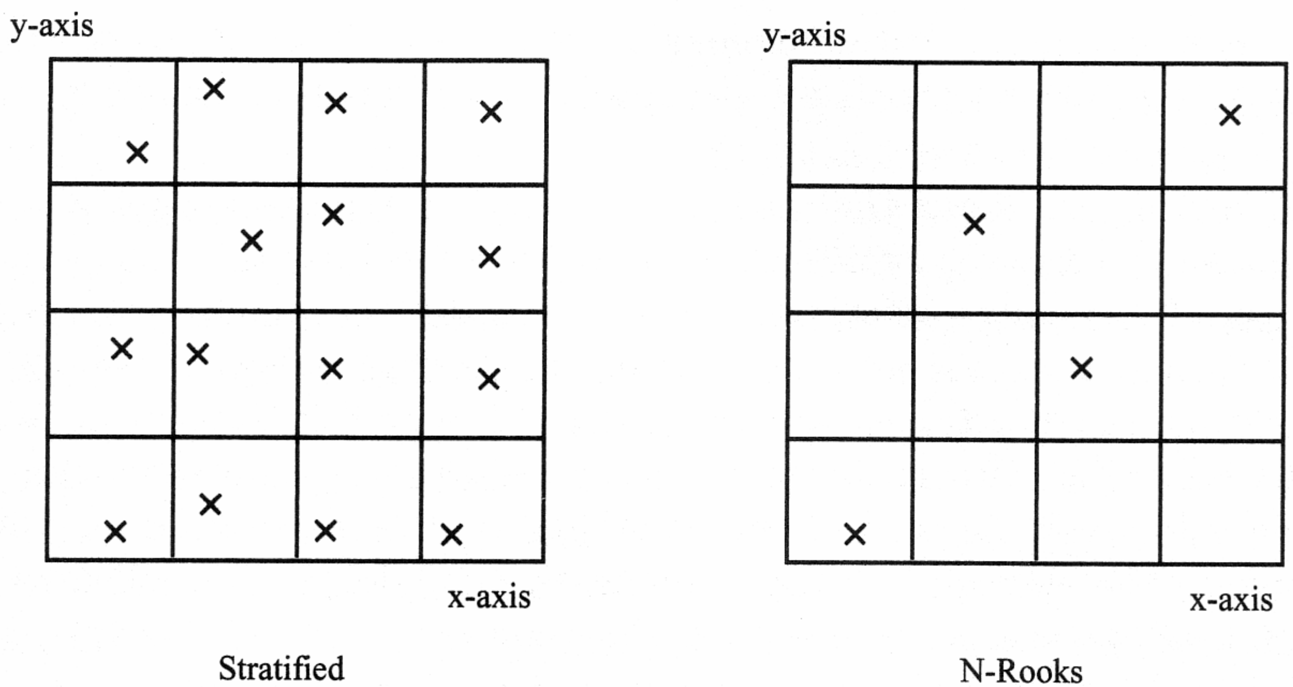
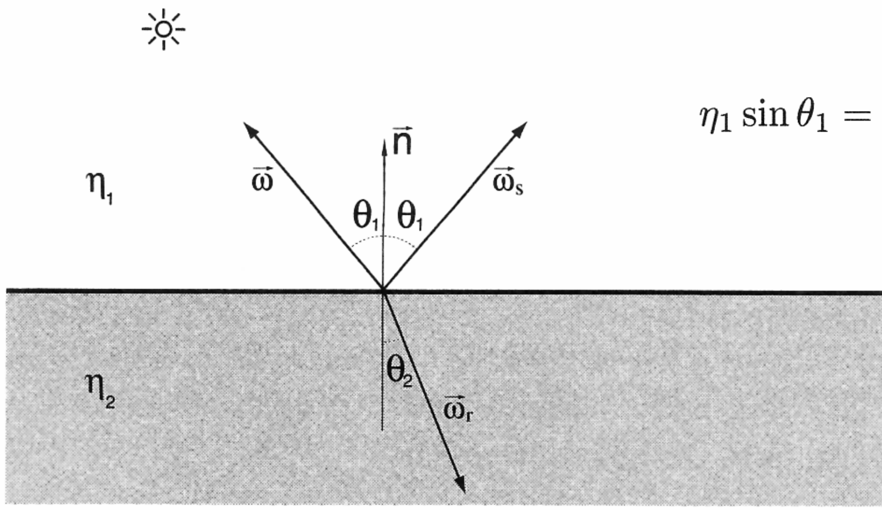


Figure 3.8. Stratified sampling versus N-Rooks sampling for two dimensions.

