Implicit Surfaces

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Curved Surface Representations

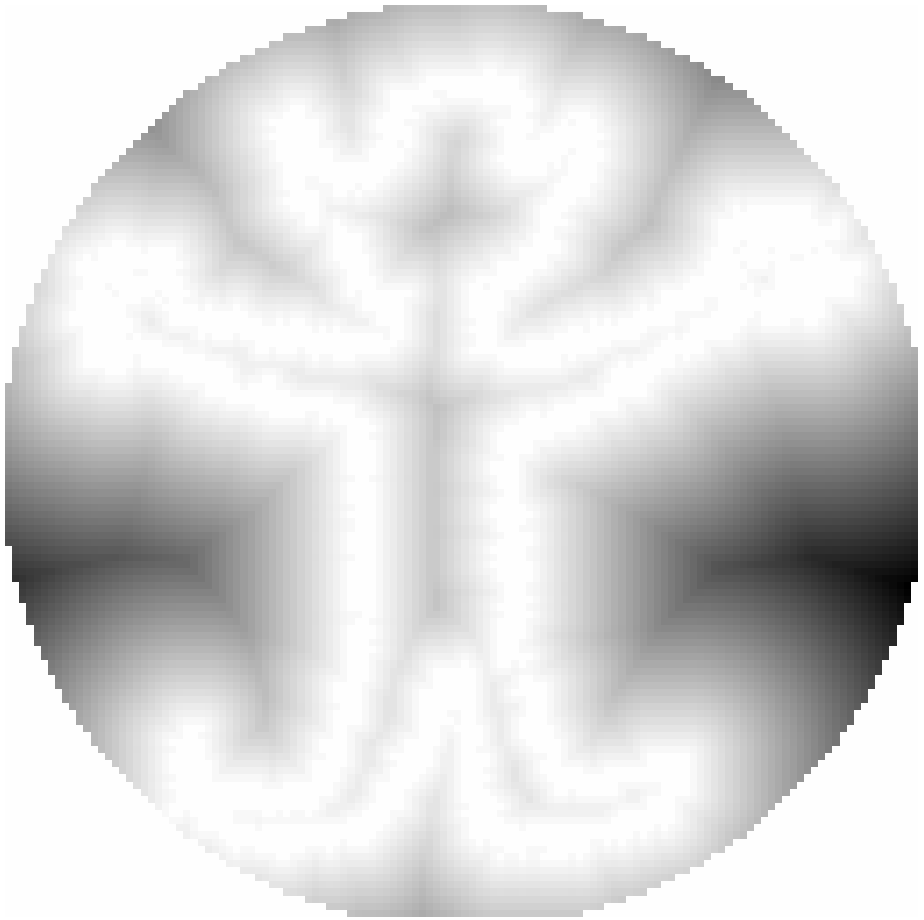
- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections

Curved Surface Representations

- Polygonal meshes
- Parametric surfaces
- Subdivision surfaces
- Implicit surfaces

Implicit Surfaces

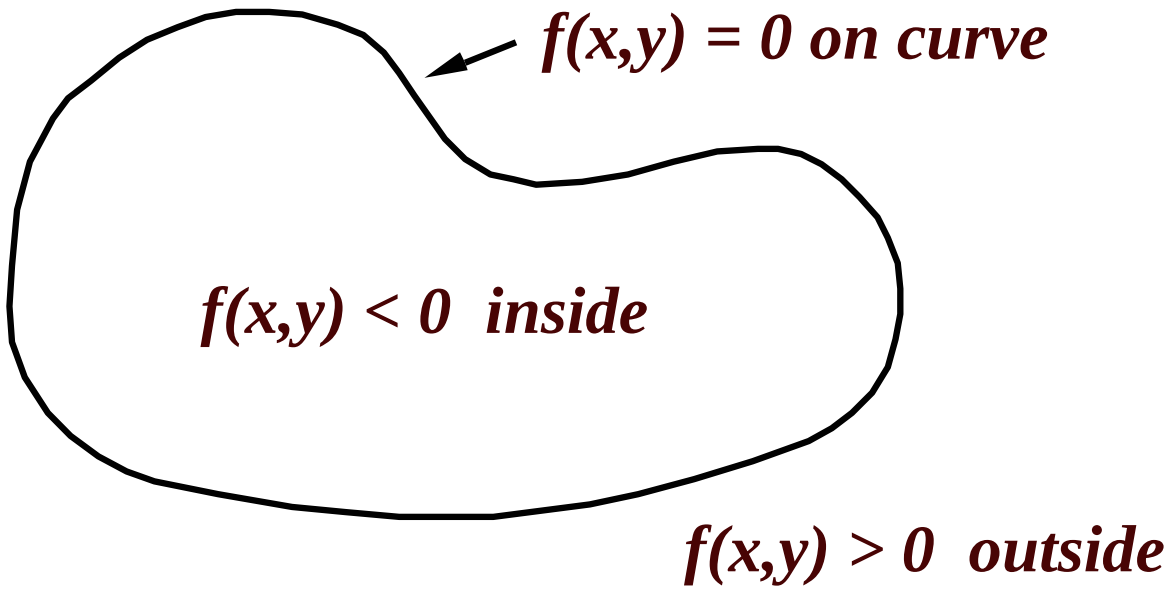
- Represent surface with function defined over all space



Kazhdan

Implicit Surfaces

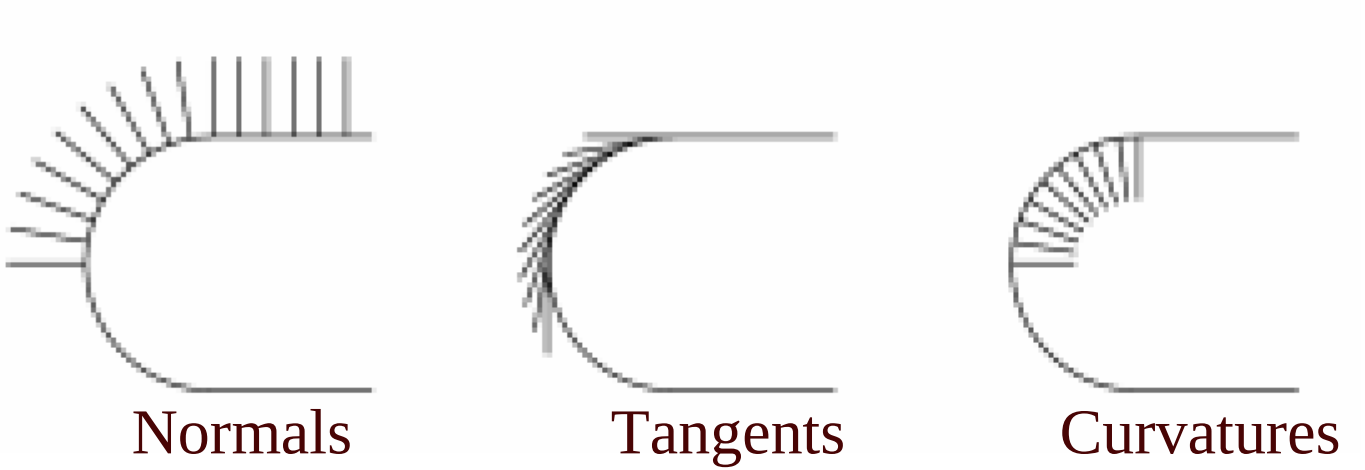
- Surface defined implicitly by function:
  - f (x, y, z) = 0 (on surface)
  - f (x, y, z) < 0 (inside)
  - f (x, y, z) > 0 (outside)



Turk

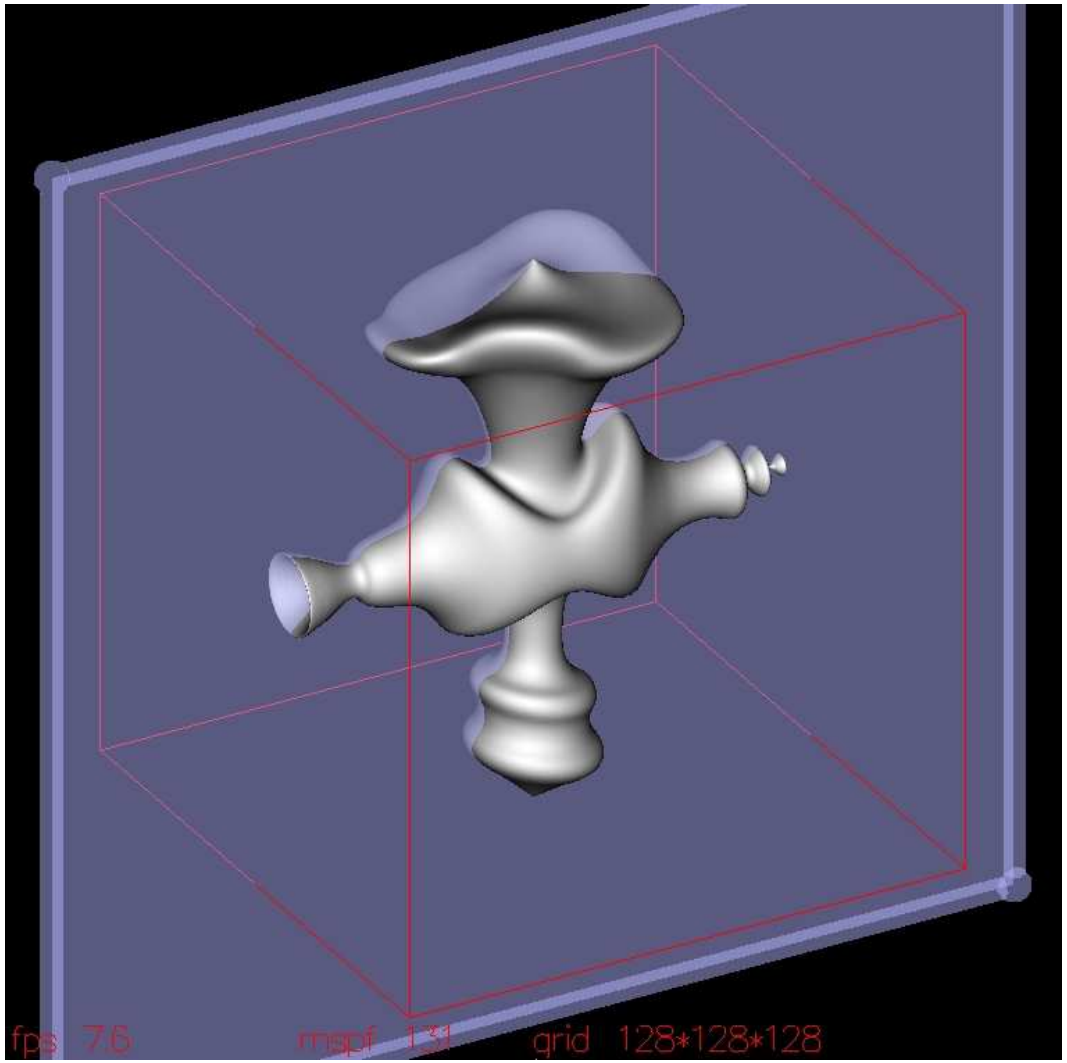
Implicit Surfaces

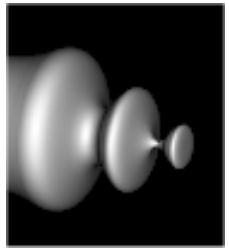
- Normals defined by partial derivatives
  - normal(x, y, z) = (df/dx, df/dy, df/dz)

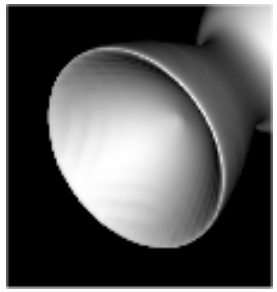


Bloomenthal

Implicit Surfaces







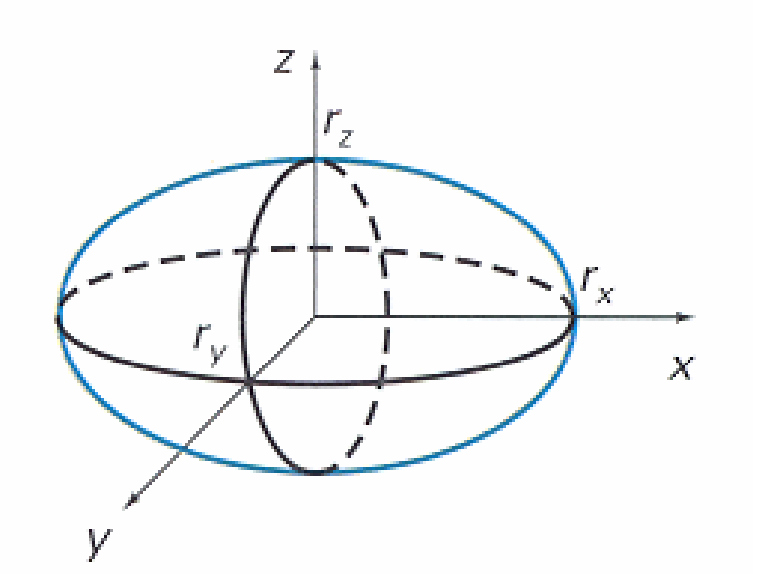
Bourke

Implicit Surface Properties

(1) Efficient check for whether point is inside

- Evaluate f(x,y,z) to see if point is inside/outside/on

$$\left(\frac{x}{r_x}\right)^2+\left(\frac{y}{r_y}\right)^2+\left(\frac{z}{r_z}\right)^2-1=0$$



H&B Figure 10.10

Implicit Surface Properties

(2) Efficient surface intersections

- Substitute to find intersections

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Substituting for P, we get:

