Photon mapping: diffuse scattering

Christopher Twigg

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1 Introduction

This is intended to be a derivation of the scaling term we should use when estimating diffuse-diffuse interreflection in the scene. Let eye be the eye, x be the surface poin that the eye ray hits, let $\vec{\omega}_i$ be an outgoing vector we have selected from the cosine-weighted probability distribution, and let y_i be the surface that these hit (as in figure 1).

2 Derivation

We want to compute

$$I(x, x \to \text{eye}) = \int_{\Omega} f_r(x, \vec{\omega}_i, x \to \text{eye}) L(x, \vec{\omega}_i)(n \cdot \vec{\omega}_i)\, d\vec{\omega}_i \tag{1}$$

We can rewrite this integral as

$$I(x, x \to \text{eye}) = \int_{\Omega} h(\vec{\omega}_i) p(\vec{\omega}_i)\, d\vec{\omega}_i \tag{2}$$

where $h(\vec{\omega}_i) = \pi f_r(x, \vec{\omega}_i, x \to \text{eye}) L(x, \vec{\omega}_i)$ and $p(\vec{\omega}_i) = \frac{1}{\pi}(n \cdot \vec{\omega}_i)$ (the $\frac{1}{\pi}$ term is a necessary normalizing constant to make $p(\vec{\omega}_i)$ into a proper distribution). We are computing this integral using the expectation

$$I(x, x \to \text{eye}) = \mathbb{E}_p(h(\vec{\omega}_i)) \tag{3}$$

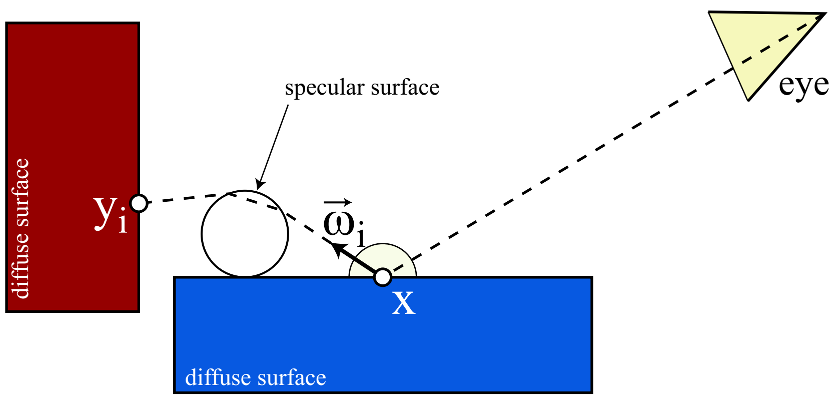

Figure 1: Sampling the diffuse-diffuse interreflection

which we can estimate using

$$\widehat{I}(x, x \to \text{eye}) = \frac{1}{N} \sum_{i=1}^{N} h(\vec{\omega}_i) \tag{4}$$

(the $\widehat{I}$ means this is an estimate of the actual quantity I). Now, recall our $h(\vec{\omega}_i)$ is

$$h(\vec{\omega}_i) = \pi f_r(x, \vec{\omega}_i, x \to \text{eye}) L(x, \vec{\omega}_i) \tag{5}$$

Since the surface is diffuse, we can simply swap in the diffuse BRDF,

$$f_r(x, \vec{\omega}_i, x \to \text{eye}) = \frac{1}{\pi} k_d \tag{6}$$

to make this

$$h(\vec{\omega}_i) = k_d L(x, \vec{\omega}_i) \tag{7}$$

Recall that $L(x, \vec{\omega}_i)$ is the radiance returned from the diffuse ray we have cast into the scene, and this gives us the final equation for diffuse-diffuse interreflection,

$$\widehat{I}(x, x \to \text{eye}) = \frac{k_d}{N} \sum_{i=1}^{N} L(x, \vec{\omega}_i) \tag{8}$$

Of course, if we are doing path tracing we use $N = 1$ and let the number of initial ray casts smooth out this quantity.