Specular Reflection and Transmission



$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2 \ .$$

Figure 2.6. The geometry of refraction and reflection.

The geometry for refraction is shown in Figure 2.6. Using Snell's law, the direction, $\vec{\omega}_r$, of the refracted ray (for a perfectly smooth surface with normal $\vec{n}$) is computed as:

$$\vec{\omega}_r = -\frac{\eta_1}{\eta_2}(\vec{\omega} - (\vec{\omega}\cdot\vec{n})\vec{n}) - \left(\sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (\vec{\omega}\cdot\vec{n})^2)}\right)\vec{n} \ . \qquad (2.32)$$

← Available in ray tracer

For the refracted ray the amount of transmitted light can be computed as $1 - F_r$.

← Use Fresnel's Eqns for F_r

The Fresnel Equations

For smooth homogeneous metals and dielectrics the amount of light reflected can be derived from Maxwell's equations, and the result is the *Fresnel equations*. Given a ray of light in a medium with index of refraction η_1 (see Figure 2.6) that strikes a material with index of refraction η_2, we can compute the amount of light reflected as:

$$\rho_{\parallel} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$\rho_{\perp} = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2} \,. \tag{2.28}$$

These coefficients take into account polarization: $\rho_{\parallel}$ is the reflection coefficient for light with the electric field being parallel to the plane of incidence, and $\rho_{\perp}$ is the reflection coefficient for light with the electric field being orthogonal to the plane of incidence. The value of the index of refraction can be found in most textbooks on optics. For commonly used materials: air ($\eta \approx 1.0$), water ($\eta \approx 1.33$), and glass ($\eta \approx 1.5 - 1.7$ depending on the type of glass). Note also that the index of refraction can be complex. This is the case for metals where the imaginary component specifies the absorption of light by the metal (i.e., the fact that metals are not transparent).

For unpolarized light the specular reflectance (also known as the Fresnel reflection coefficient F_r) becomes:

$$F_r(\theta) = \frac{1}{2}\left(\rho_{\parallel}^2 + \rho_{\perp}^2\right) = \frac{d\Phi_r}{d\Phi_i} \,. \tag{2.29}$$

For unpolarized light a good approximation to the Fresnel reflection coefficient was derived by Schlick [85]:

$$\boxed{F_r(\theta) \approx F_0 + (1 - F_0)(1 - \cos\theta)^5} \,, \tag{2.30}$$

where F_0 is the value of the real Fresnel reflection coefficient at normal incidence.

Russian Roulette

$$I = \int_0^1 f(x)dx.$$

$$I_{RR} = \int_0^P \frac{1}{P} f(\frac{x}{P})dx,$$

$$\langle I_{RR} \rangle = \begin{cases} \frac{1}{P} f(\frac{x}{P}) & \text{if } x \leq P \\ 0 & \text{if } x > P. \end{cases}$$