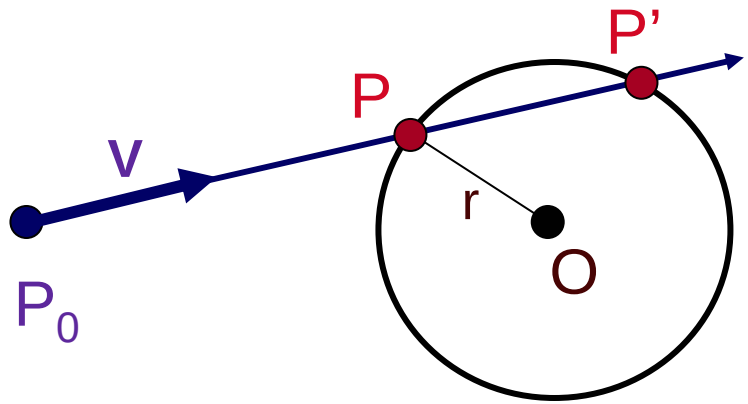
$$|P_0 + tV - O|^2 - r^2 = 0$$

Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

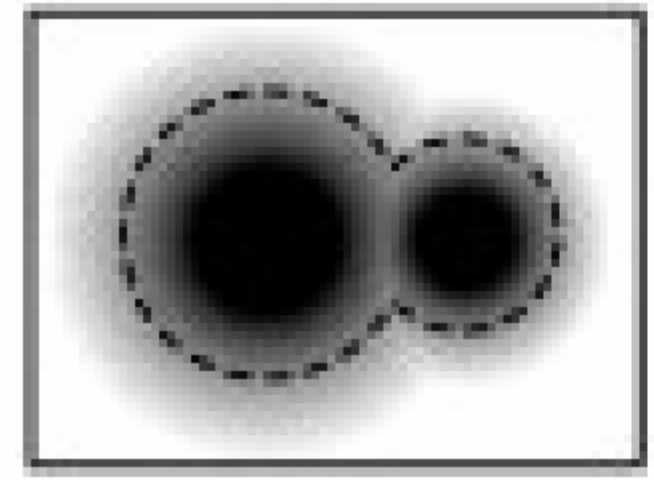
$$a = 1$$
$$b = 2\,V \bullet (P_0 - O)$$
$$c = |P_0 - C|^2 - r^2 = 0$$



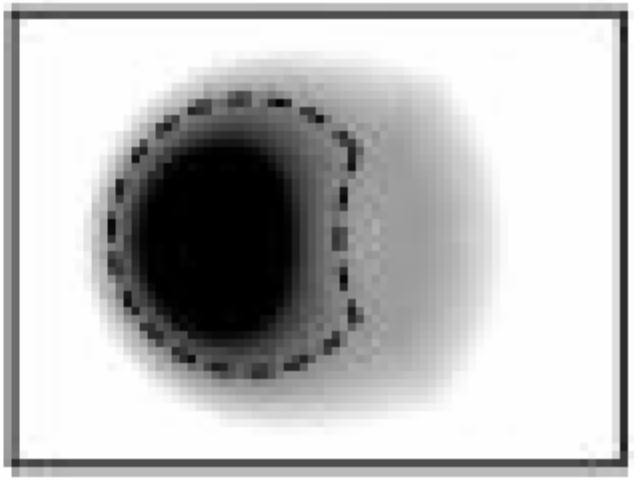
Implicit Surface Properties

(3) Efficient boolean operations (CSG)

- Union, difference, intersect



Union



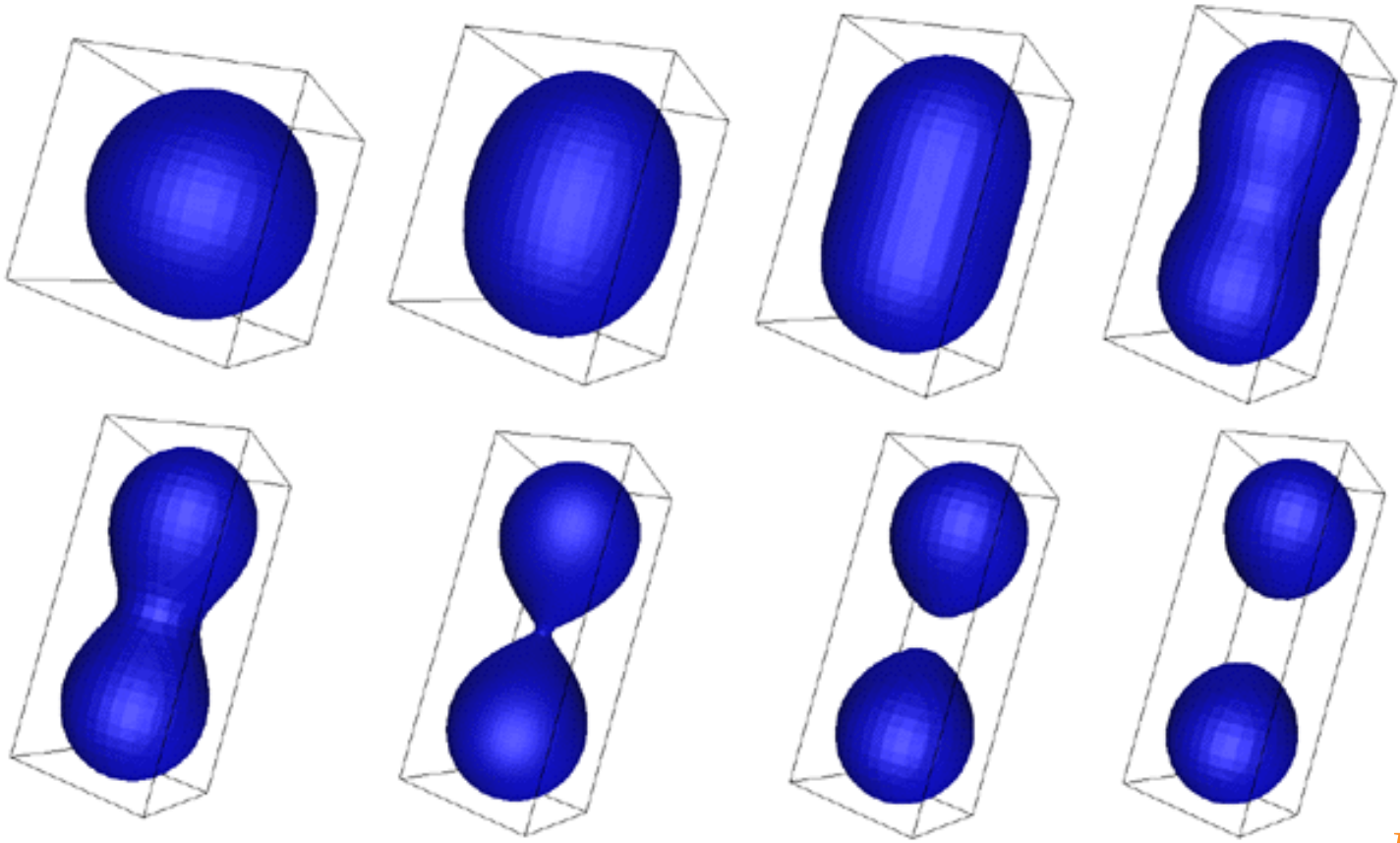
Difference

Bloomenthal

Implicit Surface Properties

(4) Efficient topology changes

- Surface is not represented explicitly!



Bourke

Implicit Surface Properties

(4) Efficient topology changes

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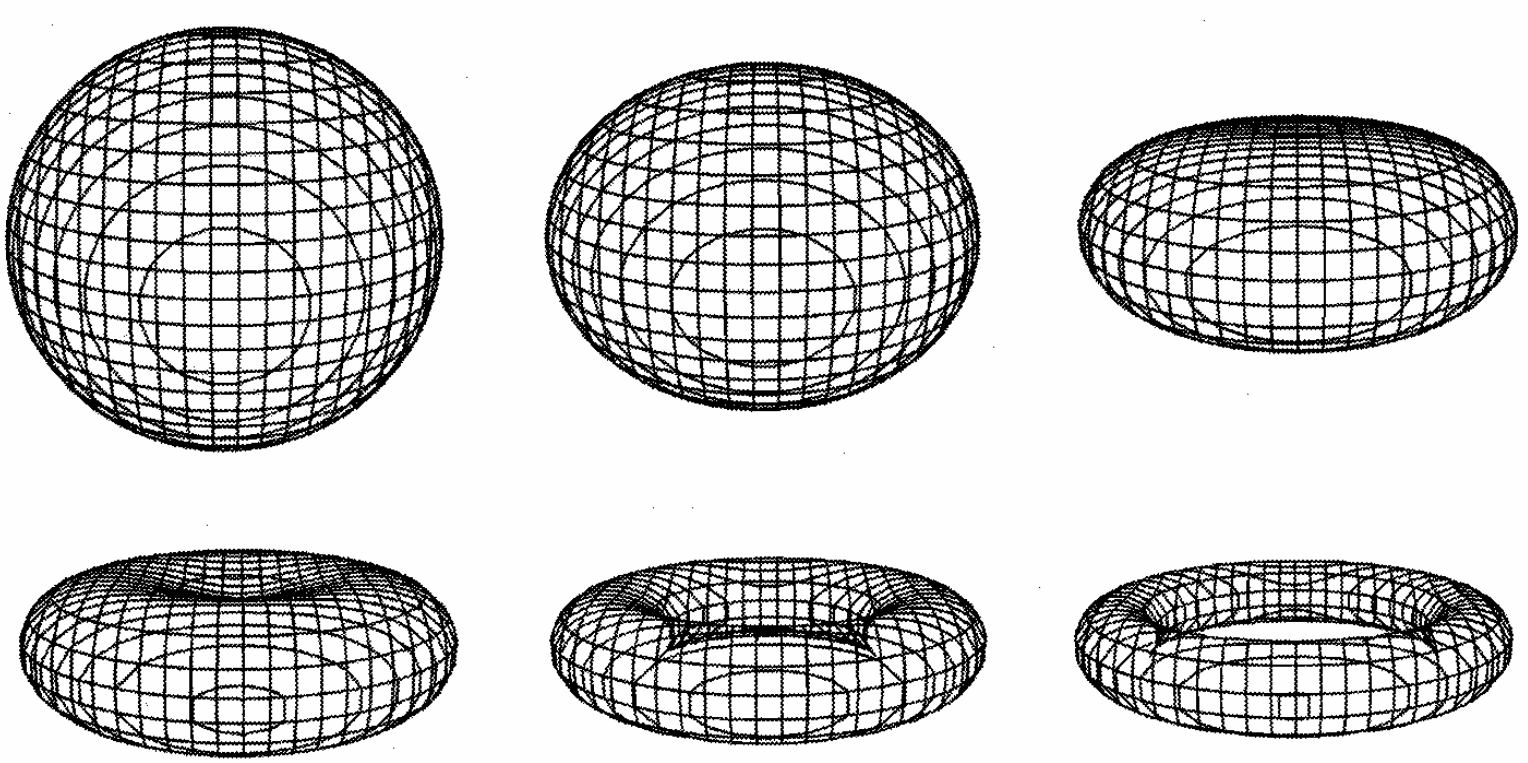
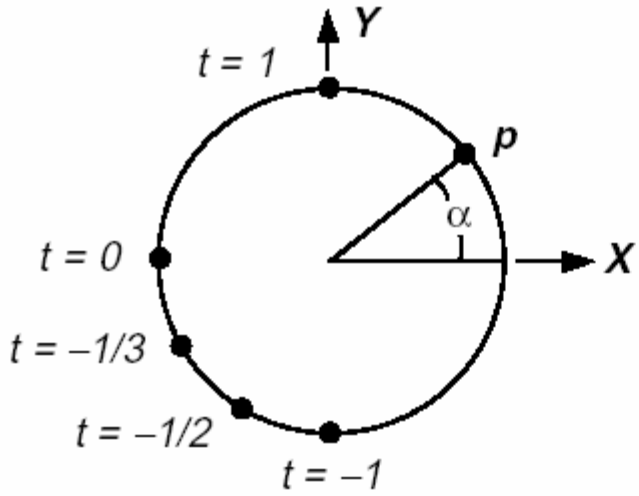


Bloomenthal

Comparison to Parametric Surfaces

- Implicit
  - Efficient intersections & topology changes
- Parametric
  - Efficient “marching” along surface & rendering



equiangular parametric
(transcendental trigonometric)

$\boldsymbol{p} = (\cos(\alpha), \sin(\alpha)), \alpha \in [0, 2\pi]$

non-equiangular parametric (rational)

$\boldsymbol{p} = (\pm(1-t^2)/(1+t^2), 2t/(1+t^2)), t \in [-1, 1]$

implicit

$\boldsymbol{p}_x^2+\boldsymbol{p}_y^2-1 = 0$

Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Blobby models
  - Skeletons
  - Procedural
  - Samples
  - Variational

Implicit Surface Representations

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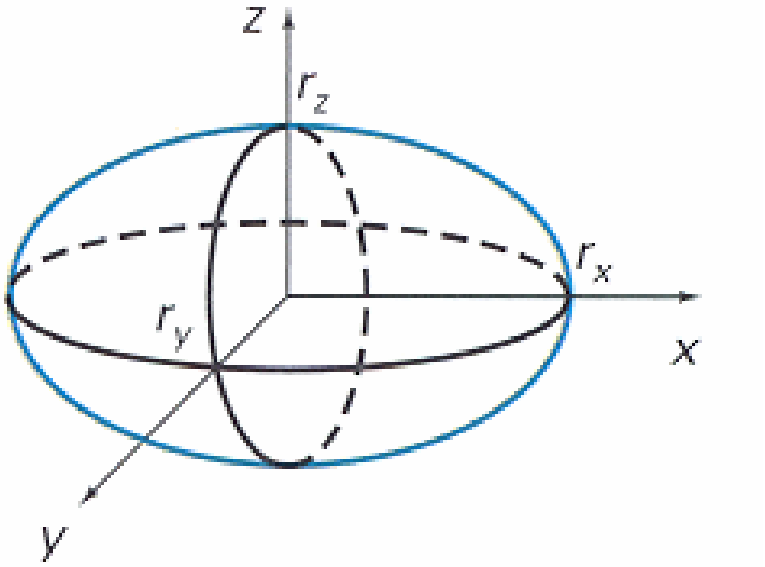
Algebraic Surfaces

