15-750 Algorithms (Spring 2002) **Prof. Manuel Blum**

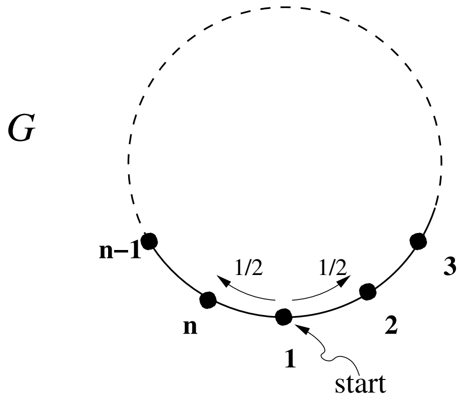
Homework #6 **due: March 15 (by 11:59pm in WeH 4126)**

Rules: as before.

Problem 1: Random Walk on a Cycle (20 points)

(this problem is due to Avrim Blum)

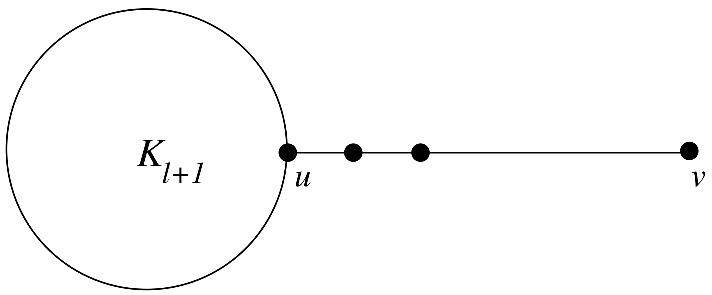
Let G denote graph which is a cycle on n vertices (see Figure below). View G as a Markov Chain, i.e. consider a random walk on G starting at vertex #1, where with probability 1/2 you move to the node's left neighbor, and with probability 1/2 to the right neighbor.



Which vertex in G is the most likely to be the last one reached in this random walk? Justify your answer. (Clearly, it cannot be vertex #1, since it is reached initially.)

Problem 2: Lollipop Graphs (30 points)

Consider a lollipop graph with $n = 2l+1$ vertices (K_{l+1} denotes a complete graph with $l+1$ vertices):



(a) Determine h_{uv} (hitting time) and c_{uv} (commute time) for u, v as marked on the figure, for $n = 5, 7, 2l+1$, using the method of Markov Chains [Section 6.3 in Motwani&Raghavan]

(b) Same as part (a), but use the method of of Electric Networks [Section 6.4 in Motwani&Raghavan]