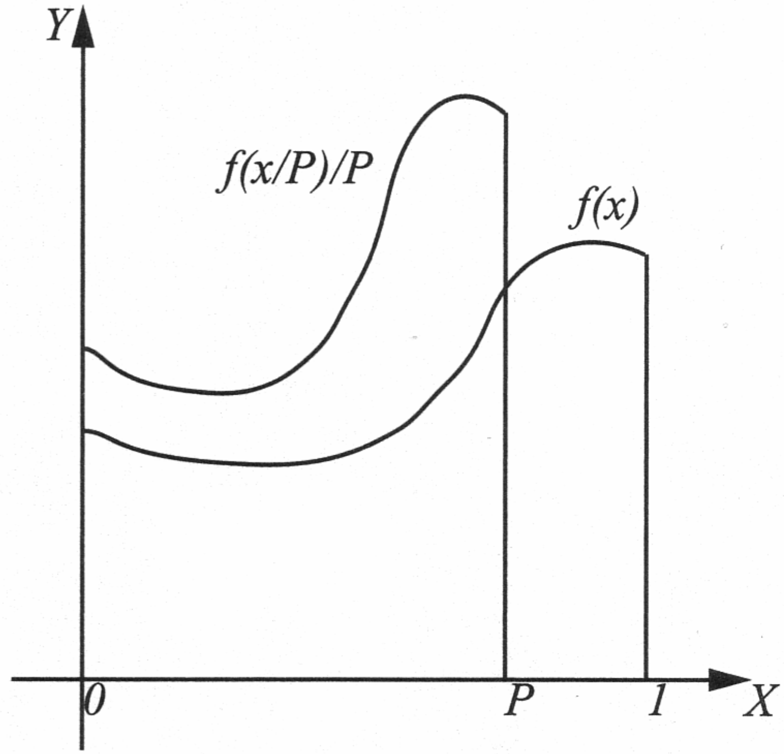


Figure 5.3. Principle of Russian roulette.

RGB Roulette

Specular or Diffuse Reflection (RGB)?

With more color bands (for example, RGB colors), the decision gets slightly more complicated. Consider again a surface with some diffuse reflection and some specular reflection, but this time with different reflection coefficients in the three color bands. To select the type of reflection we can use the same approach as described in the previous section. For the reflectance we can use the average diffuse reflectance, $\rho_{d,avg}$, and the average specular reflectance, $\rho_{s,avg}$:

$$\rho_{d,avg} = \frac{\rho_{d,r} + \rho_{d,g} + \rho_{d,b}}{3} \quad (5.4)$$

$$\rho_{s,avg} = \frac{\rho_{s,r} + \rho_{s,g} + \rho_{s,b}}{3} . \quad (5.5)$$

Here the subscripts r, g, b denote reflectance in the red, green, and blue band respectively.

Using the average reflectance values we find that:

$$\begin{array}{lll} \xi \in [0, \rho_{d,avg}] & \longrightarrow & \text{diffuse reflection} \\ \xi \in]\rho_{d,avg}, \rho_{s,avg} + \rho_{d,avg}] & \longrightarrow & \text{specular reflection} \\ \xi \in]\rho_{s,avg} + \rho_{d,avg}, 1] & \longrightarrow & \text{absorption} \end{array}$$

To account for the fact that the reflection should have used a spectral reflectance value, we need to scale the power of the reflected photon. If specular reflection is chosen we get:

$$\begin{aligned} \Phi_{s,r} &= \Phi_{i,r}\,\rho_{s,r}/\rho_{s,avg} \\ \Phi_{s,g} &= \Phi_{i,g}\,\rho_{s,g}/\rho_{s,avg} \\ \Phi_{s,b} &= \Phi_{i,b}\,\rho_{s,b}/\rho_{s,avg} \ , \end{aligned}$$

where $(\Phi_{i,r}, \Phi_{i,g}, \Phi_{i,b})$ is the spectral power of the incoming photon, and $(\Phi_{s,r}, \Phi_{s,g}, \Phi_{s,b})$ is the spectral power of the reflected photon.

It is simple to extend the selection scheme also to handle transmission, to handle more than three color bands, and to handle combinations of multiple BRDFs.

Caustics: Projection Maps

- Cast caustic map photons toward specular objects
- Use object bounds and rejection testing
- Simple case: spheres