- Implicit function is polynomial
  - $f(x,y,z)=ax^d+by^d+cz^d+dx^{d-1}y+dx^{d-1}z+dy^{d-1}x+...$

$$\left(\frac{x}{r_x}\right)^2+\left(\frac{y}{r_y}\right)^2+\left(\frac{z}{r_z}\right)^2-1=0$$



H&B Figure 10.10

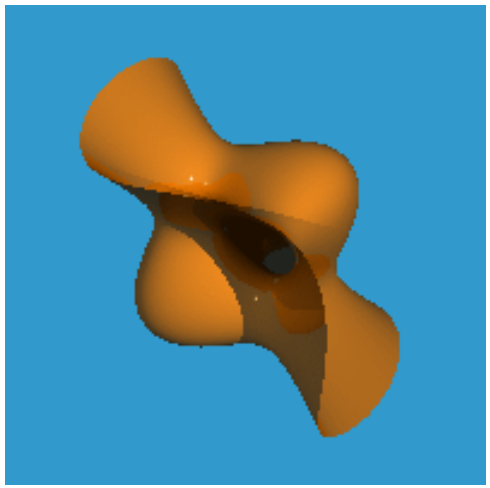
Algebraic Surfaces

- Most common form: quadrics
  - $f(x,y,z)=ax^2+by^2+cz^2+2dxy+2eyz+2fxz+2gx+2hy+2jz+k$

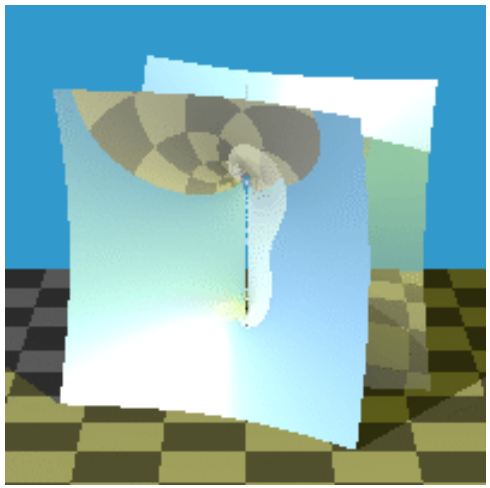
- Examples
  - Sphere
  - Ellipsoid
  - Torus
  - Paraboloid
  - Hyperboloid

Algebraic Surfaces

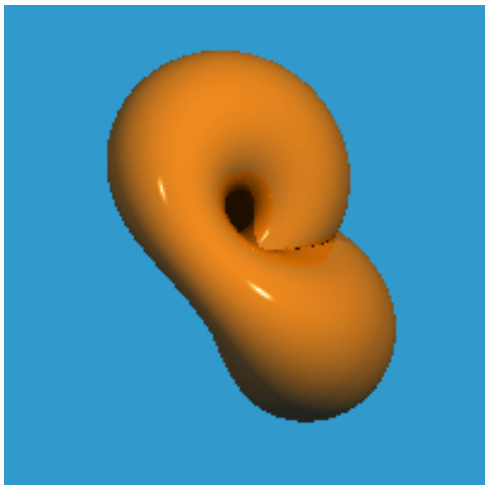
- Higher degree algebraics



Cubic



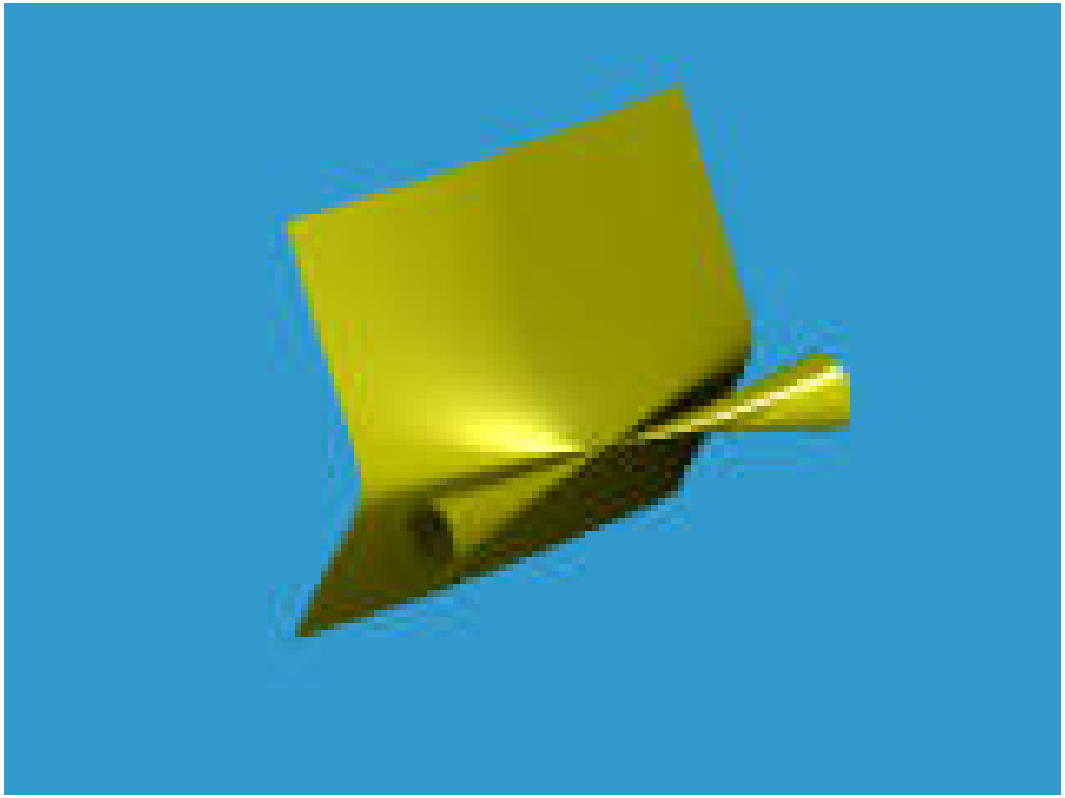
Quartic



Degree six

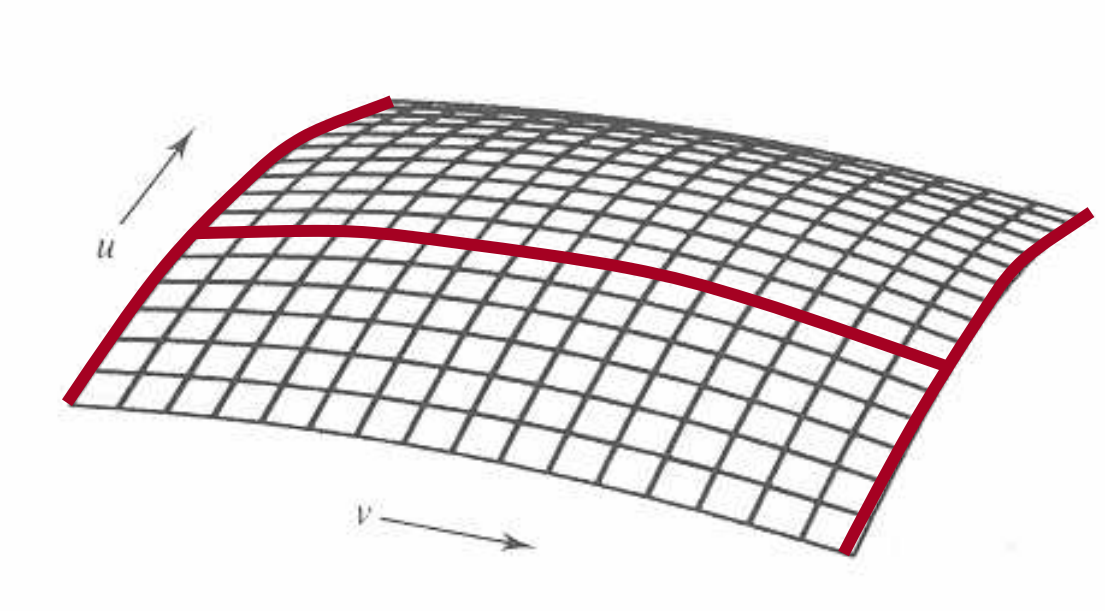
Algebraic Surfaces

- Function extends to infinity
  - Must trim to get desired patch (this is difficult!)



Algebraic Surfaces

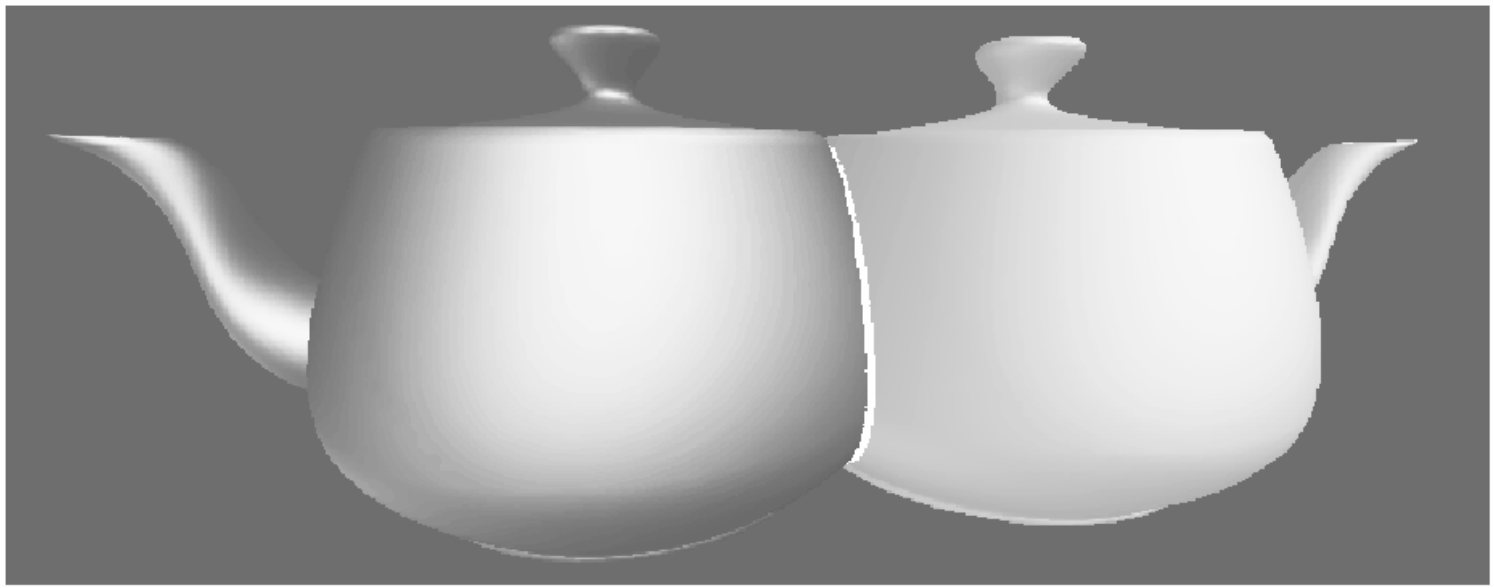
- Equivalent parametric surface
  - Tensor product patch of degree m and n curves yields algebraic function with degree 2mn



Bicubic patch has degree 18!

Algebraic Surfaces

- Intersection
  - Intersection of degree m and n algebraic surfaces yields curve with degree mn



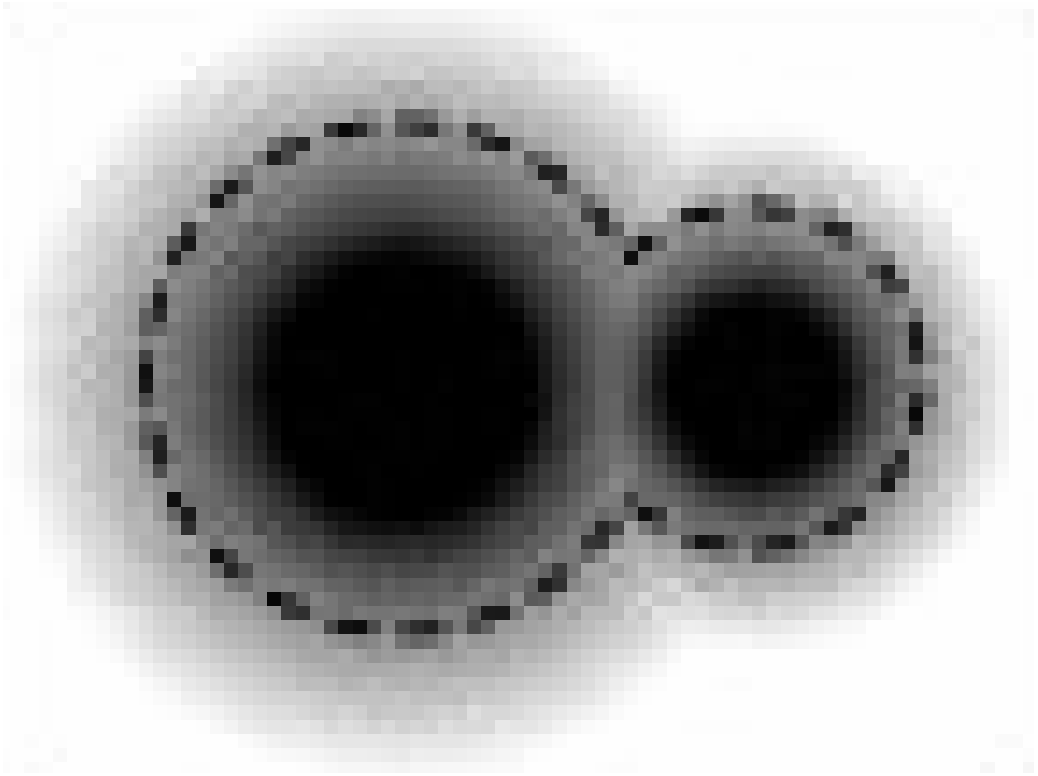
Intersection of bicubic patches has degree 324!

Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - ➢ Blobby models
  - Skeletons
  - Procedural
  - Samples
  - Variational

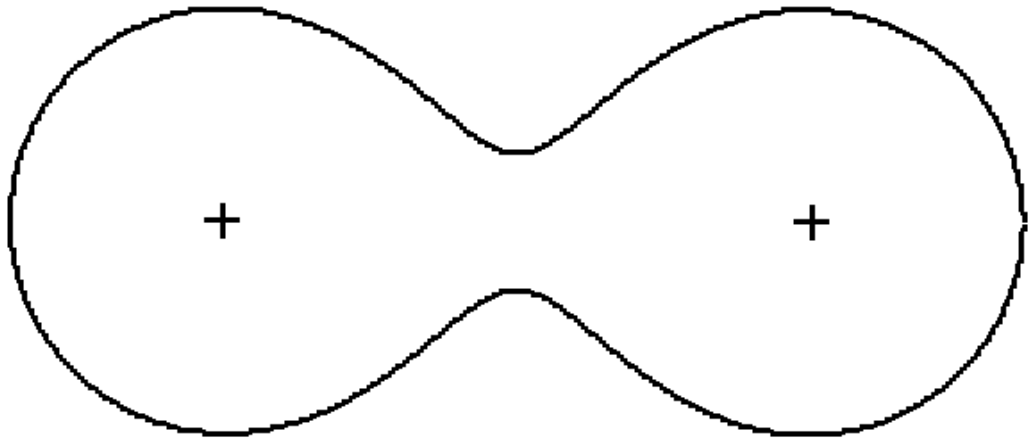
Blobby Models

- Implicit function is sum of spherical basis functions



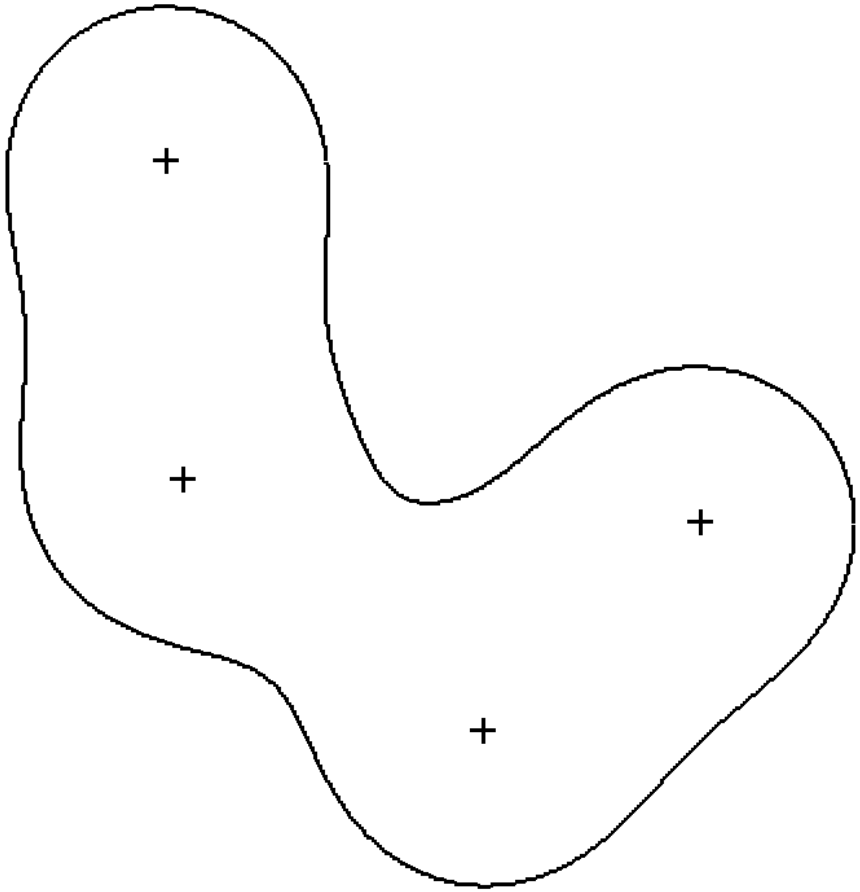
Blobby Models

- Sum of two blobs



Blobby Models

- Sum of four blobs

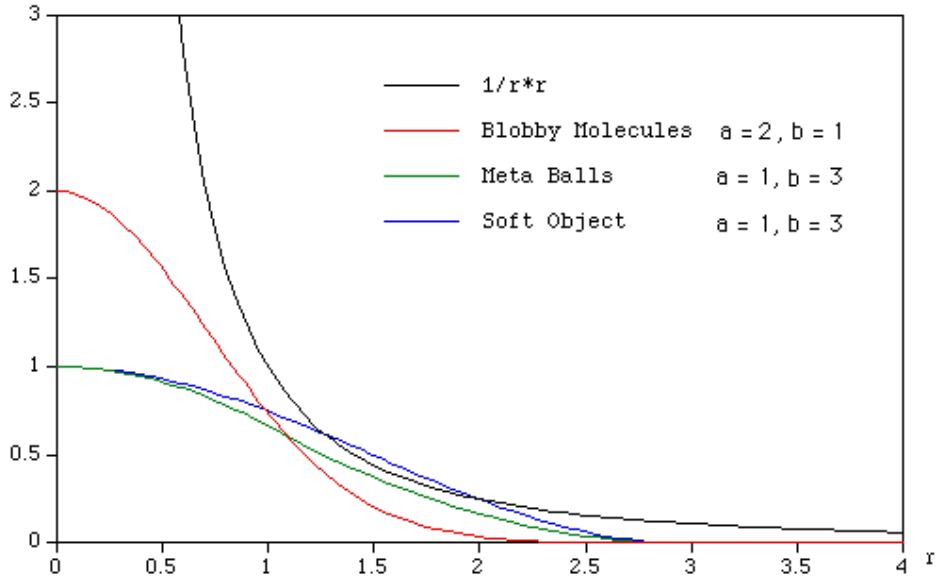


Turk

Blobby Models

- Blobby molecules

$$D(r) = ae^{-br^2}$$



- Meta balls

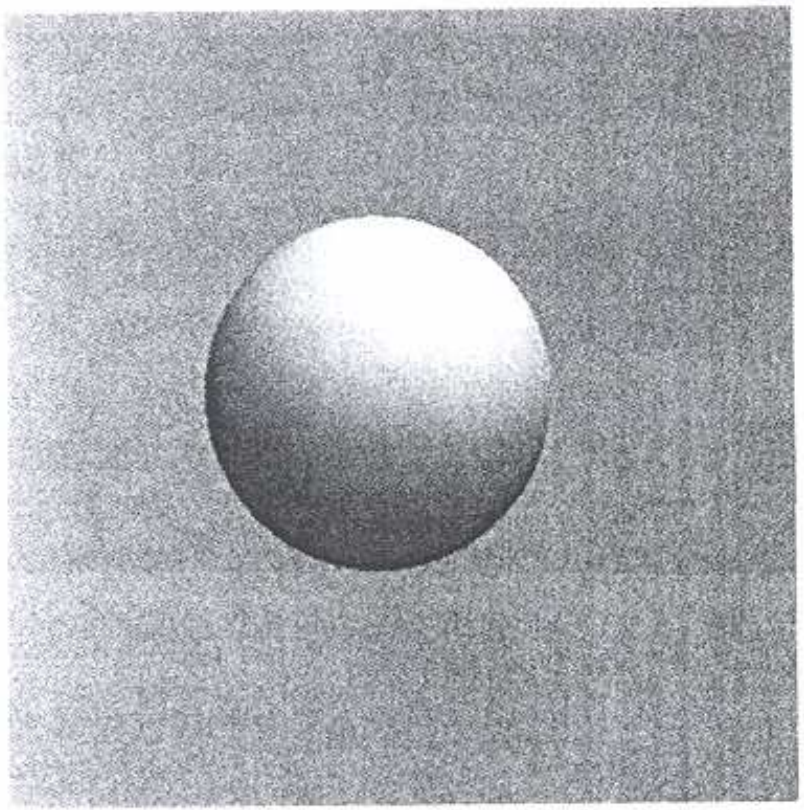
$$D(r) = \begin{cases} a(1-\frac{3r^2}{b^2}) & 0 \le r \le b/3 \\ \frac{3a}{2}(1-\frac{r}{b})^2 & b/3 \le r \le b \\ 0 & b \le r \end{cases}$$

- Soft objects

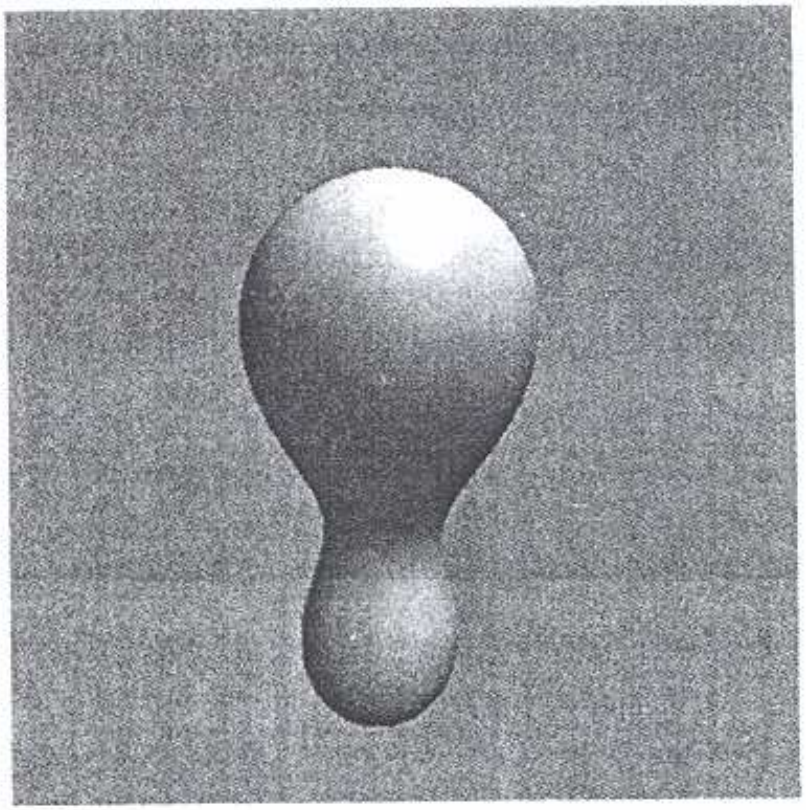
$$D(r) = \begin{cases} a(1-\frac{4r^6}{9b^6}+\frac{17r^4}{9b^4}-\frac{22r^2}{9b^2}) & r \le b \\ 0 & r \ge b \end{cases}$$

