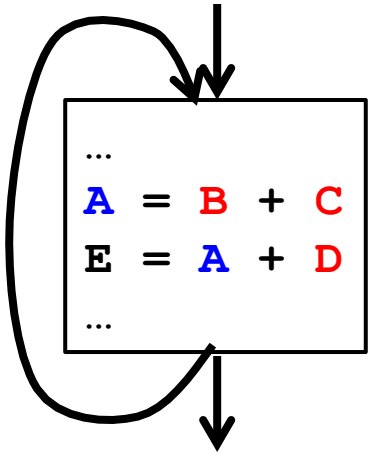
Introduction to

Static Single Assignment (SSA)

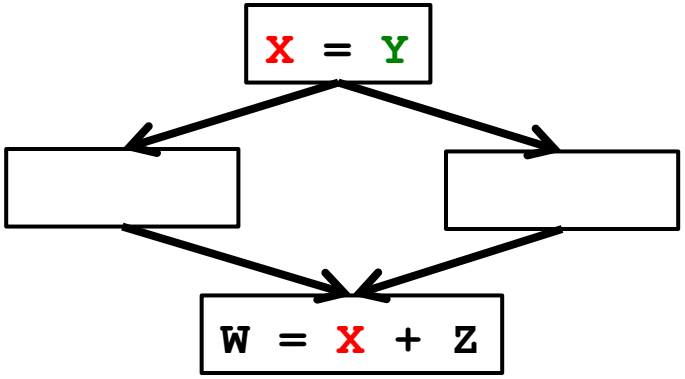
(Slide content courtesy of Seth Goldstein.)

Recurring Theme: Where Is a Variable Defined or Used?

- Example: Loop-Invariant Code Motion
  - Are **B**, **C**, and **D** only defined outside the loop?
  - Other definitions of **A** inside the loop?
  - Uses of **A** inside the loop?

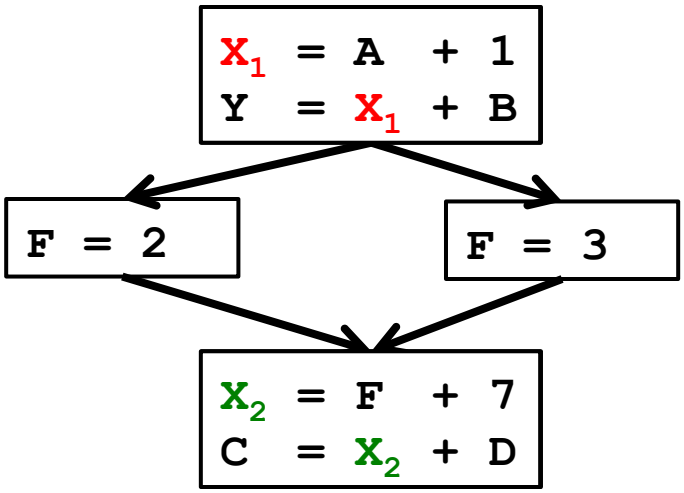


- Example: Copy Propagation
  - For a given use of **X**:
    - Are all reaching definitions of **X**:
      - copies from same variable: e.g., **X = Y**
    - Where **Y** is not redefined since that copy?
  - If so, substitute use of **X** with use of **Y**



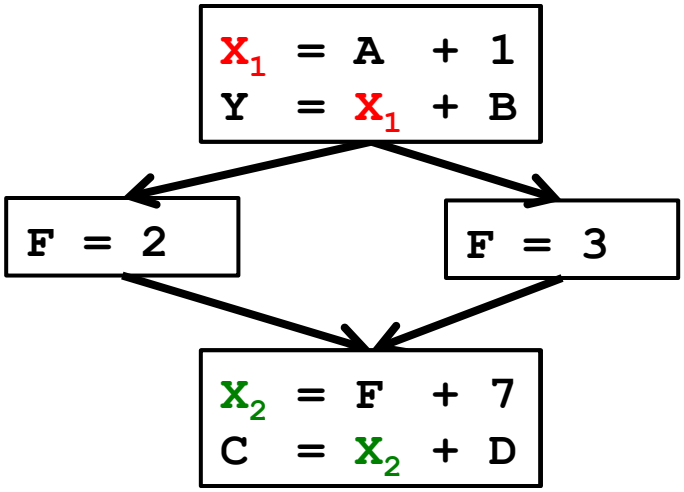
- It would be nice if we could *traverse directly* between related uses and def's
  - this would enable a form of *sparse* code analysis (skip over "don't care" cases)

Appearances of Same Variable Name May Be Unrelated



- The values in reused storage locations may be provably independent
  - in which case the compiler can optimize them as separate values
- Compiler could use renaming to make these different versions more explicit

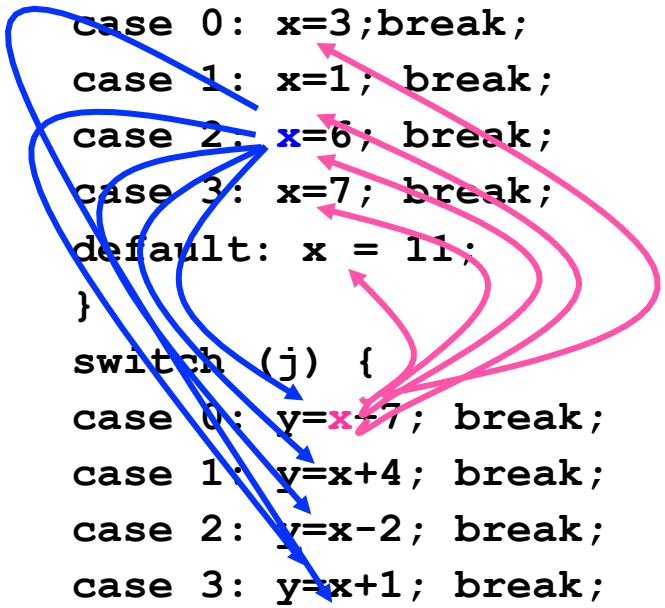
Definition-Use and Use-Definition Chains



- Use-Definition (UD) Chains:
  - for a given definition of a variable X, what are all of its uses?
- Definition-Use (DU) Chains:
  - for a given use of a variable X, what are all of the reaching definitions of X?

Unfortunately DU and UD Chains Can Be Expensive

```
foo(int i, int j) {
   …
   switch (i) {
   case 0: x=3;break;
   case 1: x=1; break;
   case 2: x=6; break;
   case 3: x=7; break;
   default: x = 11;
   }
   switch (j) {
   case 0: y=x-7; break;
   case 1: y=x+4; break;
   case 2: y=x-2; break;
   case 3: y=x+1; break;
   default: y=x+9;
   }
   …
```



In general,

N defs

M uses

⇒ O(NM) space and time

One solution: limit each variable to ONE definition site

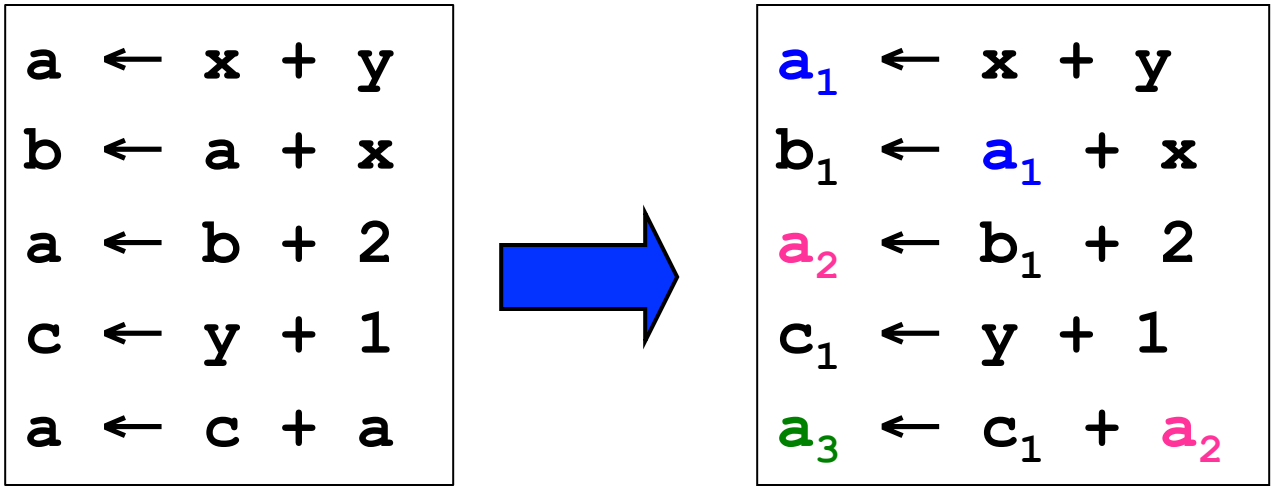
Unfortunately DU and UD Chains Can Be Expensive

```
foo(int i, int j) {
   …
   switch (i) {
   case 0: x=3; break;
   case 1: x=1; break;
   case 2: x=6;
   case 3: x=7;
   default: x = 11;
   }
   x1 is one of the above x's
   switch (j) {
   case 0: y=x1+7;
   case 1: y=x1+4;
   case 2: y=x1-2;
   case 3: y=x1+1;
   default: y=x1+9;
   }
   …
```

One solution: limit each variable to ONE definition site

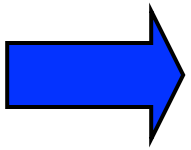
Static Single Assignment (SSA)

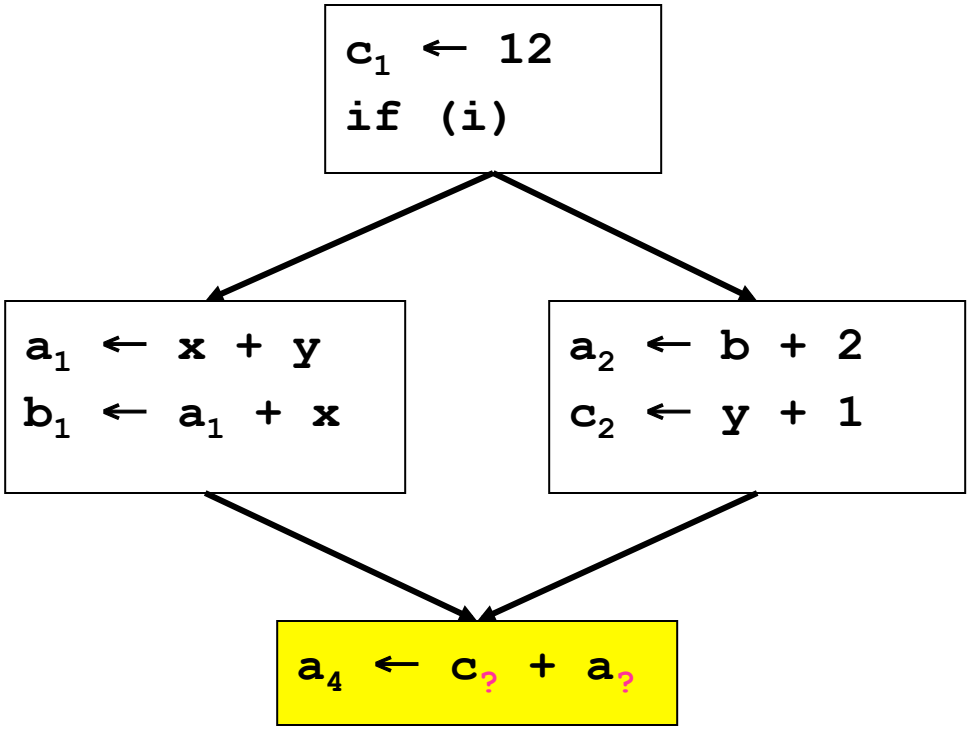
- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block (reminiscent of Value Numbering):
  - Visit each instruction in program order:
    - LHS: assign to a *fresh version* of the variable
    - RHS: use the *most recent version* of each variable



What about Joins in the CFG?