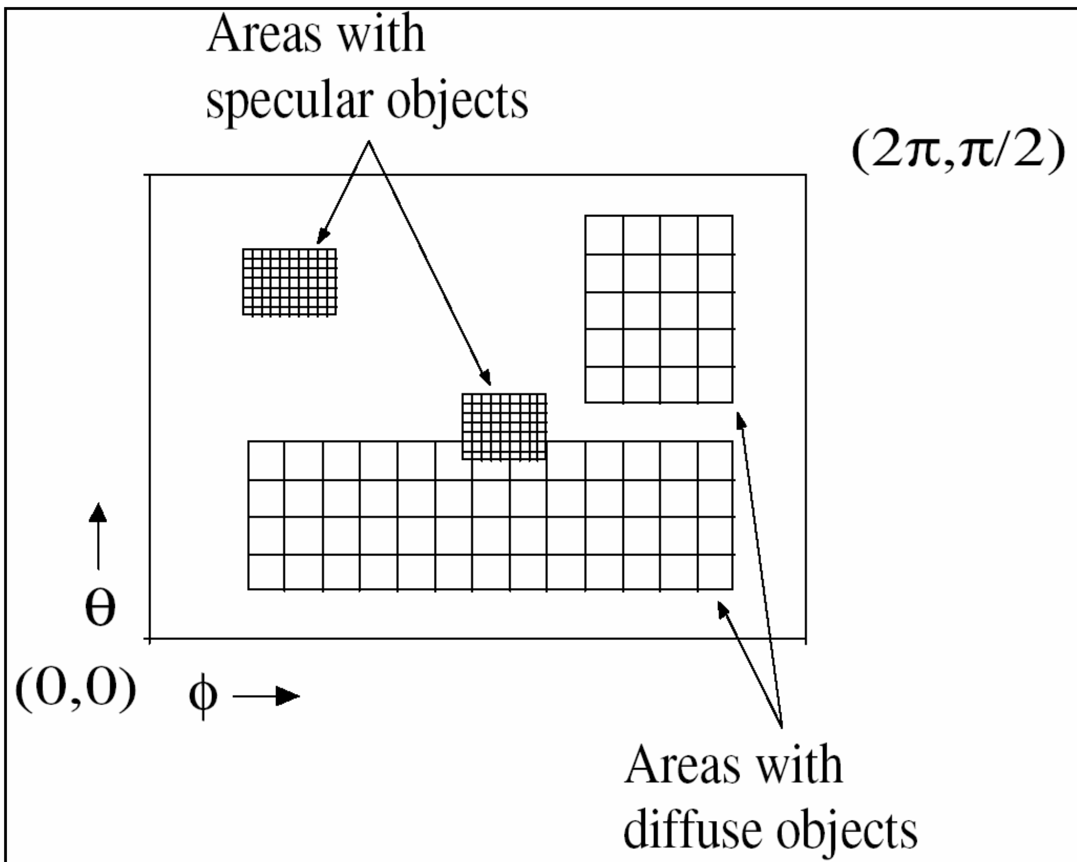


Figure 2: The projection map is filled with information about the surface characteristics of the objects around the light source position.