Bourke

Blobby Model of Face

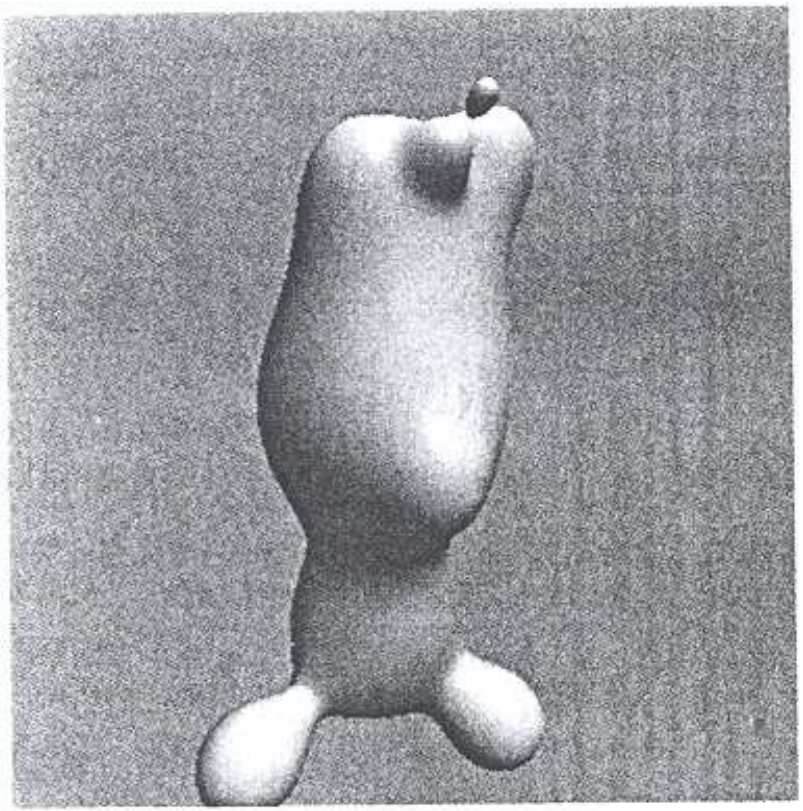


(a) $N = 1$

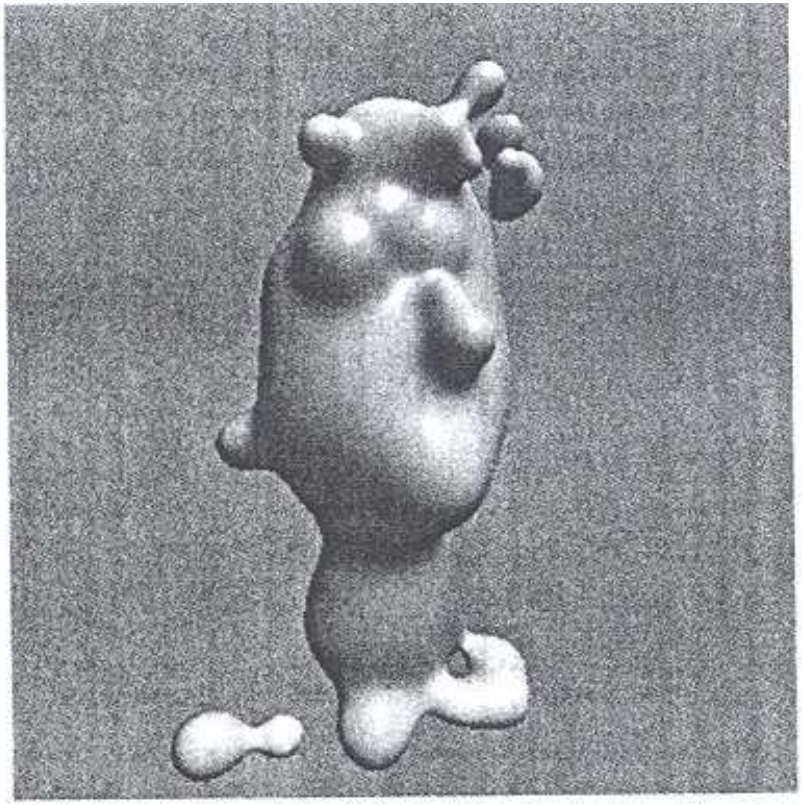


(b) $N = 2$

Blobby Model of Face

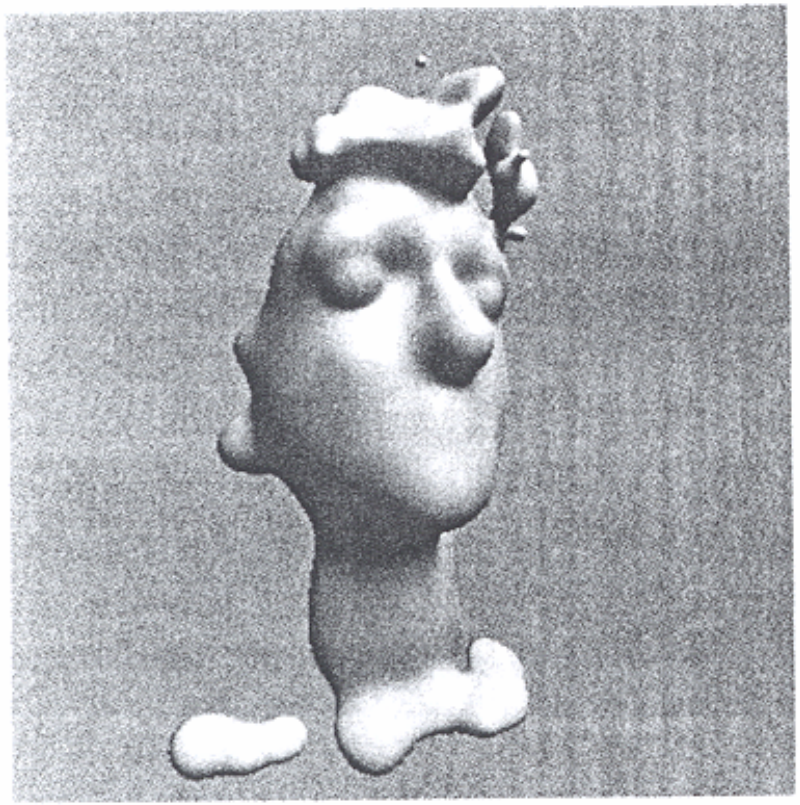


(c) $N = 10$



(d) $N = 35$

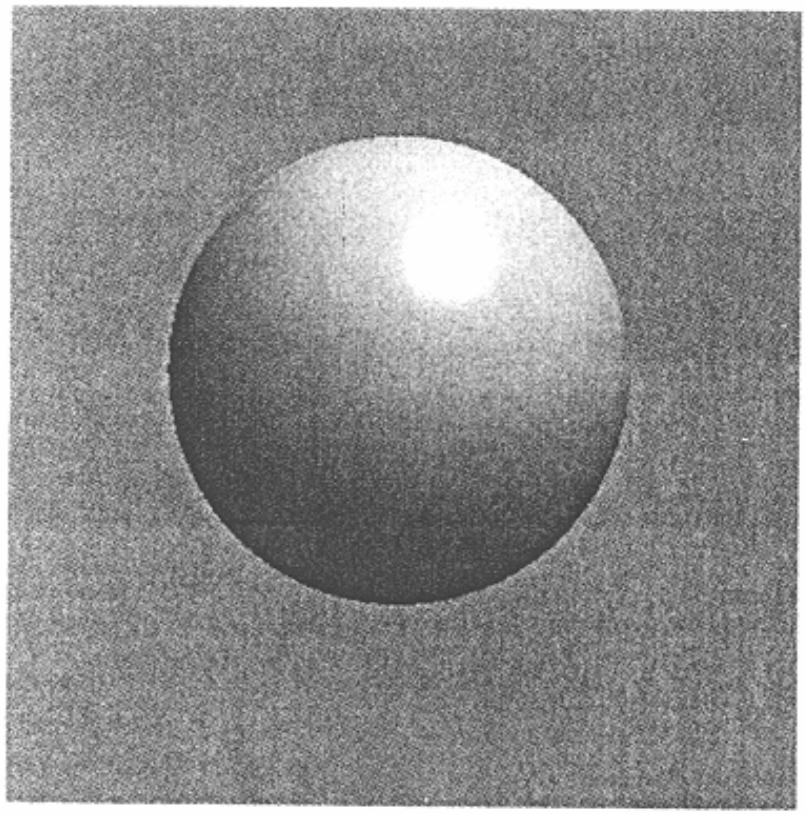
Blobby Model of Face



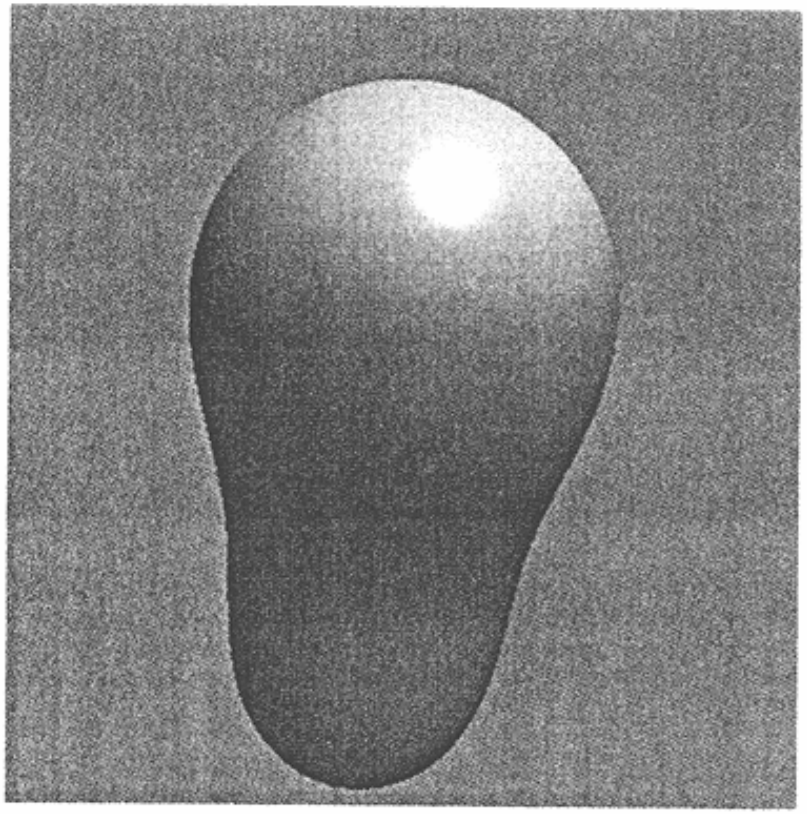
(e) $N = 70$

(f) $N = 243$

Blobby Model of Head

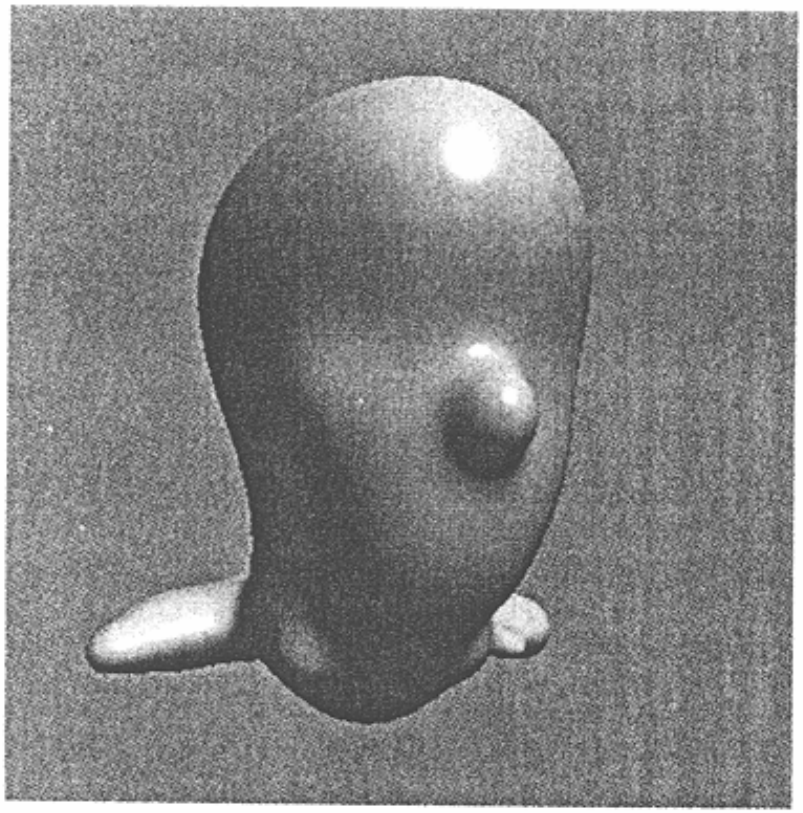


(a) $N = 1$

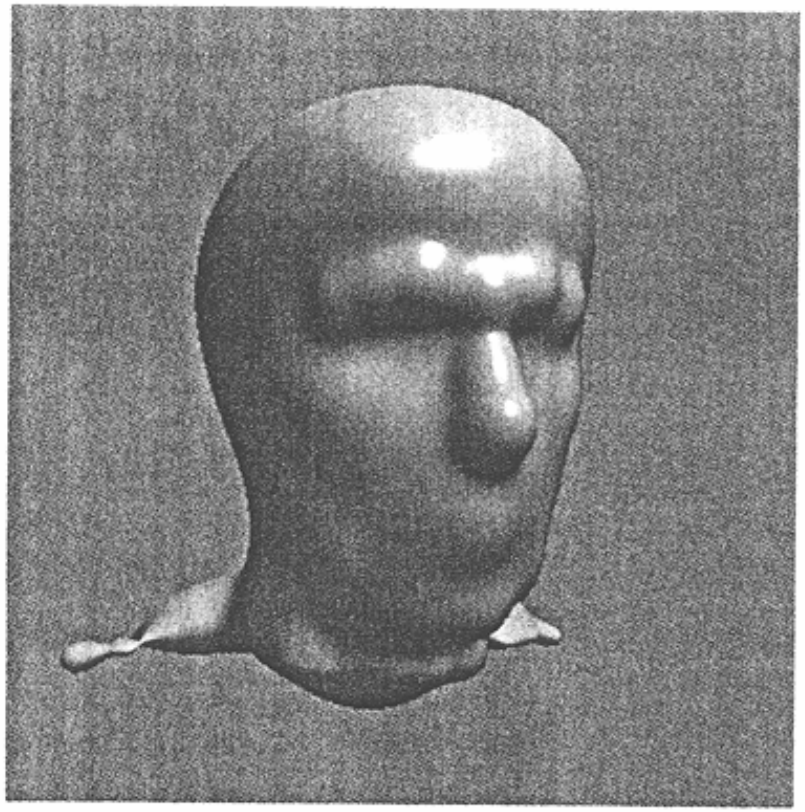


(b) $N = 2$

Blobby Model of Head

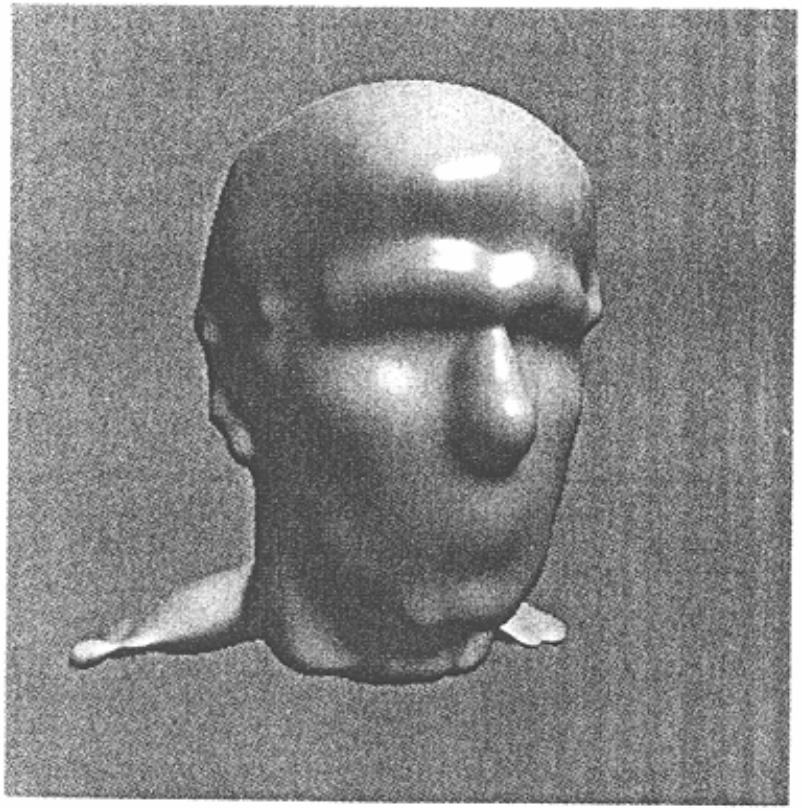


(c) $N = 20$

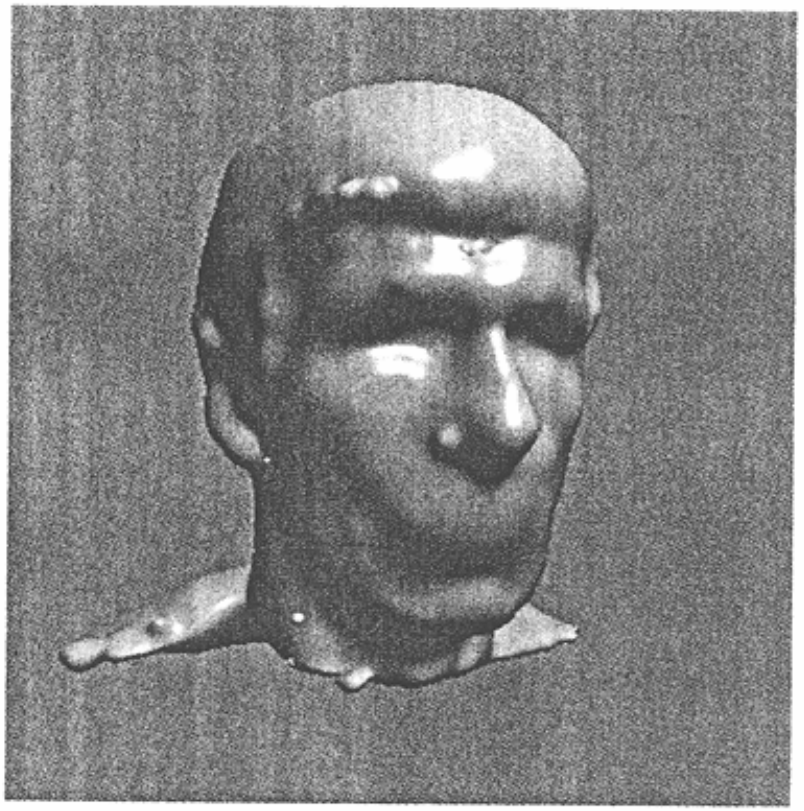


(d) $N = 60$

Blobby Model of Head



(e) $N = 120$