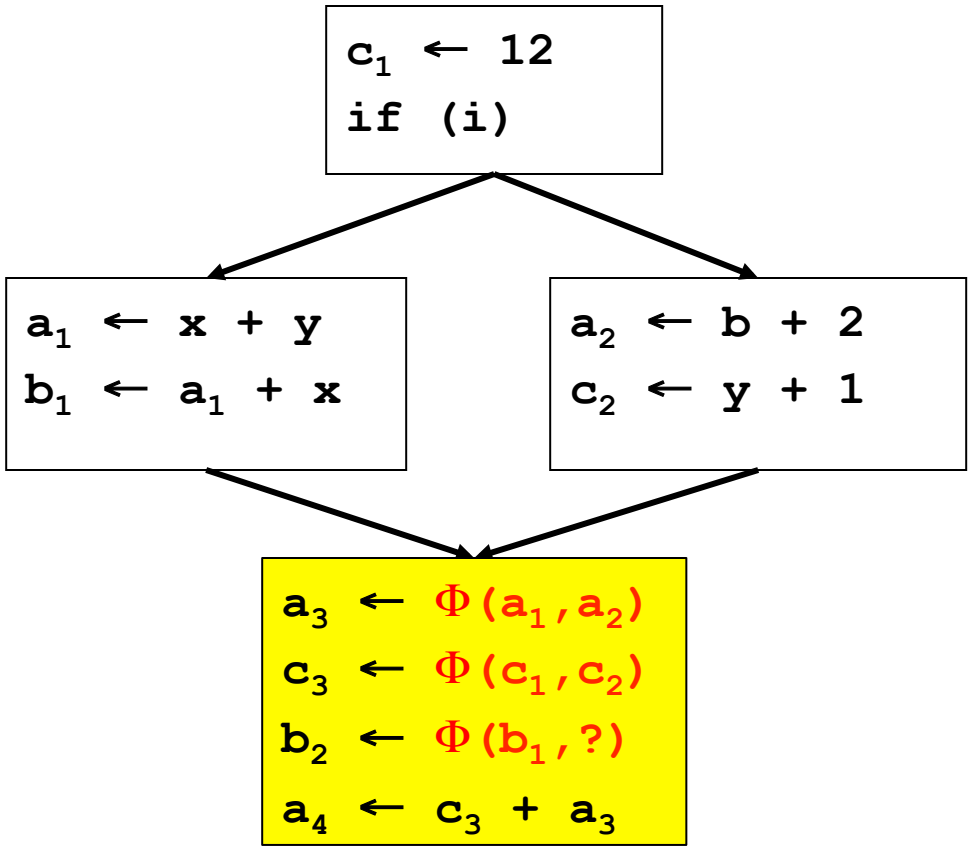
```
c ← 12
if (i) {
   a ← x + y
   b ← a + x
} else {
   a ← b + 2
   c ← y + 1
}
a ← c + a
```





→ Use a notational fiction: a Φ function

Merging at Joins: the Φ function



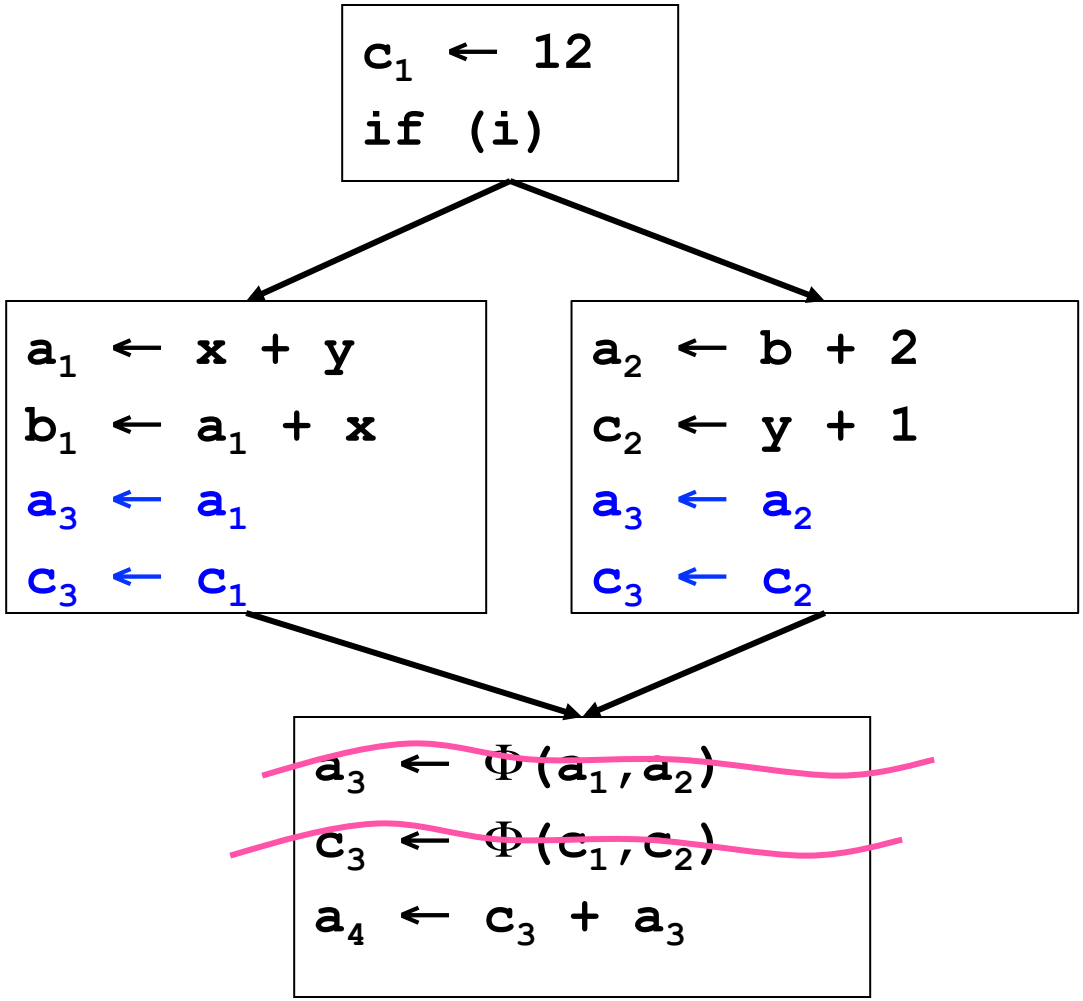
The Φ function

- Φ merges multiple definitions along multiple control paths into a single definition.
- At a basic block with *p* predecessors, there are *p* arguments to the Φ function.

$$x_{new} \leftarrow \Phi(x_1, x_1, x_1, \ldots, x_p)$$

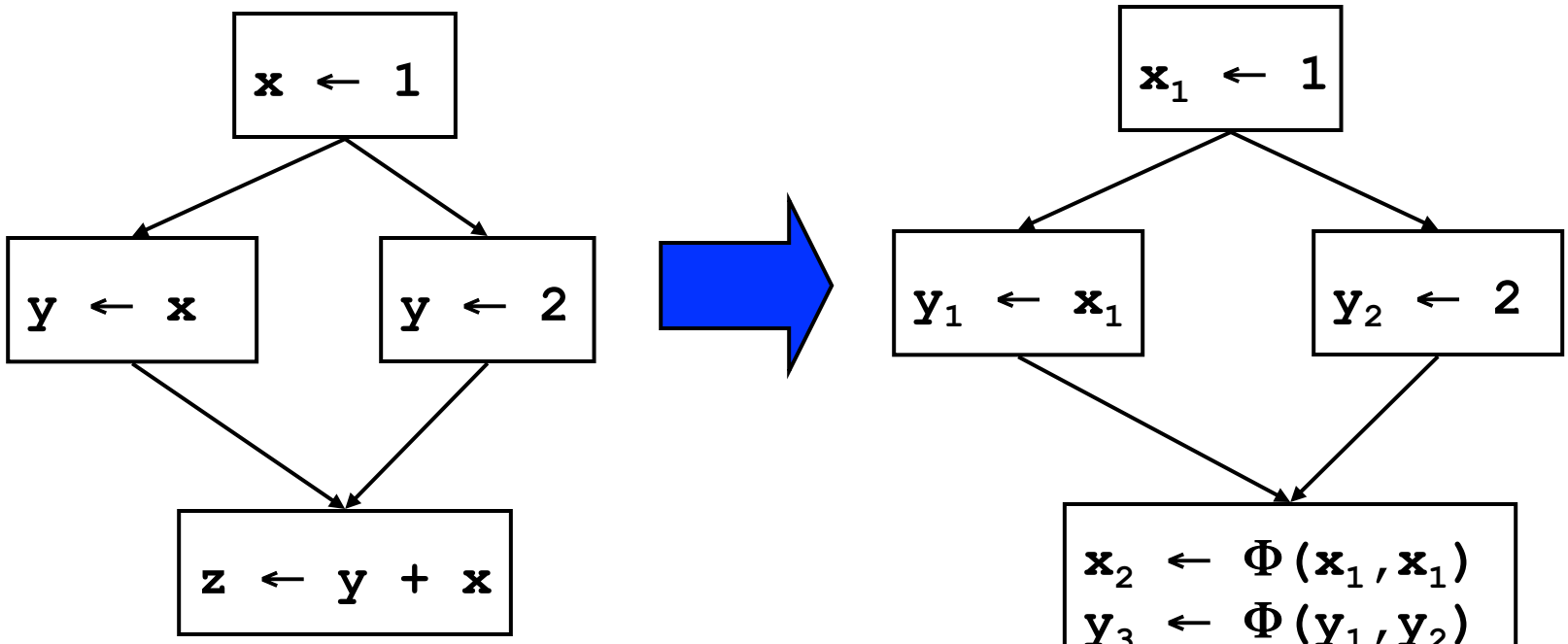
- How do we choose which x_i to use?
  - We don't really care!
  - If we care, use moves on each incoming edge