Jensen & Christensen, 1995

Photon Mapping Summary

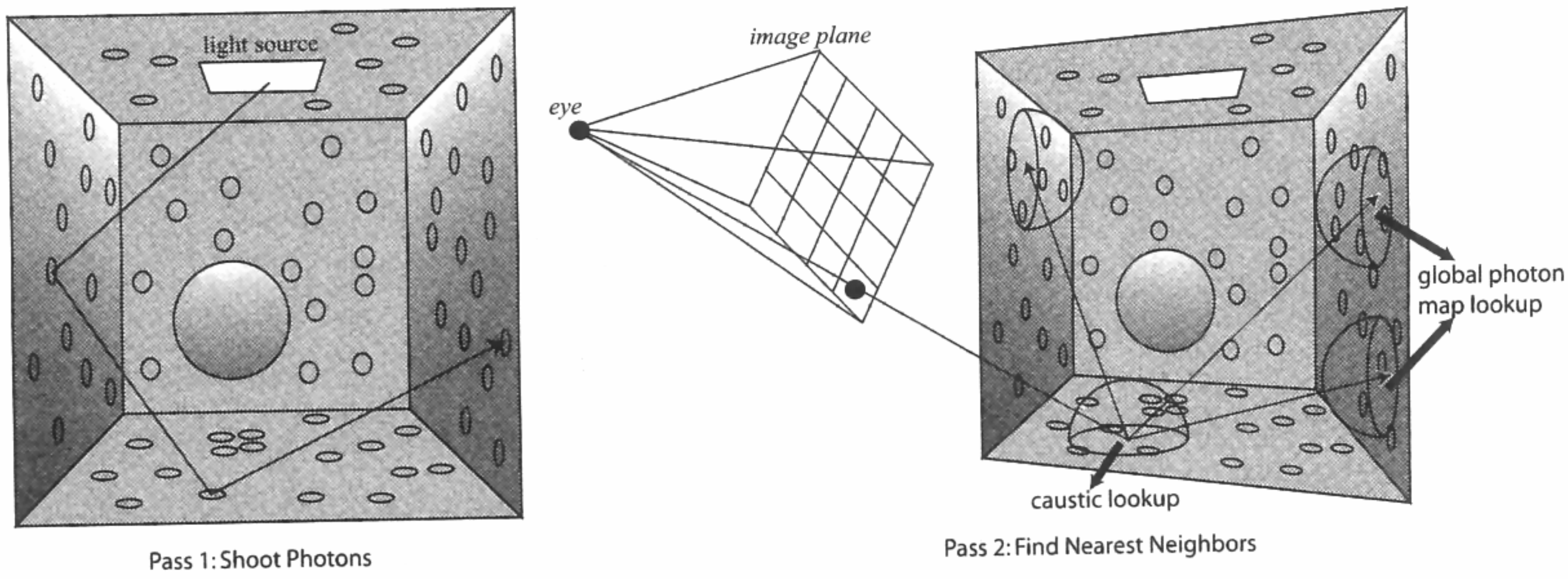


Figure 7.10. Two passes of photon mapping in a Cornell box with a glass sphere. In Pass 1, photons are traced and deposited on nonspecular surfaces. In Pass 2, global illumination is indirectly computed using the global photon map (as shown). For each indirect ray, the N closest photons in the global photon map are found. Caustics are also found by doing a similar look-up in the caustic map at the visible point. Direct illumination, specular, and glossy reflections (not shown) are computed using ray tracing.

Solving the Rendering Equation

- Must compute reflected radiance, L_r

As shown in Section 2.5 the outgoing radiance, L_o, at a given surface location, x, can be computed as:

$$L_o(x, \vec{\omega}) = L_e(x, \vec{\omega}) + L_r(x, \vec{\omega}) \ , \tag{9.1}$$

where the reflected radiance, L_r, is computed by the following integral:

$$L_r(x, \vec{\omega}) = \int_\Omega f_r(x, \vec{\omega}', \vec{\omega}) L_i(x, \vec{\omega}')(\vec{\omega}' \cdot \vec{n}) \, d\vec{\omega}' \ . \tag{9.2}$$

To evaluate this integral efficiently it is worth considering the properties of the BRDF, f_r, and the incoming radiance, L_i.

Solving the Rendering Equation

- Split BRDF into diffuse and specular parts:

The BRDF is often a combination of two components: a smooth (diffuse) and a sharp (specular) component. This information is very useful when evaluating the BRDF, and we therefore split the BRDF into the sum of two terms: a specular/glossy term, $f_{r,S}$, and a diffuse term, $f_{r,D}$ (note that these do not have to be Lambertian $f_{r,d}$ or perfect specular $f_{r,s}$):

$$f_r(x, \vec{\omega}', \vec{\omega}) = f_{r,S}(x, \vec{\omega}', \vec{\omega}) + f_{r,D}(x, \vec{\omega}', \vec{\omega}) \,. \tag{9.3}$$

Solving the Rendering Equation

- Split incoming radiance into three parts:

Similarly the incoming radiance is the sum of three components:

$$L_i(x, \vec{\omega}') = L_{i,l}(x, \vec{\omega}') + L_{i,c}(x, \vec{\omega}') + L_{i,d}(x, \vec{\omega}')\,, \qquad (9.4)$$

where

- $L_{i,l}(x, \vec{\omega}')$ is direct illumination from the light sources.
- $L_{i,c}(x, \vec{\omega}')$ is caustics—indirect illumination from the light sources via specular reflection or transmission.
- $L_{i,d}(x, \vec{\omega}')$ is indirect illumination from the light sources that has been reflected diffusely at least once.

Solving the Rendering Equation

- Splittings produce ***four illumination components***:

$$
\begin{aligned}
L_r(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}' \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L_{i,l}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}' + \\
&\quad \int_\Omega f_{r,S}(x,\vec{\omega}',\vec{\omega})(L_{i,c}(x,\vec{\omega}') + L_{i,d}(x,\vec{\omega}'))(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}' + \\
&\quad \int_\Omega f_{r,D}(x,\vec{\omega}',\vec{\omega}) L_{i,c}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}' + \\
&\quad \int_\Omega f_{r,D}(x,\vec{\omega}',\vec{\omega}) L_{i,d}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}' .
\end{aligned}
$$