(f) $N = 451$

Blobby Models

Objects resulting from CSG of
implicit soft objects and other primitives

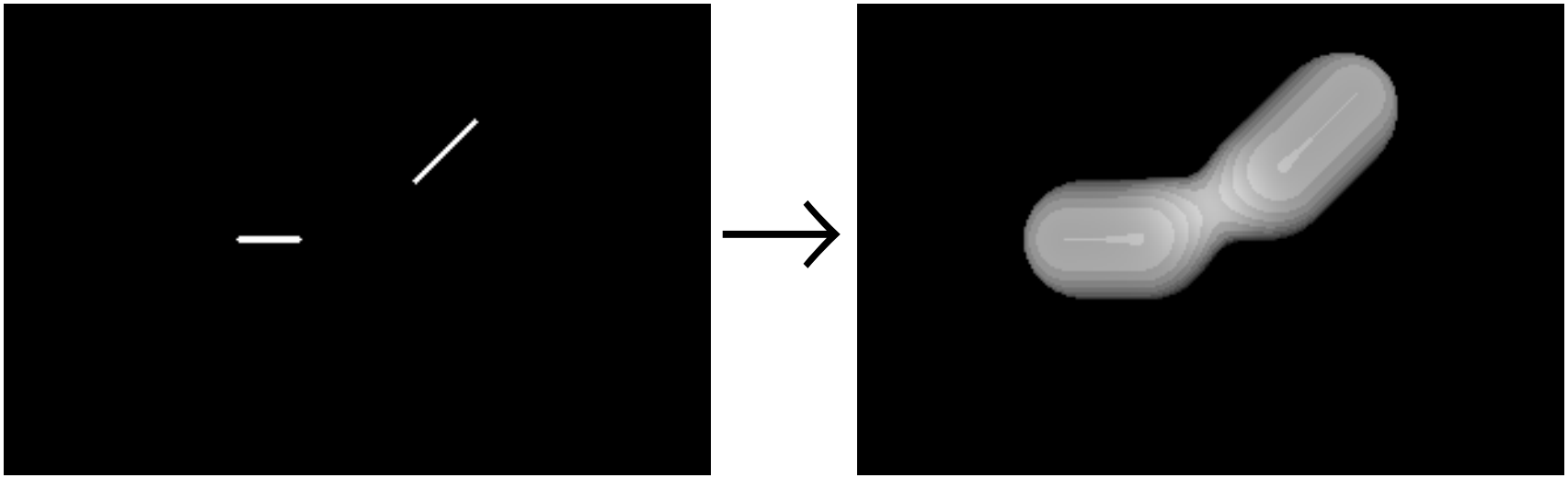
Menon

Implicit Surface Representations

- How do we define implicit function?
  - Algebraics
  - Blobby models
  - ➢ Skeletons
  - Procedural
  - Samples
  - Variational

Skeletons

- Bulge problem



Skeletons

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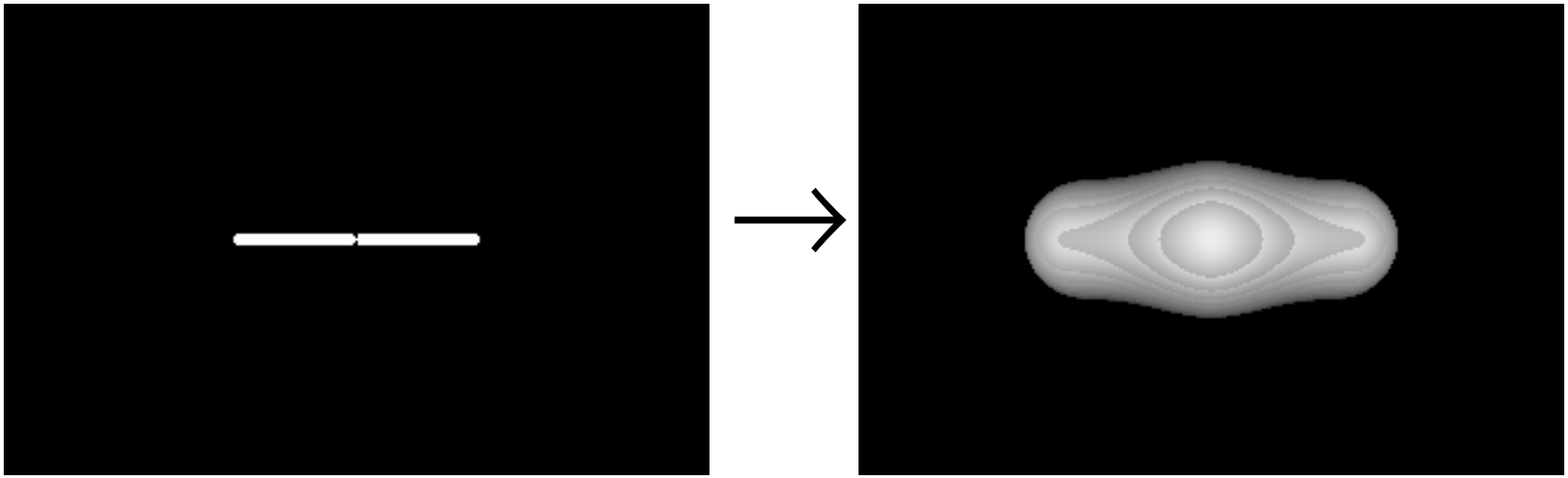
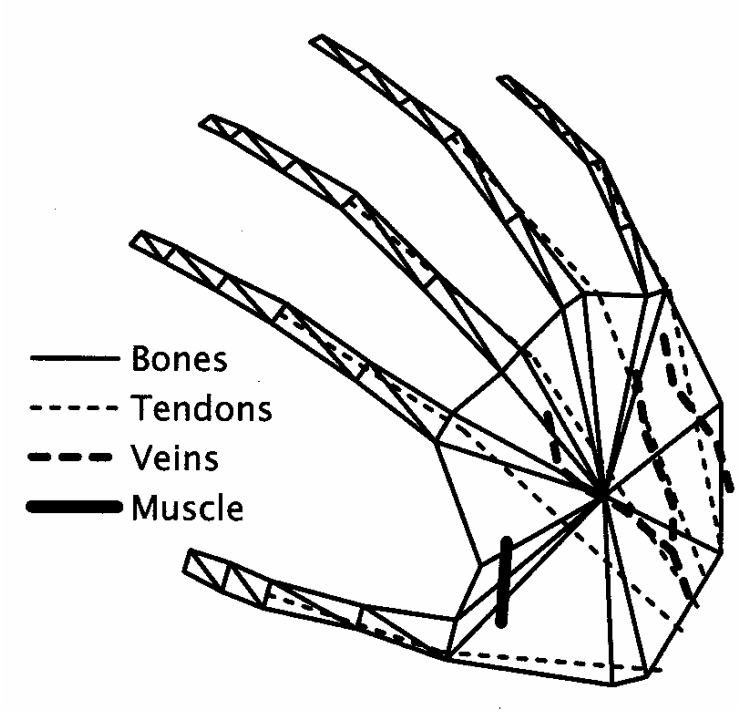


Skeletons

- Convolution surfaces



Bloomenthal

Implicit Surface Representations

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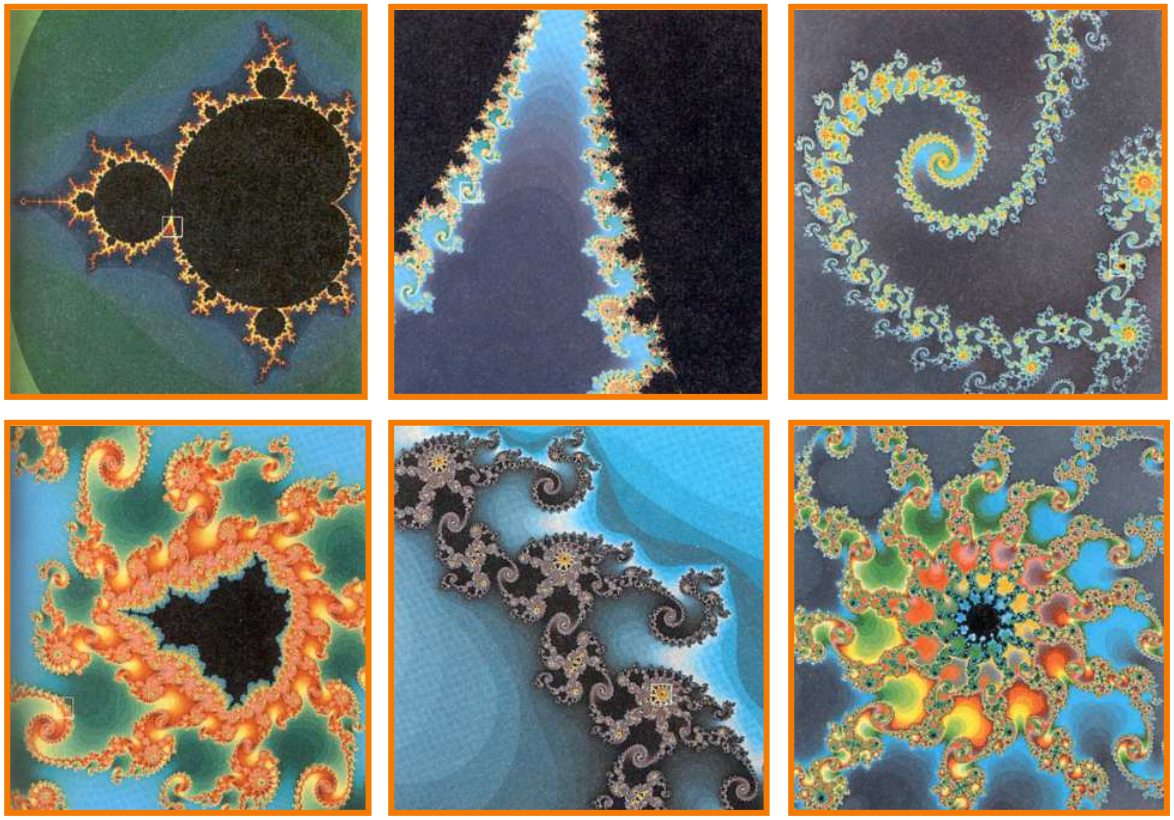
Procedural Implicits