"Implementing" Φ



Trivial SSA

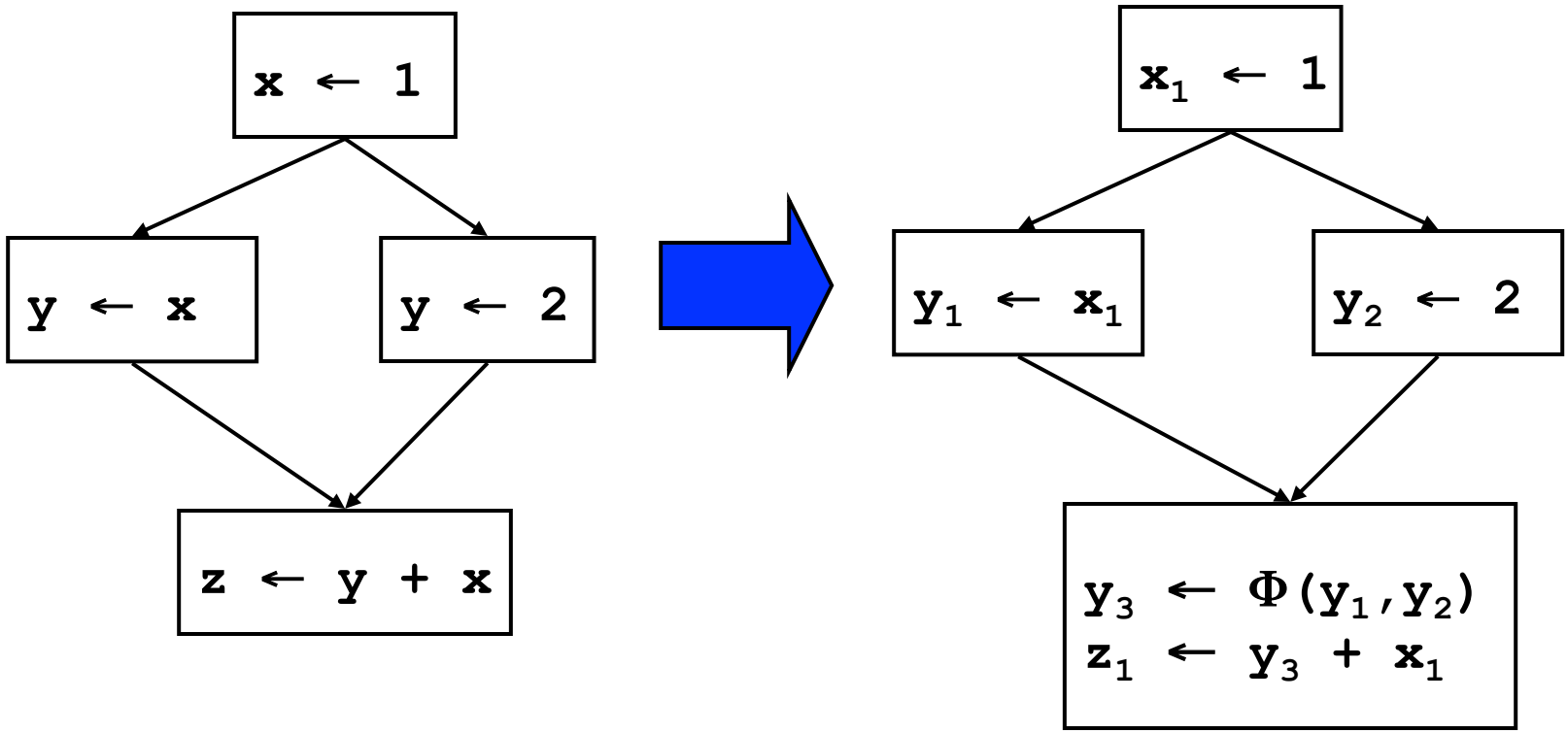
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



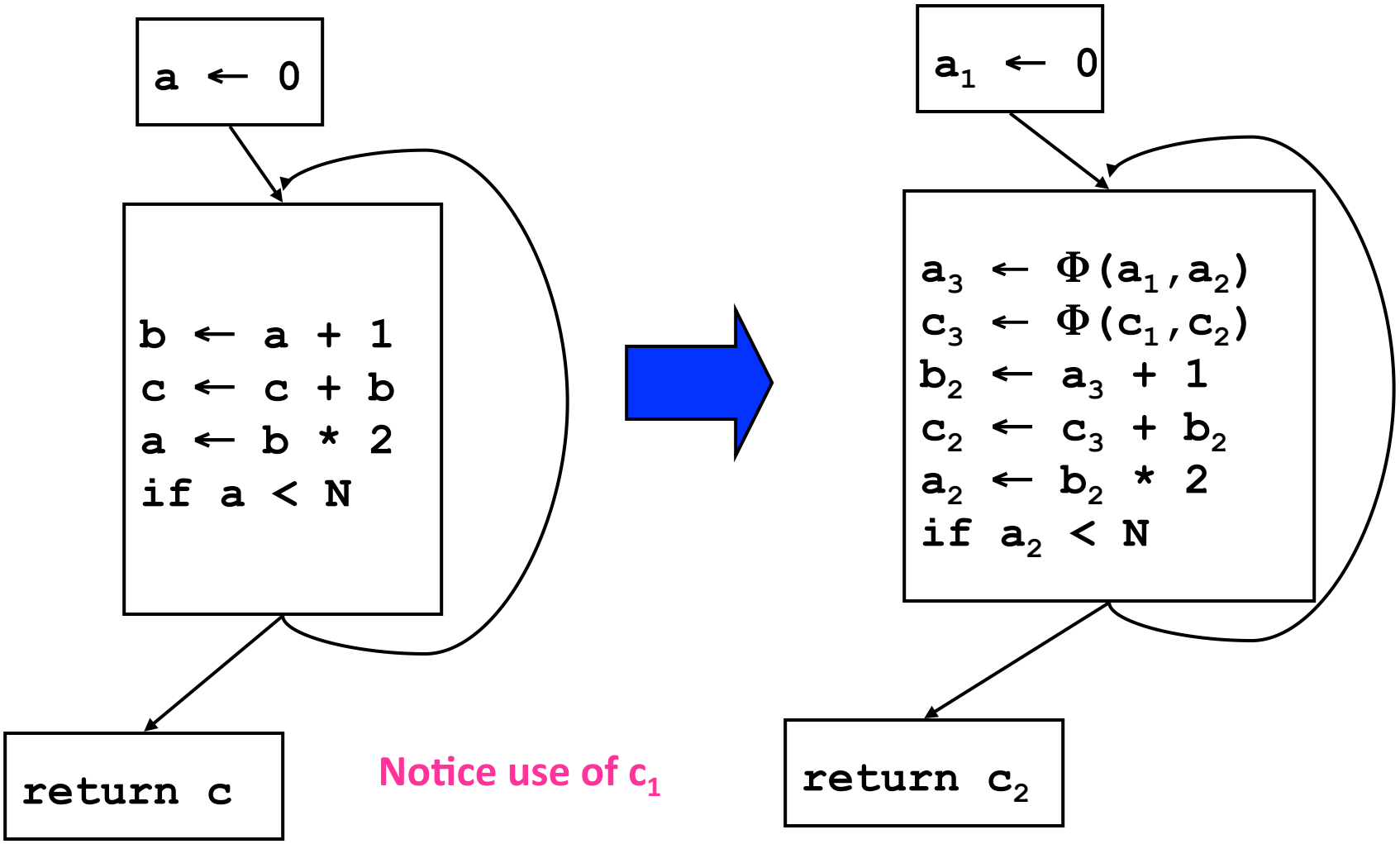
Too many Φ functions inserted.

Minimal SSA

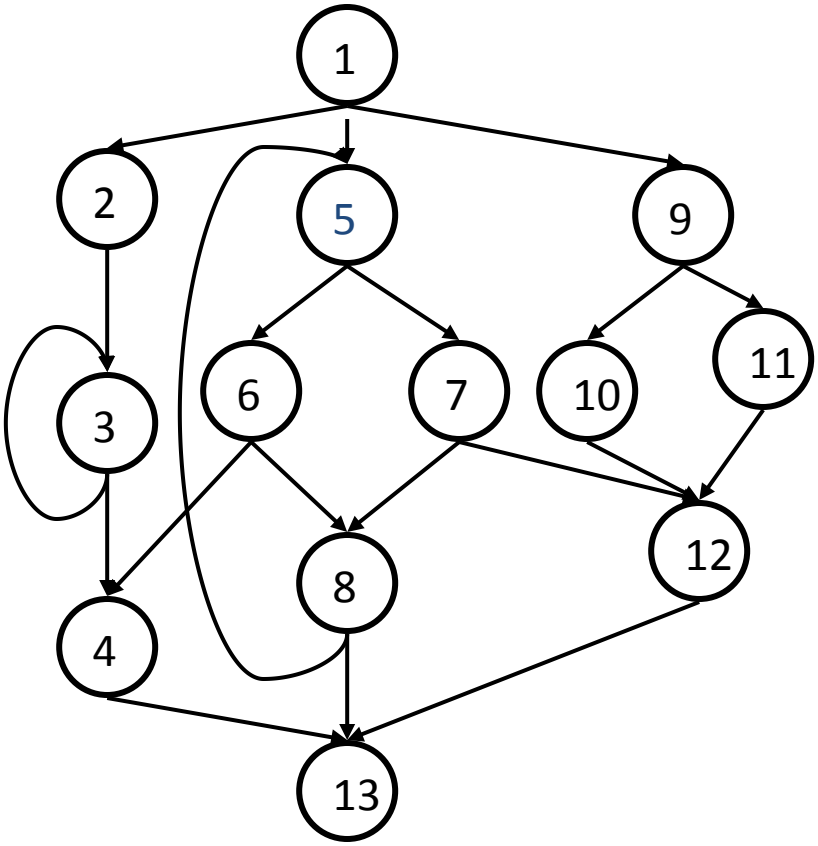
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables with multiple outstanding defs.