The Four Illumination Components

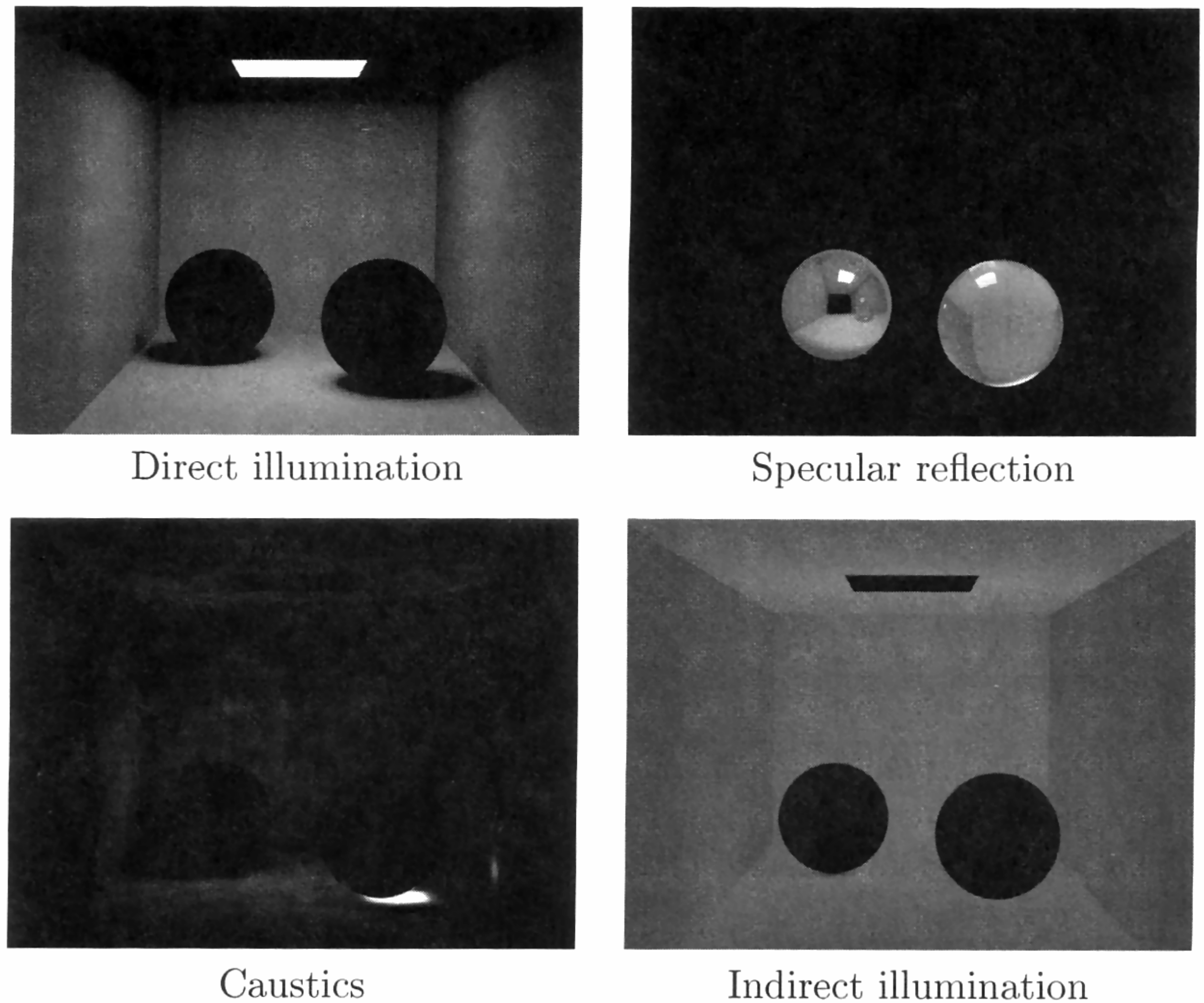
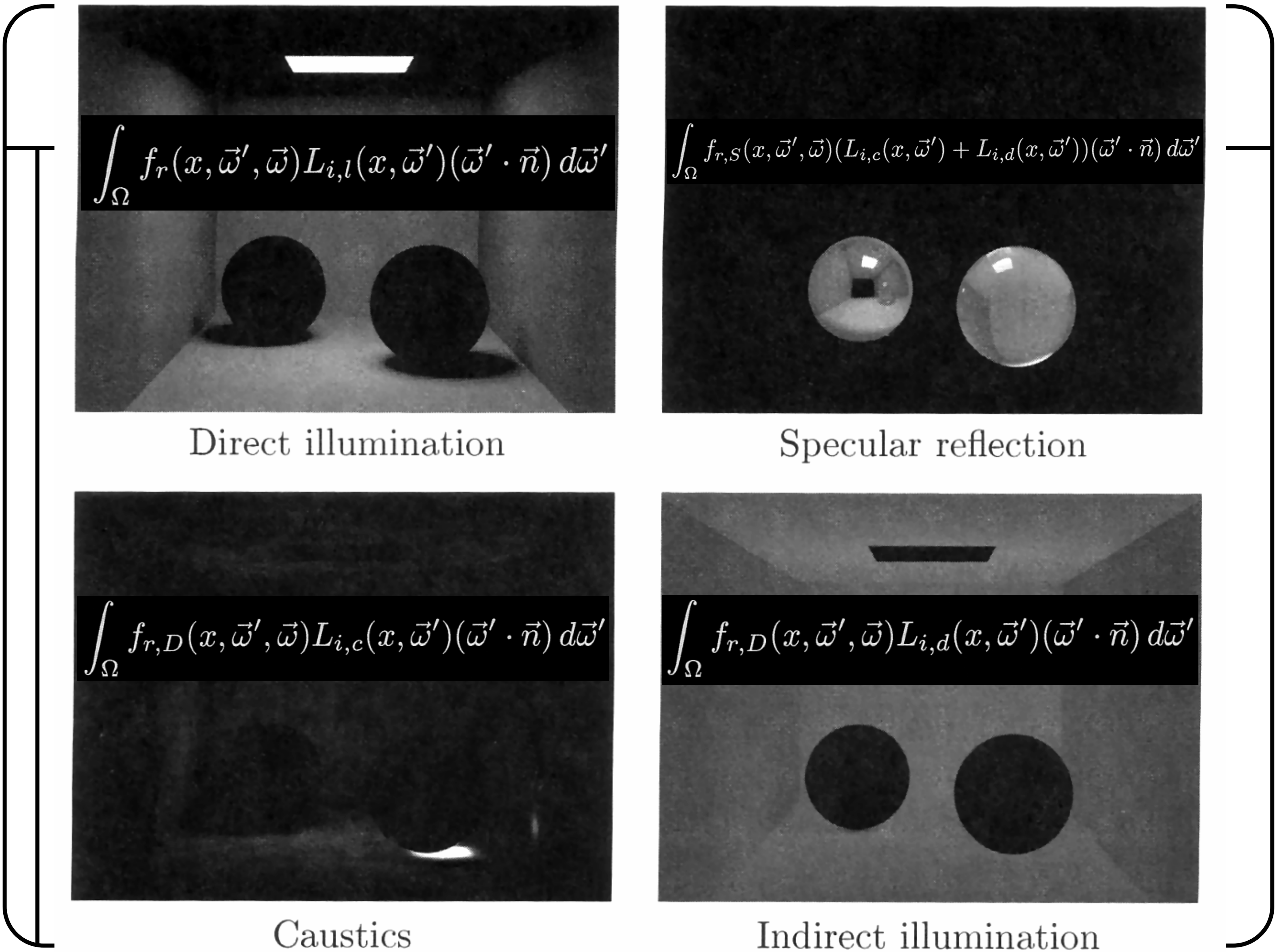


Figure 9.8. The different components of the rendered solution.


$\int_\Omega f_r(x,\vec{\omega}',\vec{\omega})L_{i,l}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}'$
Direct illumination
$\int_\Omega f_{r,S}(x,\vec{\omega}',\vec{\omega})(L_{i,c}(x,\vec{\omega}')+L_{i,d}(x,\vec{\omega}'))(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}'$
Specular reflection
$\int_\Omega f_{r,D}(x,\vec{\omega}',\vec{\omega})L_{i,c}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}'$
Caustics
$\int_\Omega f_{r,D}(x,\vec{\omega}',\vec{\omega})L_{i,d}(x,\vec{\omega}')(\vec{\omega}'\cdot\vec{n})\,d\vec{\omega}'$
Indirect illumination