Rules: Work in groups of 3. Submit one set of solutions per group, and remember to put all your names on it. We'd prefer if you typed your answers, but if your handwriting is neat and legible, written work is fine. You may refer to whatever sources you want, but *CITE* your references! If you get a great idea from somebody, give appropriate credit: it won't lower your grade, and it's the right thing to do. If you have any questions or errors to report, email `ryanw@cs.cmu.edu`.

(This homework is due March 31st.)

1 True-False Questions on Planar Graphs.

Justify your answers with a proof/counter-example.

We say that G is 2-edge-connected if every cut of G has at least two edges crossing it.

1. (True or False) If G is planar and 2-edge-connected, then the two faces bordering an edge are distinct.

2. (True or False) Let H be a subgraph of planar graph G. Suppose you embed H in the plane in any way you like. Then H can be extended to a planar embedding of G.

3. (True or False) Let G be planar, and C be a cycle of G. Then G can be embedded into in the plane, with C as the outer cycle (i.e. it borders the outer face).

2 Treaping Out.

Let $E_{n,1}$ be the expected distance from the root to the first node (the leftmost node) in a treap. Give an exact expression for $E_{n,1}$.

3 Vertex Cover on Planar Graphs.

A *vertex cover* of G is a subset S of its vertices such that, for all edges $\{u, v\}$ in G, either $u \in S$ or $v \in S$. That is, the vertex subset "covers" all edges of G.

Finding a vertex cover of minimum cardinality is a canonical NP-hard problem. Even for planar graphs, it is NP-hard. It can be approximately solved rather well; this problem will be about *exact* algorithms for minimum vertex cover.

The best known exact algorithms for minimum vertex cover use $O(2^{cn})$ time in the worst case, where c is a constant less than 1. However, planarity permits us a much better running time:

Give an algorithm for solving minimum vertex cover on planar graphs that runs in $O(2^{c\sqrt{n}})$ time, for some constant $c > 0$.

Your proof of correctness should be pretty straightforward, so a sketch is sufficient. However, you should prove (using a recurrence relation, perhaps) that the runtime is indeed bounded by $2^{c\sqrt{n}}$.