15-750 Algorithms (Spring 2002) **Prof. Manuel Blum**

Homework #7 **due: March 22 (by 11:59pm in WeH 4126)**

Rules: as before.

Problem 1: Graph coloring (20 points)

(this is Problem 6.5 from M&R; you should solve it <u>without</u> using the reference given in the book)

Let G be a 3-colorable graph. Consider the following algorithm for coloring the vertices of G with 2 colors, so that no triangle of G is monochromatic. The algorithm begins with an arbitrary 2-coloring of G. While there is a monochromatic triangle in G, it chooses one such triangle, and changes the color of a randomly chosen vertex of that triangle. Derive an upper bound on the expected number of such re-coloring steps before the algorithm finds a 2-coloring with the desired property.

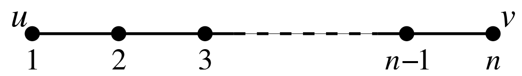
Problem 2: A long directed random walk (10 points)

(this is Problem 6.16 from M&R)

Show that the expected time for a random walk to visit every vertex of a strongly connected directed graph is not bounded above by any polynomial function of n, the number of vertices. In other words, construct a directed graph that is strongly connected and where the expected cover time is super-polynomial.

Problem 3: Take a walk on a ... straight line (20 points)

(a) What is the *exact* value for the expected time for a random walk to go from u to v, where u and v are opposite ends of the n-node straight line graph:



Compute this quantity both directly and using the method of resistive networks. Run a simulation to determine the expected time empirically: perform a series of experiments for $n = 11$ with $t = 1, 10, 100, \ldots$ trials (where each trial is a complete random walk from u to v), and take the arithmetic mean from each experiment. How many trials do you need to get the mean within 1% of the exact computed value (i.e. two decimal digits correct)?

(b) What is the *exact* value for the expected time for a random walk to go from u to v, where u and v are adjacent vertices of the 2-way infinite straight line graph:

Compute this quantity however you like. Run a simulation, as in part (a). What do you observe? Understand and explain.

(please turn over)

Problem 4: A "universal" universal sequence (extra credit)

(this problem is due to Grant Reaber)

Recall from the lecture on universal sequences, that for each n there exists a universal sequence of length polynomial in n, which traverses every labeled graph[1] with n vertices.

Is it true, that there exists a single (infinite) universal sequence S such that for each n the initial portion of S, of length polynomial in n, traverses every labeled, connected graph with n vertices. Prove your answer.

[1] The graphs considered are connected, undirected, without self-loops or multiple edges.