Another Example



When Do We Insert Φ?



CFG

If there is a def of **a** in block 5, which nodes need a Φ()?

When do we insert Φ?

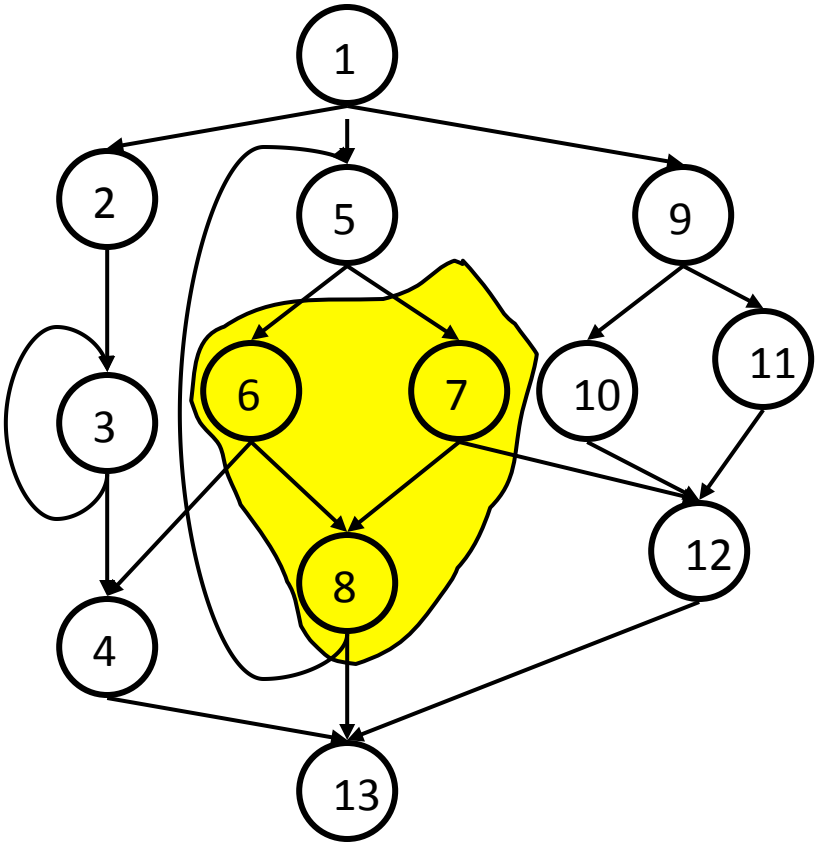
- We insert a Φ function for variable **A** in block Z iff:
  - **A** was defined more than once before
    - (i.e., **A** defined in **X** and Y AND $X \neq Y$)
  - There exists a non-empty path from x to z, P_{xz}, and a non-empty path from y to z, P_{yz}, s.t.
    - $P_{xz} \cap P_{yz} = \{ z \}$
    - $z \notin P_{xq}$ or $z \notin P_{yr}$ where $P_{xz} = P_{xq} \rightarrow z$ and $P_{yz} = P_{yr} \rightarrow z$
- Entry block contains an implicit def of all vars
- Note: A = Φ(…) is a def of A

Dominance Property of SSA

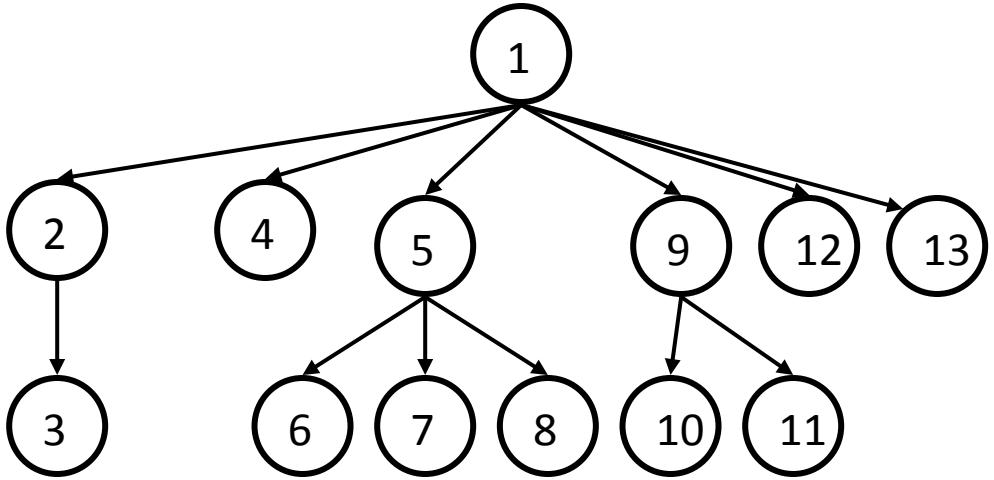
- In SSA, definitions dominate uses.
  - If x_i is used in $x \leftarrow \Phi(\ldots, x_i, \ldots)$, then BB($x_i$) dominates i^{th} predecessor of BB(PHI)
  - If x is used in y ← … x …, then BB(x) dominates BB(y)

- We can use this for an efficient algorithm to convert to SSA

Dominance



CFG

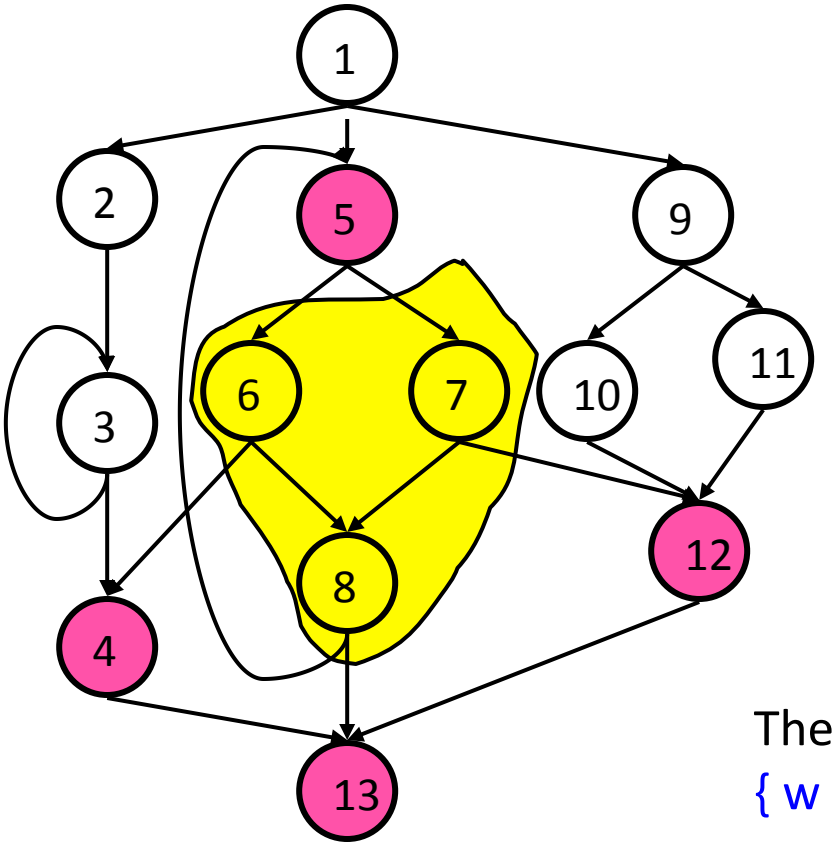


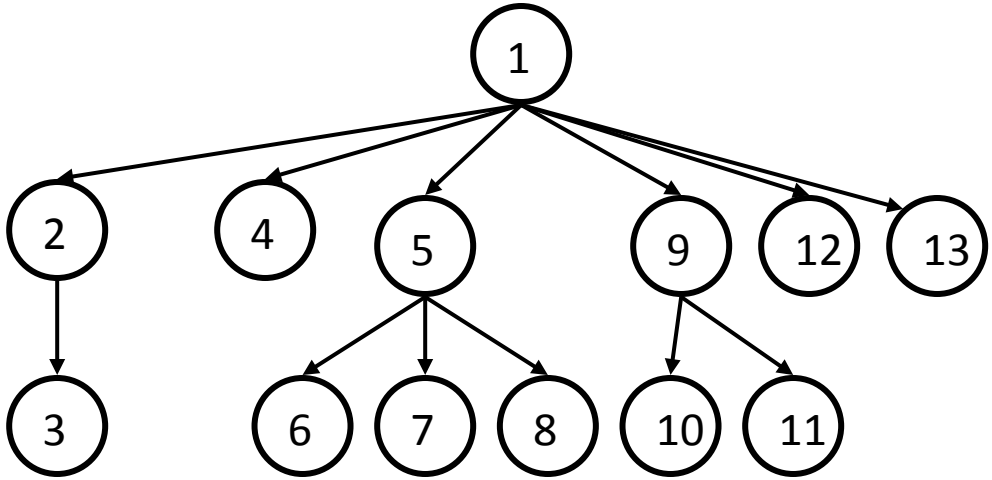
D-Tree

If there is a def of a in block 5, which nodes need a $\Phi()$?