- f(x,y,z) is result of procedure
  - Example: Mandelbrot set



H&B Figure 10.100

Implicit Surface Representations

- How do we define implicit function?
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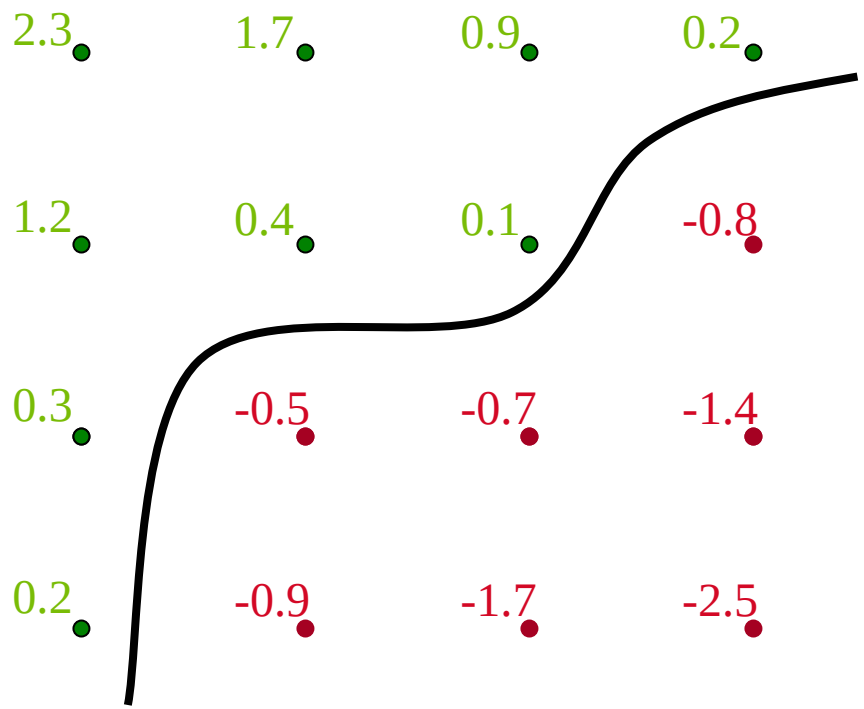
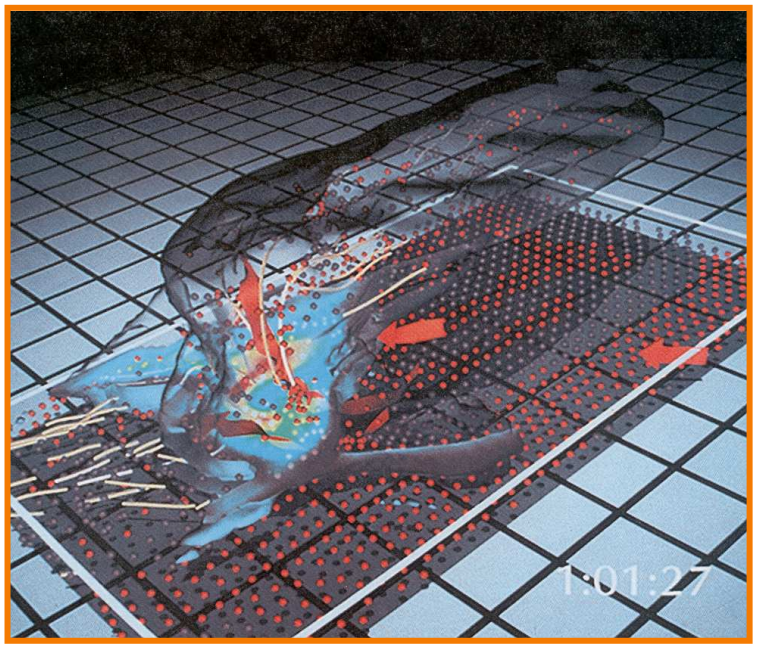
Sampled Functions

- Most common example: voxels
  - Interpolate samples stored on regular grid
  - Isosurface at f(x,y,z) =0 defines surface



Sampled Functions

- Acquired from simulations or scans



Airflow Inside a Thunderstorm
(Bob Wilhelmson,
University of Illinois at Urbana-Champaign)

Visible Human
(National Library of Medicine)

Implicit Surface Representations

- How do we define implicit function?
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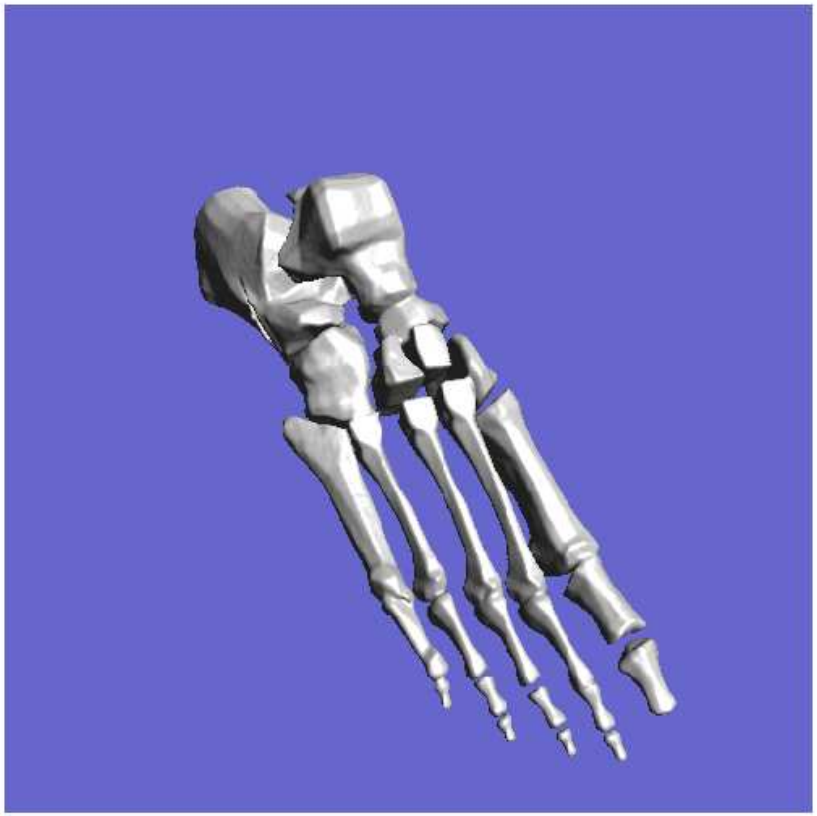
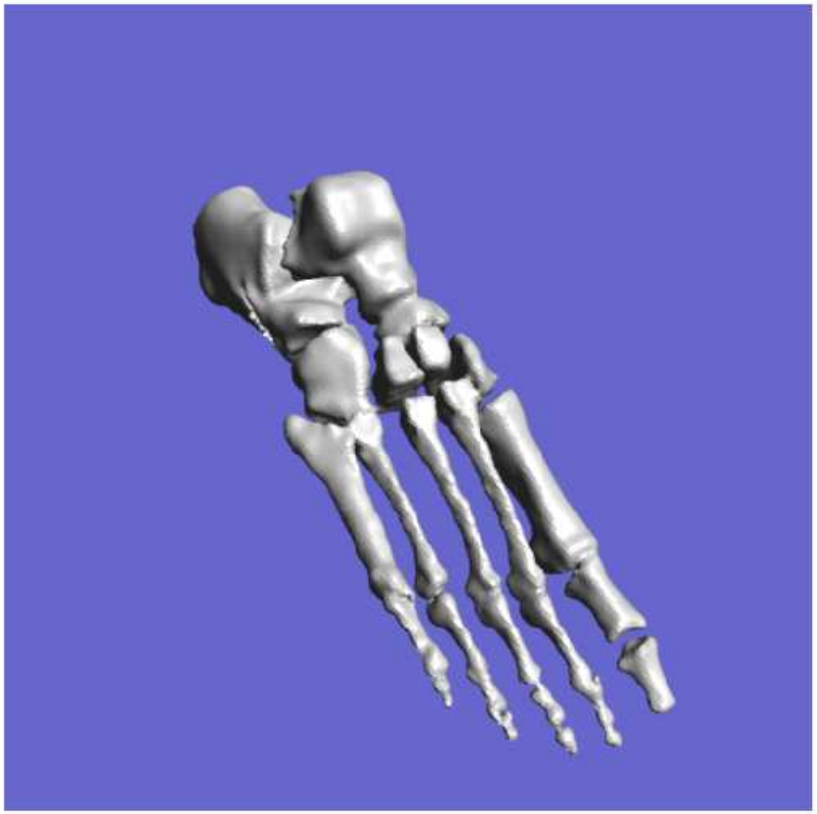
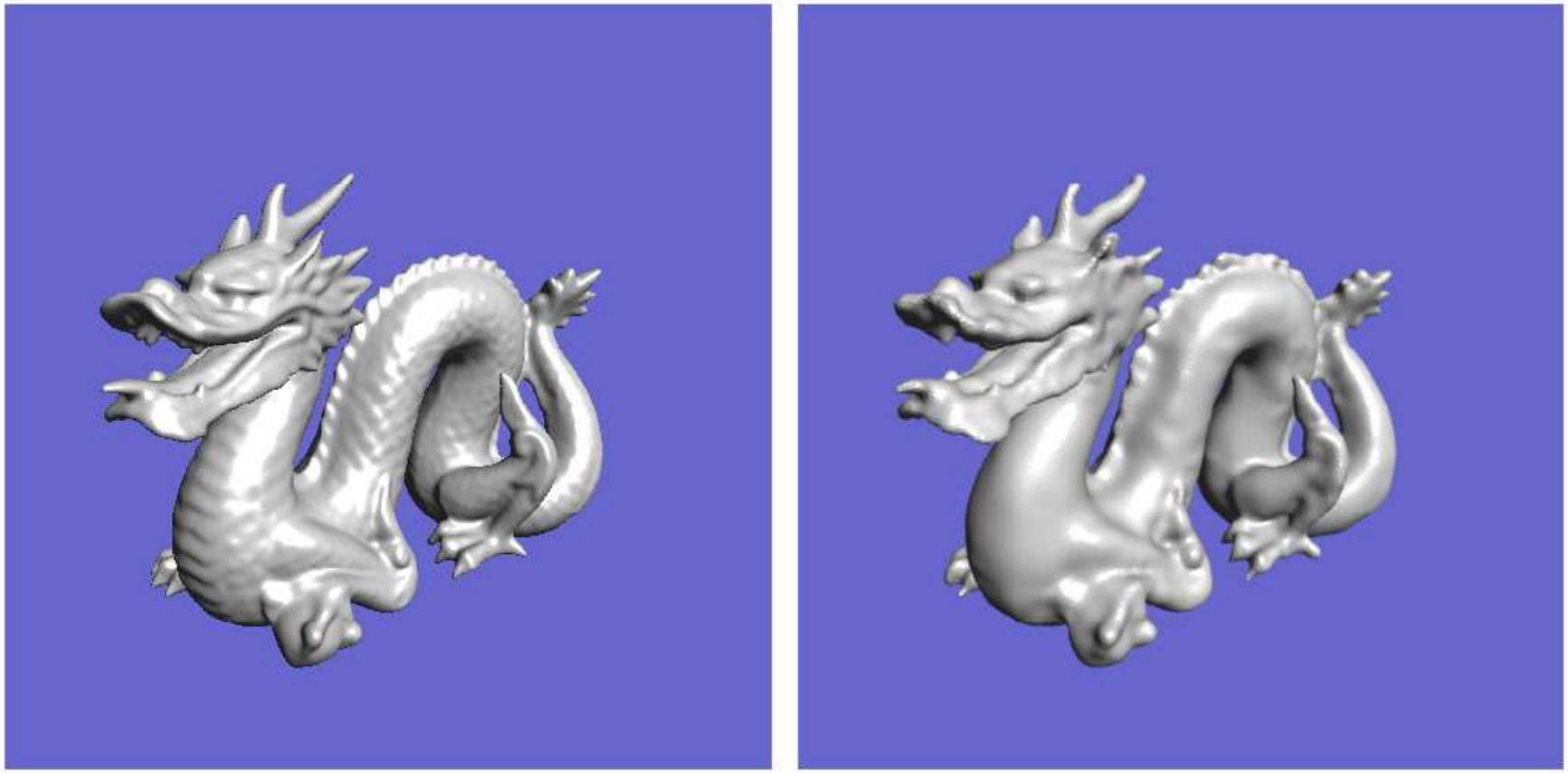
Variational Implicit Surfaces





Bloomenthal

Variational Implicit Surfaces



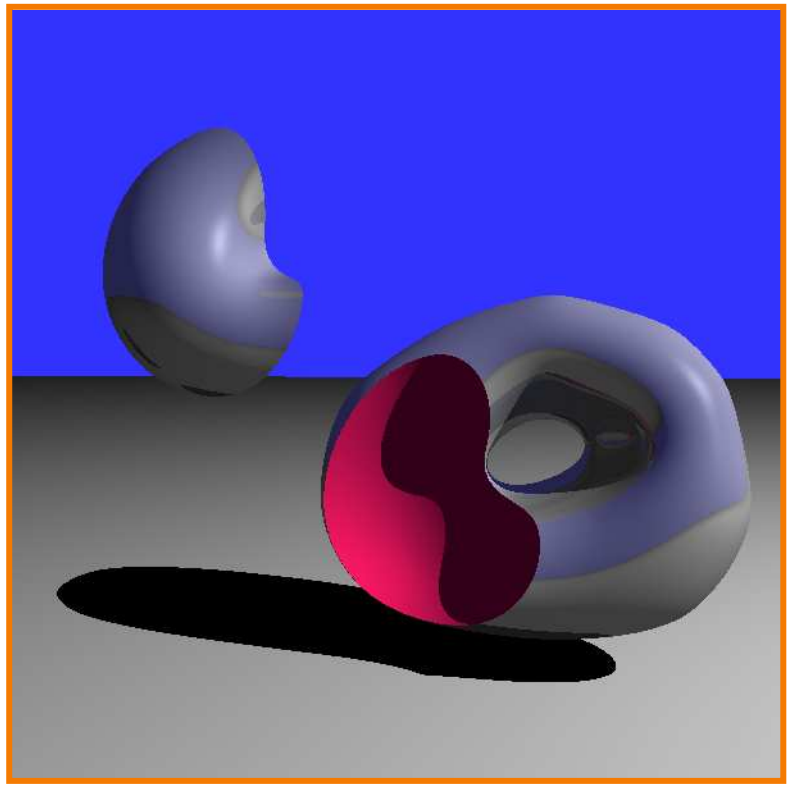
Bloomenthal

Implicit Surface Summary

- Advantages:
  - Easy to test if point is on surface
  - Easy to compute intersections/unions/differences
  - Easy to handle topological changes
- Disadvantages:
  - Indirect specification of surface
  - Hard to describe sharp features
  - Hard to enumerate points on surface
    - » Slow rendering

