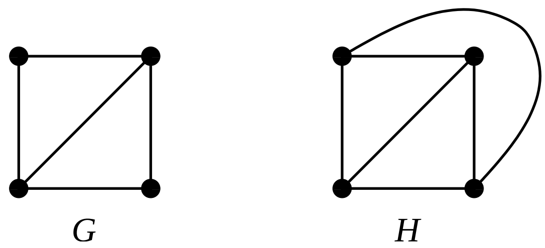
15-750 Algorithms (Spring 2002) **Prof. Manuel Blum**

Homework #4 **due: Feb. 15 (by 11:59pm in WeH 4126)**

Problem 1: Coloring outerplanar graphs (20 points)

A graph is called *outerplanar* if it has an embedding in the plane, such that every vertex lies on the unbounded face. For example, in the following figure graph G is outerplanar, but H is not (although both G and H are planar).



A *k-coloring* of a graph $G = (V, E)$ is a labeling $c : V \to \{1, \ldots, k\}$ (think of the labels as *colors*). A k-coloring is *proper* if $(u, v) \in E$ implies $c(u) \neq c(v)$. We say that a graph G is *k-colorable* if it has a proper k-coloring.

Is it true, that every outerplanar graph is 3-colorable? If no, give a counterexample; if yes, prove it and give an algorithm which in linear time finds a proper 3-coloring of any outerplanar graph G.

Problem 2: Height of treaps (30 points)

In lecture #10 we calculated for an element m of a treap the expected length of the path from the root down to m, where the expectation was taken over all random priority assignments.

In this problem we are interested in the expected *height* of a random treap, where the *height of a treap* is defined as the length of the longest path from the root to a leaf (in particular, the height of a treap with only one element is 0).

(a) Calculate the exact expected height of a treap with n = 2, 3, 4 and 5 elements.

(b) Give fairly tight bounds on the expected height in the general case.

Assume that the priorities are (distinct) integers $\{1, \ldots, n\}$, and take the expectation over all $n!$ permutations of priorities.