x strictly dominates w (x sdom w) iff x dom w AND x ≠ w

Dominance Frontier



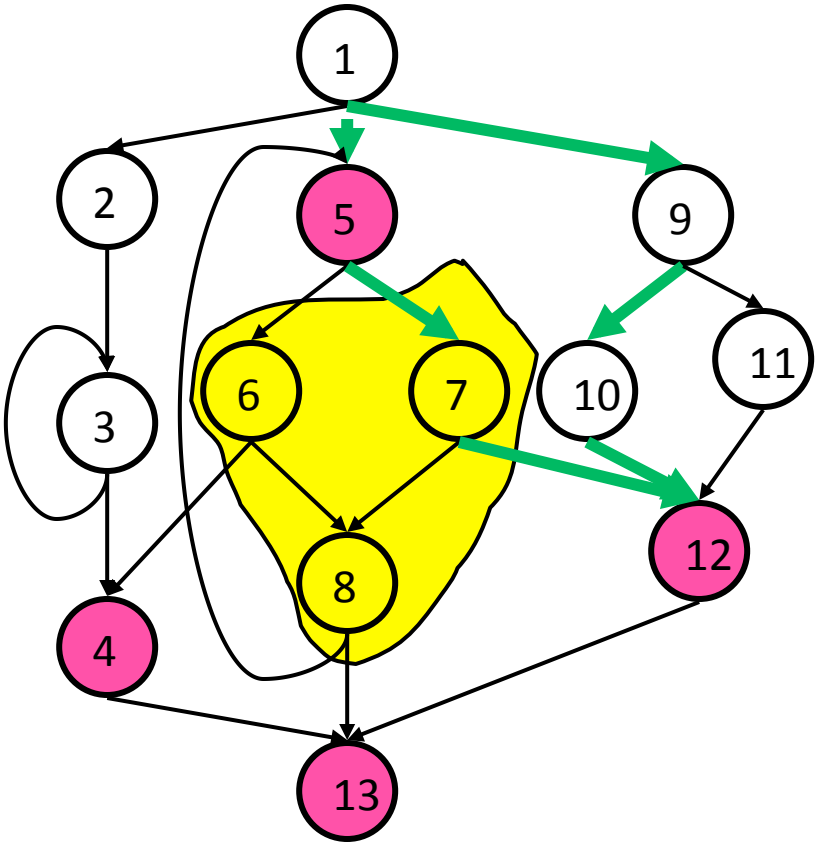


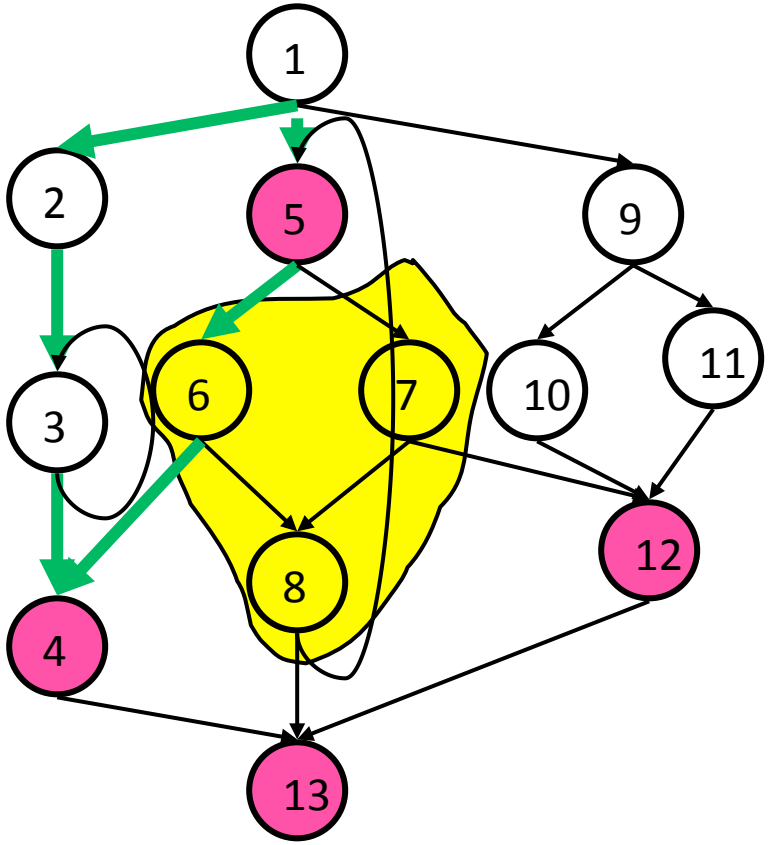
The Dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

CFG D-Tree

x strictly dominates w (x sdom w) iff x dom w AND x ≠ w

Dominance Frontier and Path Convergence





Using Dominance Frontier to Compute SSA

- place all $\Phi()$
- Rename all variables

Using Dominance Frontier to Place Φ()

- Gather all the defsites of every variable
- Then, for every variable
  - foreach defsite
    - foreach node in DominanceFrontier(defsite)
      - if we haven't put Φ() in node, then put one in
      - if this node didn't define the variable before, then add this node to the defsites

- This essentially computes the Iterated Dominance Frontier on the fly, inserting the minimal number of Φ() neccesary

Using Dominance Frontier to Place Φ()

```
foreach node n {
   foreach variable v defined in n {
     orig[n] ∪= {v}
     defsites[v] ∪= {n}
   }
}
foreach variable v {
   W = defsites[v]
   while W not empty {
     n = remove node from W
     foreach y in DF[n]
     if y ∉ PHI[v] {
       insert "v ← Φ(v,v,…)" at top of y
       PHI[v] = PHI[v] ∪ {y}
       if v ∉ orig[y]: W = W ∪ {y}
     }
   }
}
```

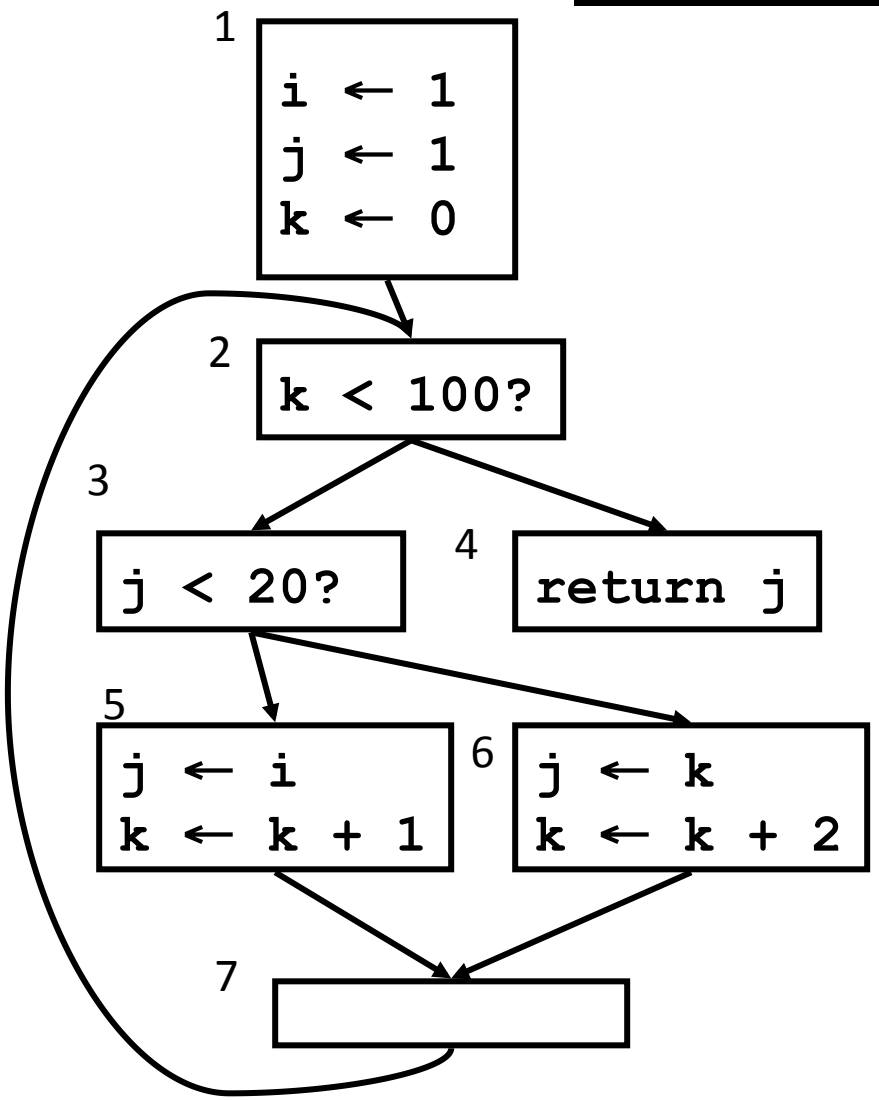
Renaming Variables

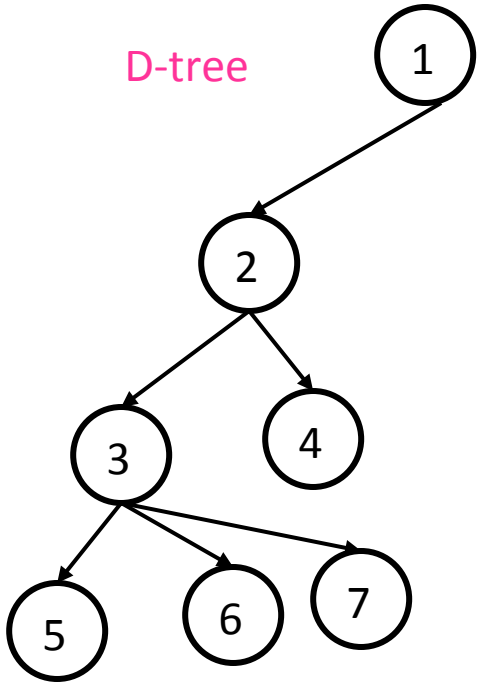
- Algorithm:
  - Walk the D-tree, renaming variables as you go
  - Replace uses with more recent renamed def

- For straight-line code this is easy
- What if there are branches and joins?
  - use the closest def such that the def is above the use in the D-tree

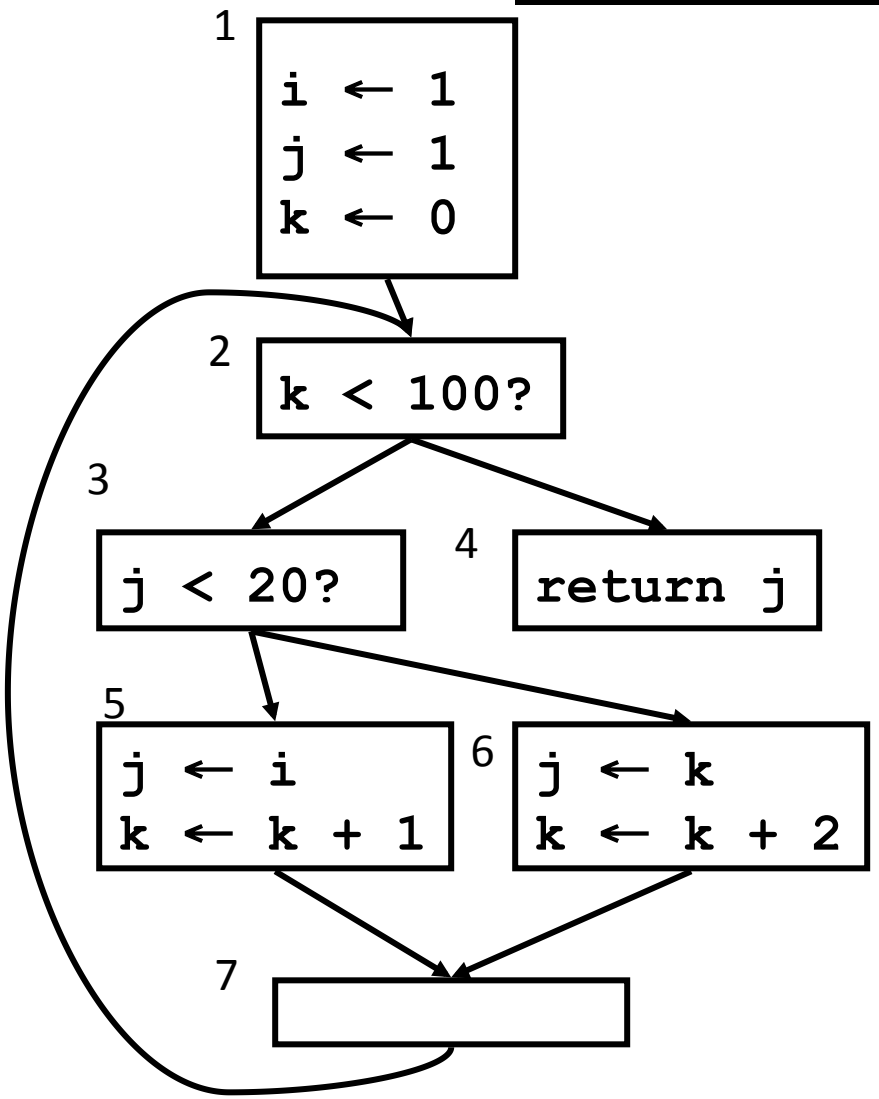
- Easy implementation:
  - for each var: rename (v)
  - rename(v): replace uses with top of stack
    at def: push onto stack
    call rename(v) on all children in D-tree
    for each def in this block pop from stack