Rendering Implicit Surfaces

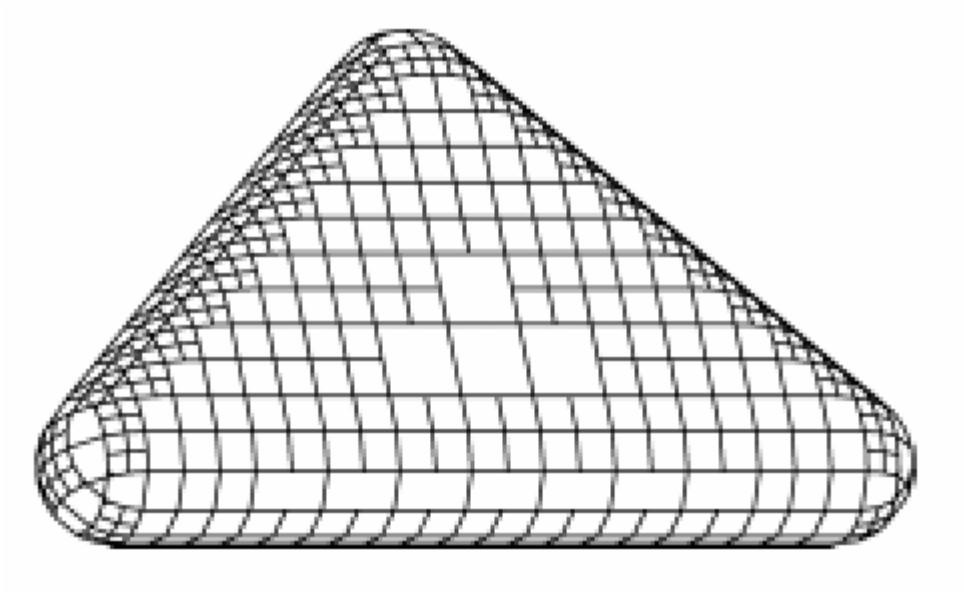
- How do we render images of implicit surfaces?
  - Polygonization
  - Ray tracing
  - Contours
  - Floating particles



Turk

Rendering with Polygons

- Polygonization is not always easy



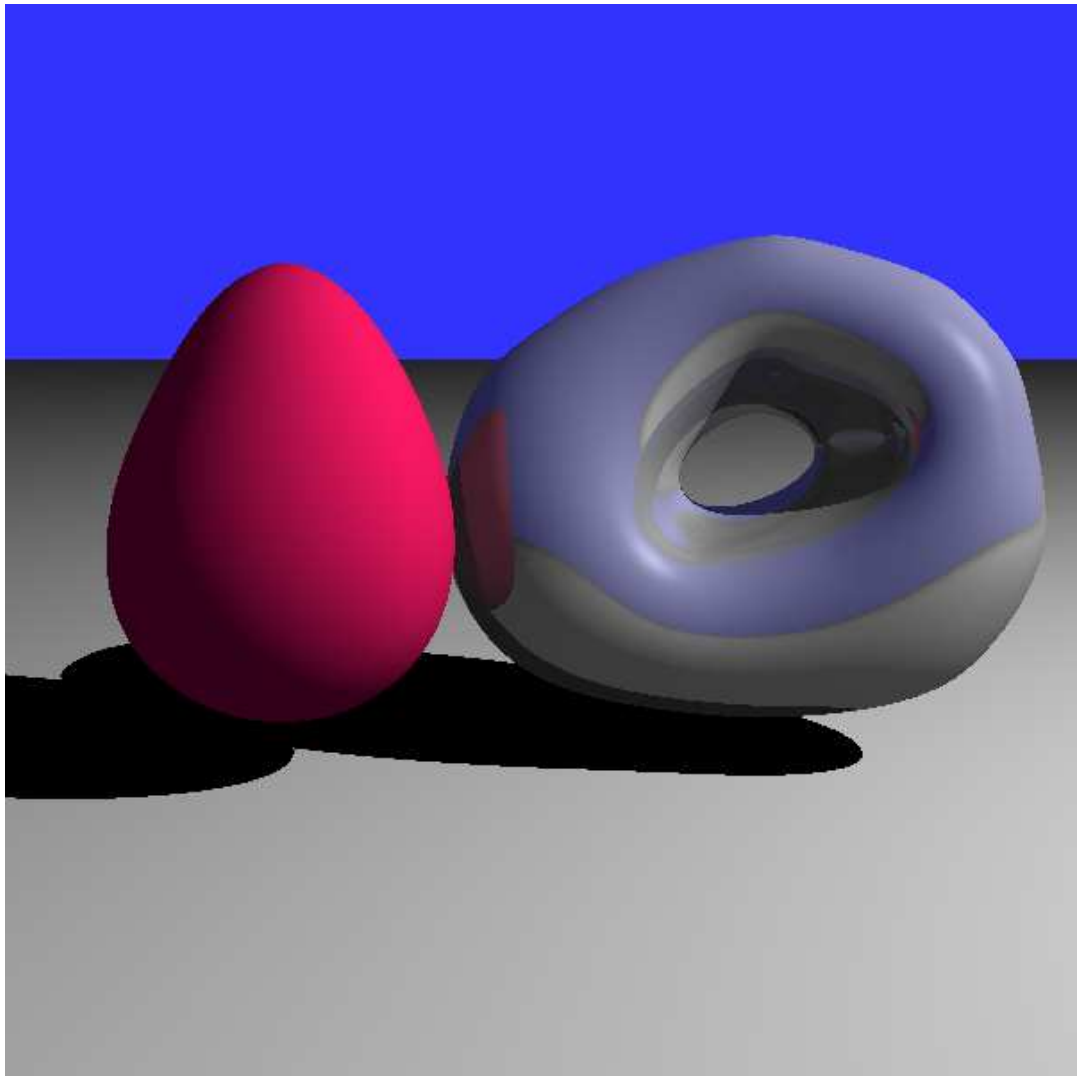
Adaptive Polygonalization

Bloomenthal

Marching Cubes

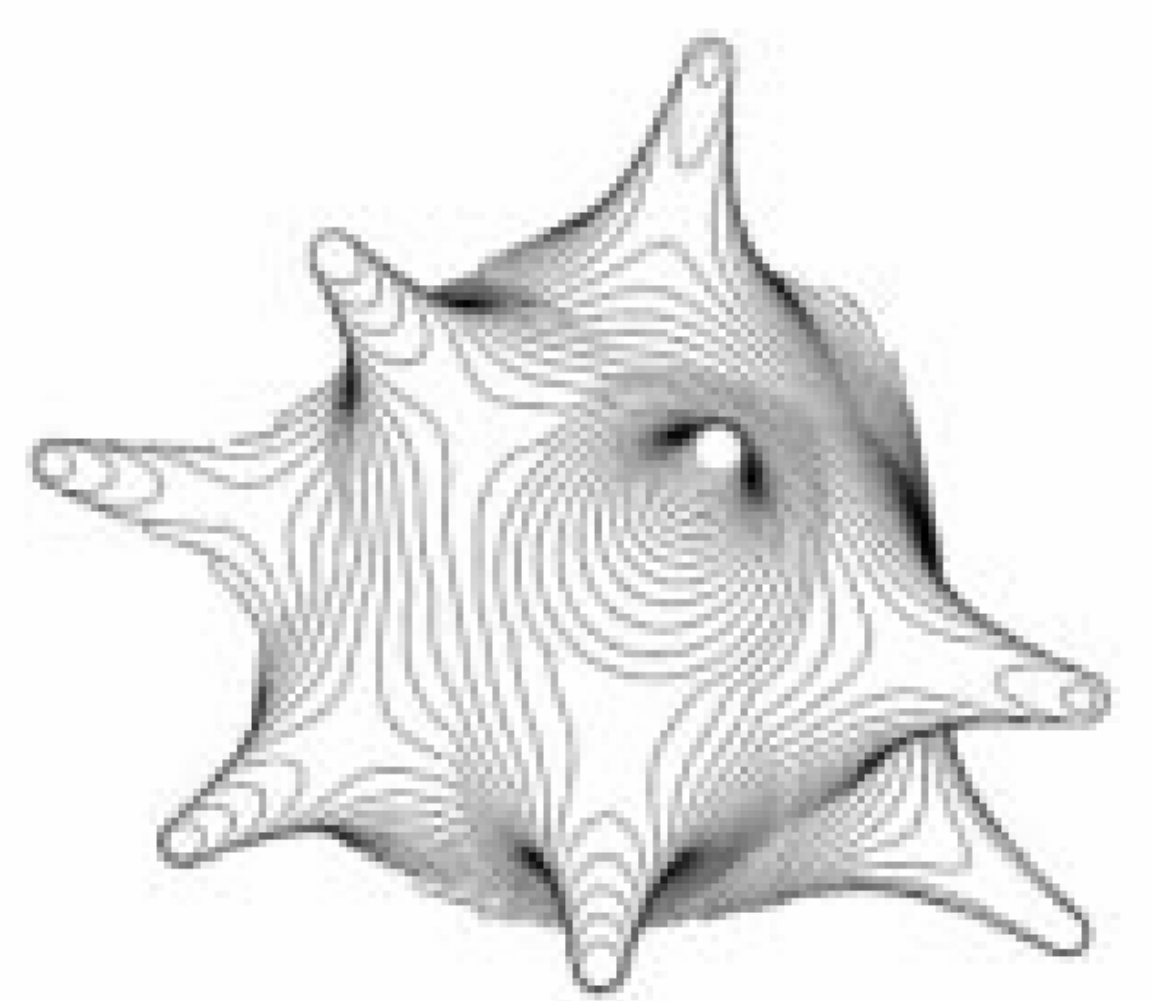
Lorensen

Rendering with Ray Tracing



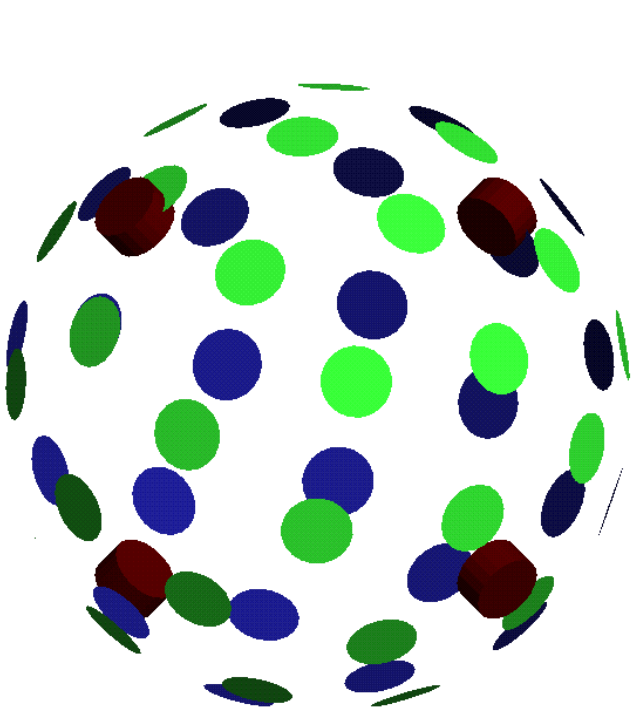
Turk

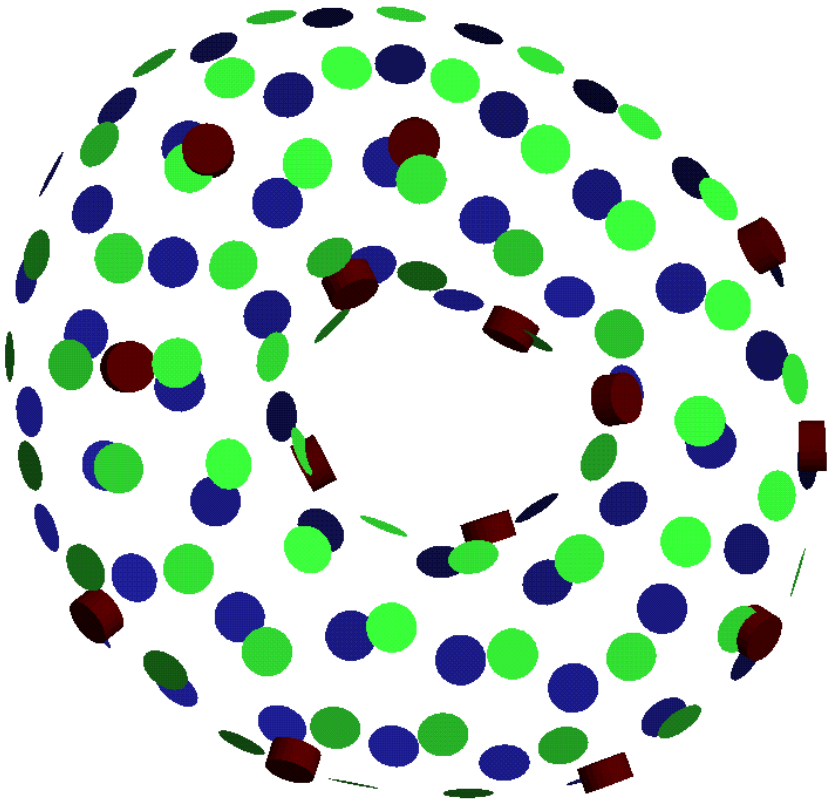
Rendering with Contours



Bloomenthal

Rendering with Floating Particles





Turk

Summary

Feature	Polygonal Mesh	Implicit Surface	Parametric Surface	Subdivision Surface
Accurate	No	Yes	Yes	Yes
Concise	No	Yes	Yes	Yes
Intuitive specification	No	No	Yes	No
Local support	Yes	No	Yes	Yes
Affine invariant	Yes	Yes	Yes	Yes
Arbitrary topology	Yes	Yes	No	Yes
Guaranteed continuity	No	Yes	Yes	Yes
Natural parameterization	No	No	Yes	No
Efficient display	Yes	No	Yes	Yes
Efficient intersections	No	Yes	No	No