Compute Dominance Tree



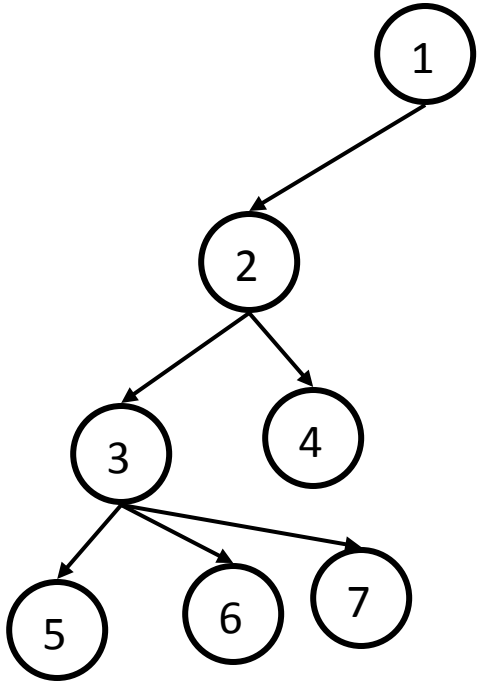


Compute Dominance Frontiers

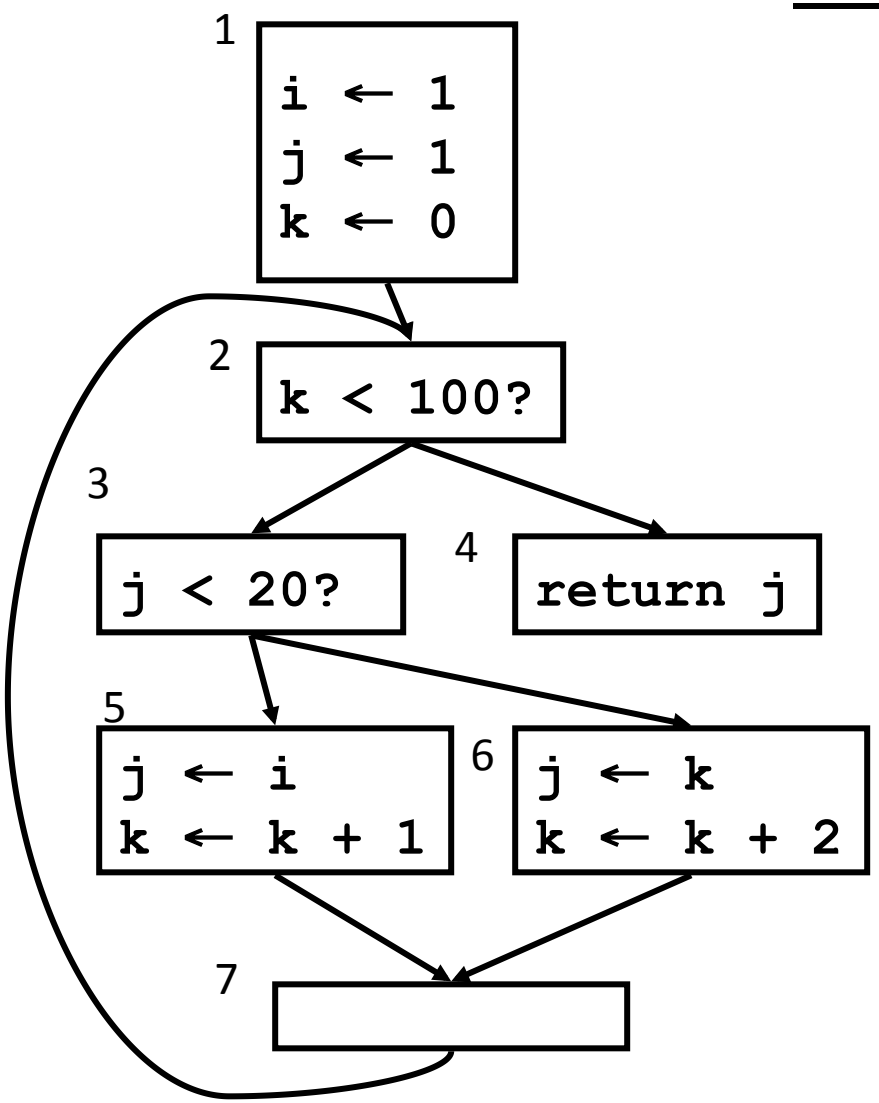


DFs

1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}



Insert Φ()



	DFs
1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

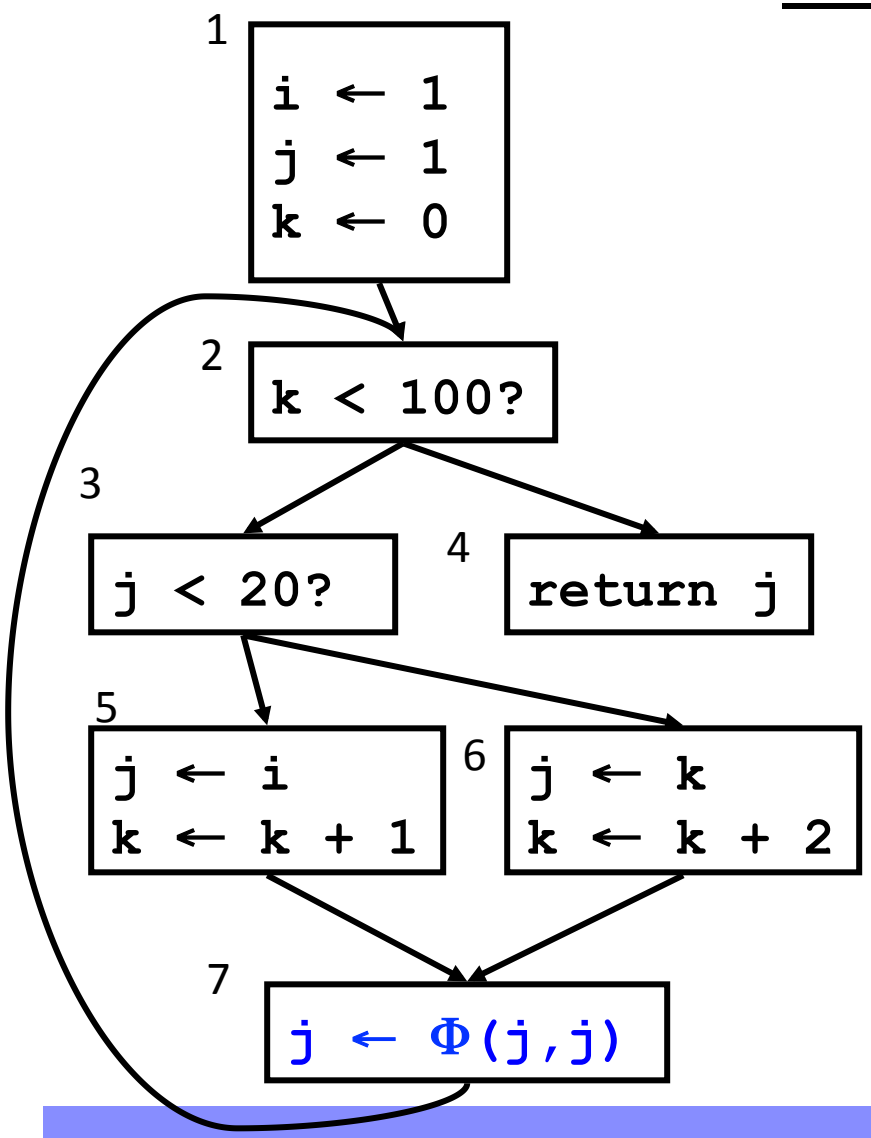
DFs

var i: W={1}

var j: W={1,5,6}

DF{1} DF{5}

Insert Φ()



	DFs		orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

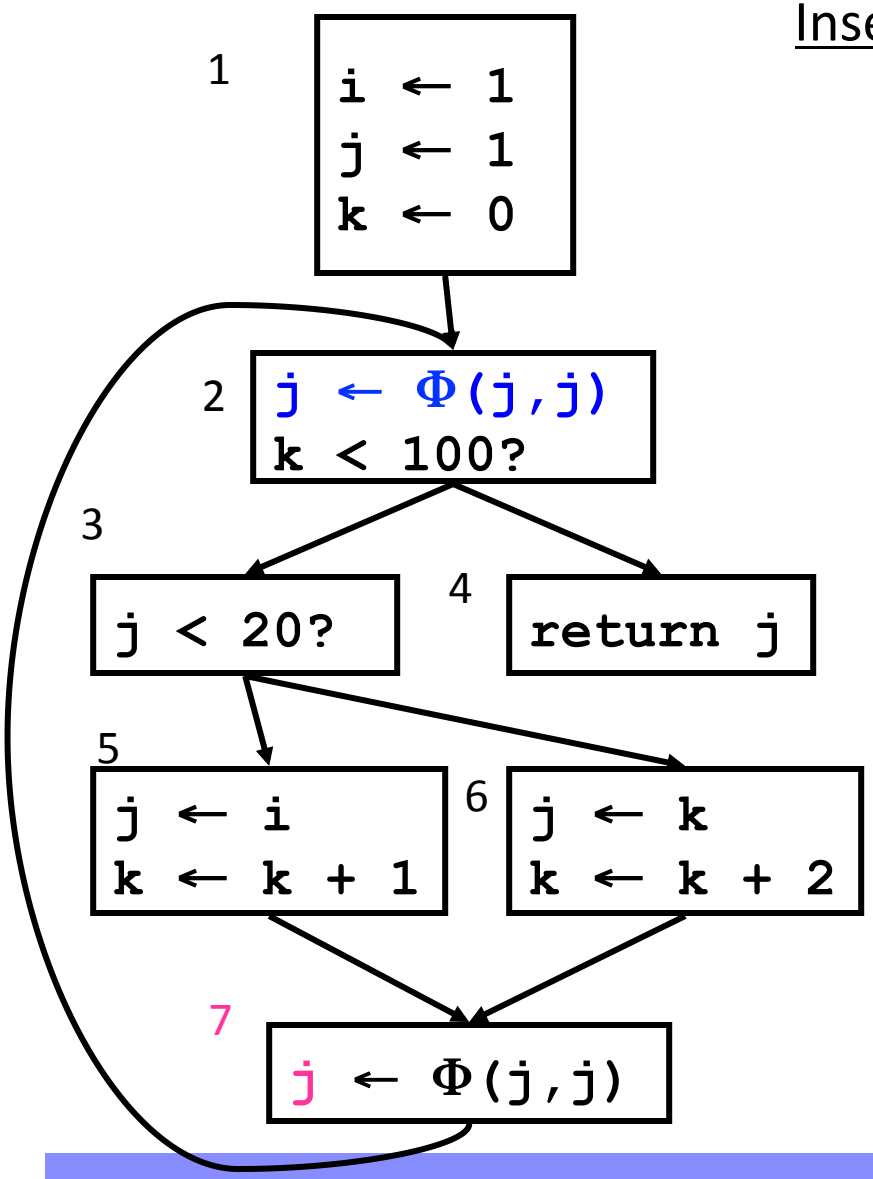
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6}

DF{1} DF{5}



Insert Φ()

	DFs		orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

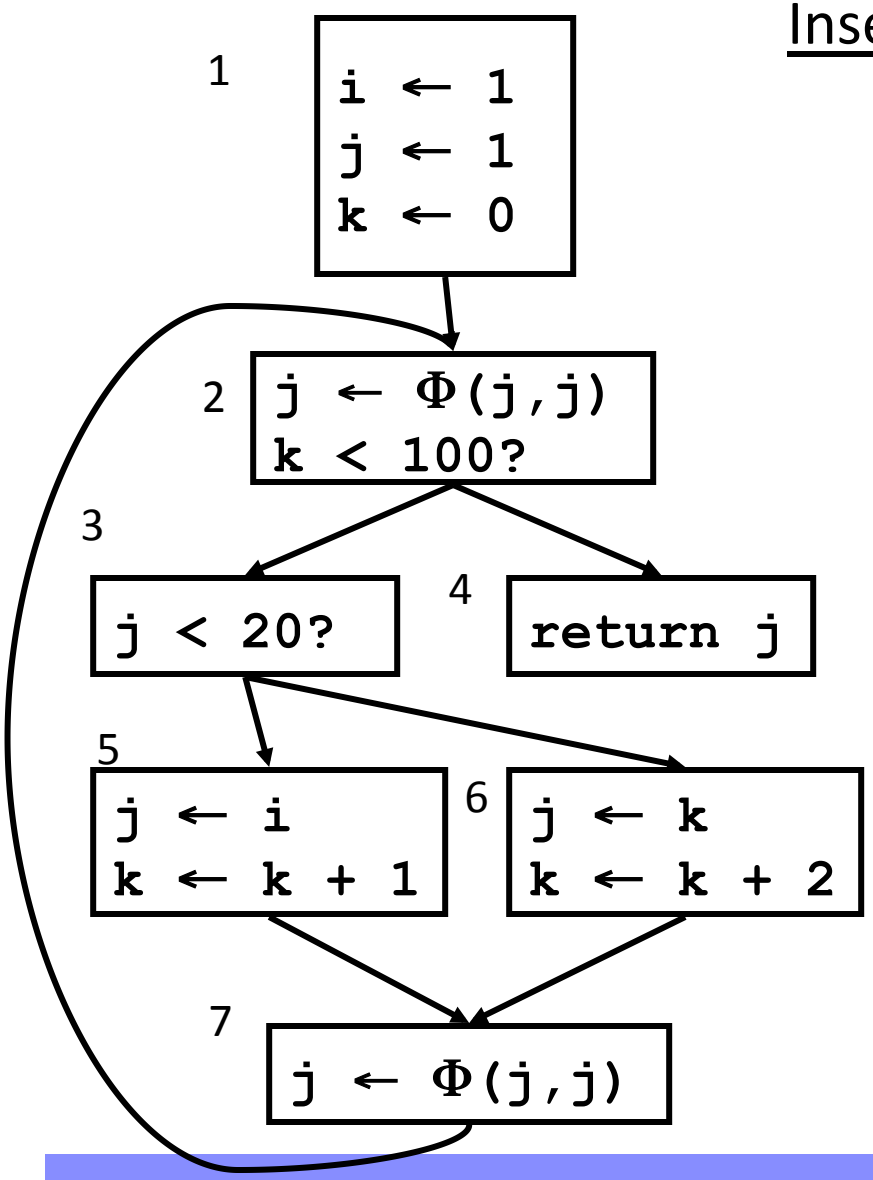
defsites[v]

i	{1}
j	{1,5,6,7}
k	{1,5,6}

DFs

var j: W={1,5,6,7}

DF{1} DF{5} DF{7}



Insert Φ()

	DFs		orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

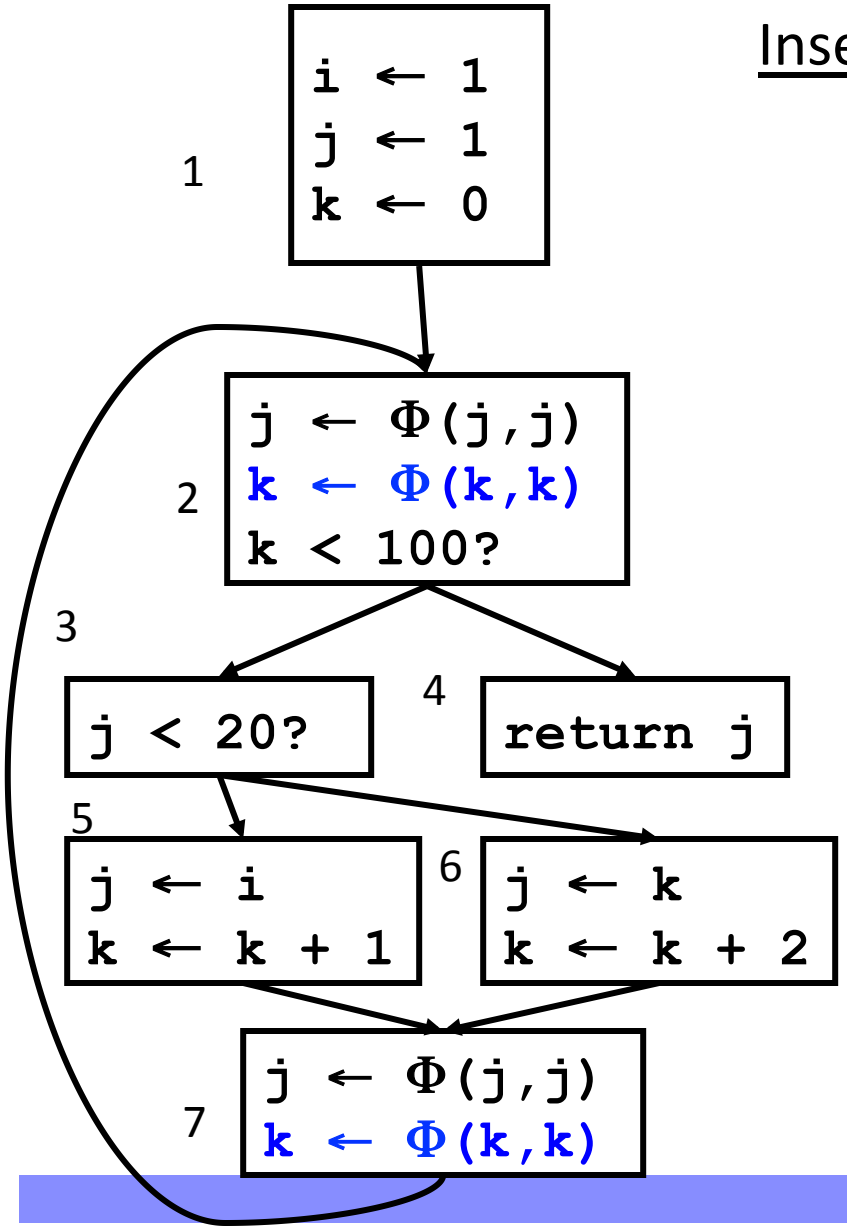
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6,7}

DF{1} DF{5} DF{7} DF{6}



Insert Φ()

DFs

1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

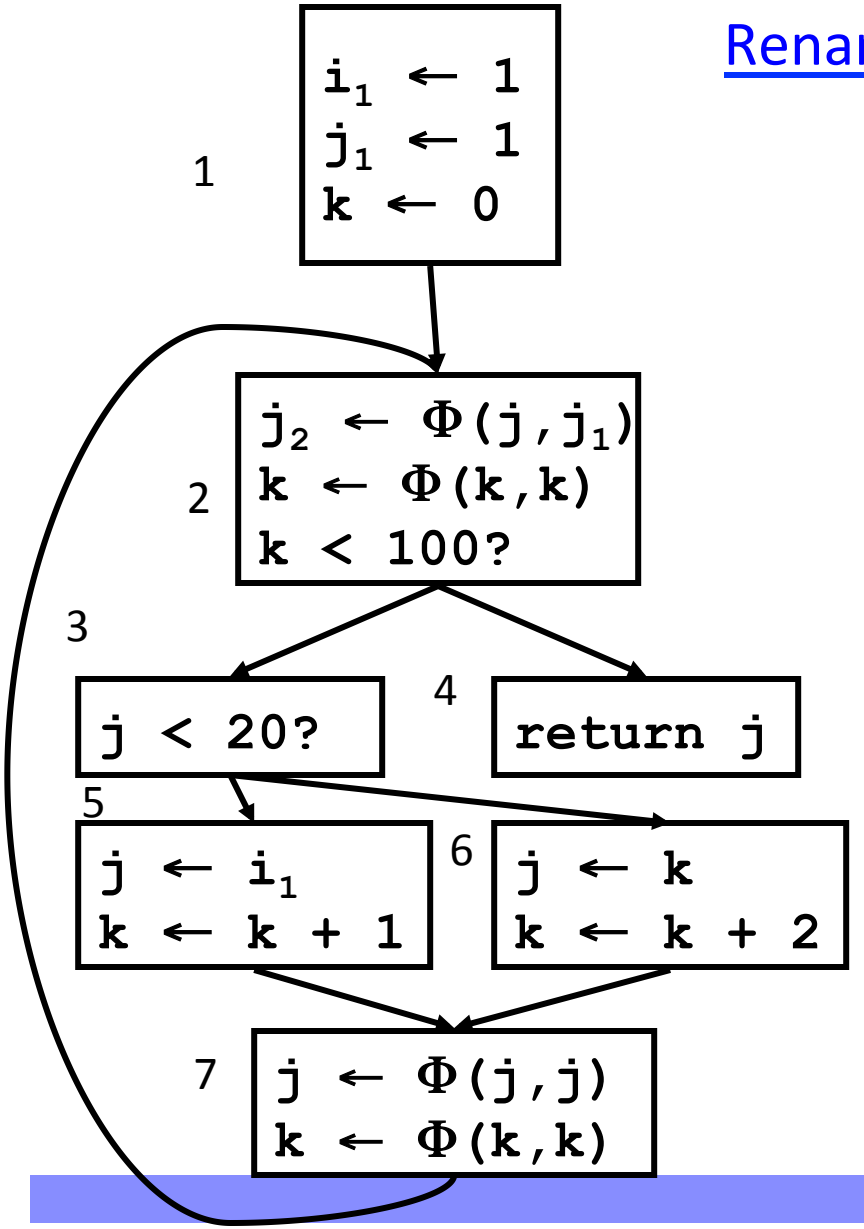
defsites[v]

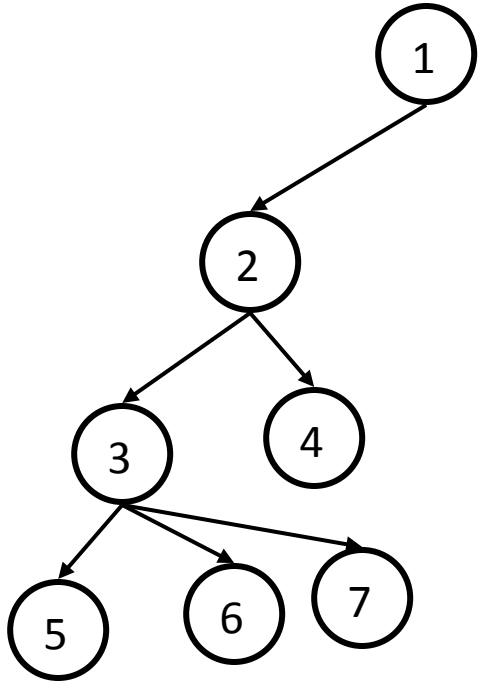
i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

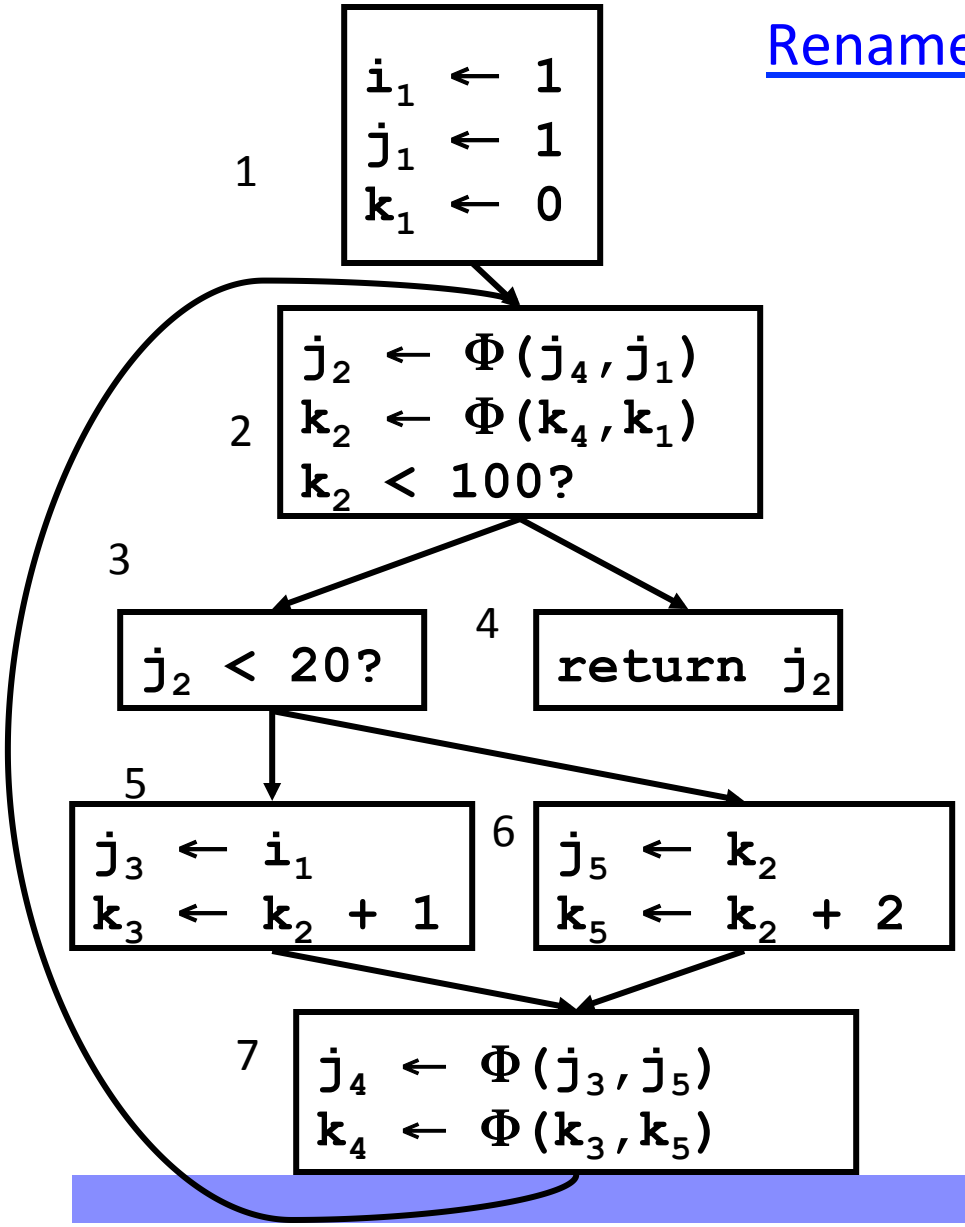
var k: W={1,5,6}

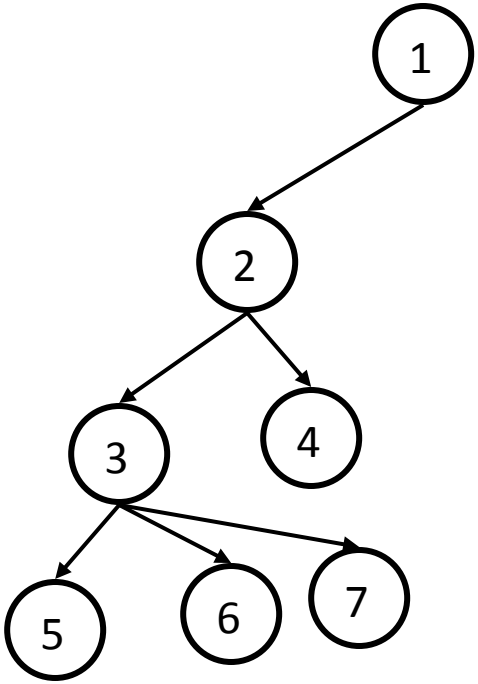
Rename Vars


1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k \leftarrow 0$
2
$j_2 \leftarrow \Phi(j,j_1)$
$k \leftarrow \Phi(k,k)$
$k < 100?$
3
$j < 20?$
4
return j
5
$j \leftarrow i_1$
$k \leftarrow k + 1$
6
$j \leftarrow k$
$k \leftarrow k + 2$
7
$j \leftarrow \Phi(j,j)$
$k \leftarrow \Phi(k,k)$

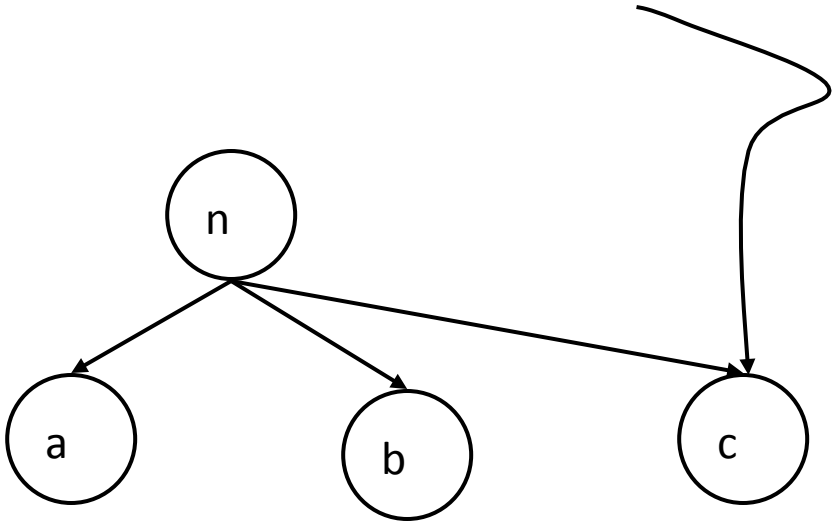

1
2
3
4
5
6
7

Rename Vars


1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$
2
$j_2 \leftarrow \Phi(j_4, j_1)$
$k_2 \leftarrow \Phi(k_4, k_1)$
$k_2 < 100?$
3
$j_2 < 20?$
4
return j_2
5
$j_3 \leftarrow i_1$
$k_3 \leftarrow k_2 + 1$
6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$
7
$j_4 \leftarrow \Phi(j_3, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

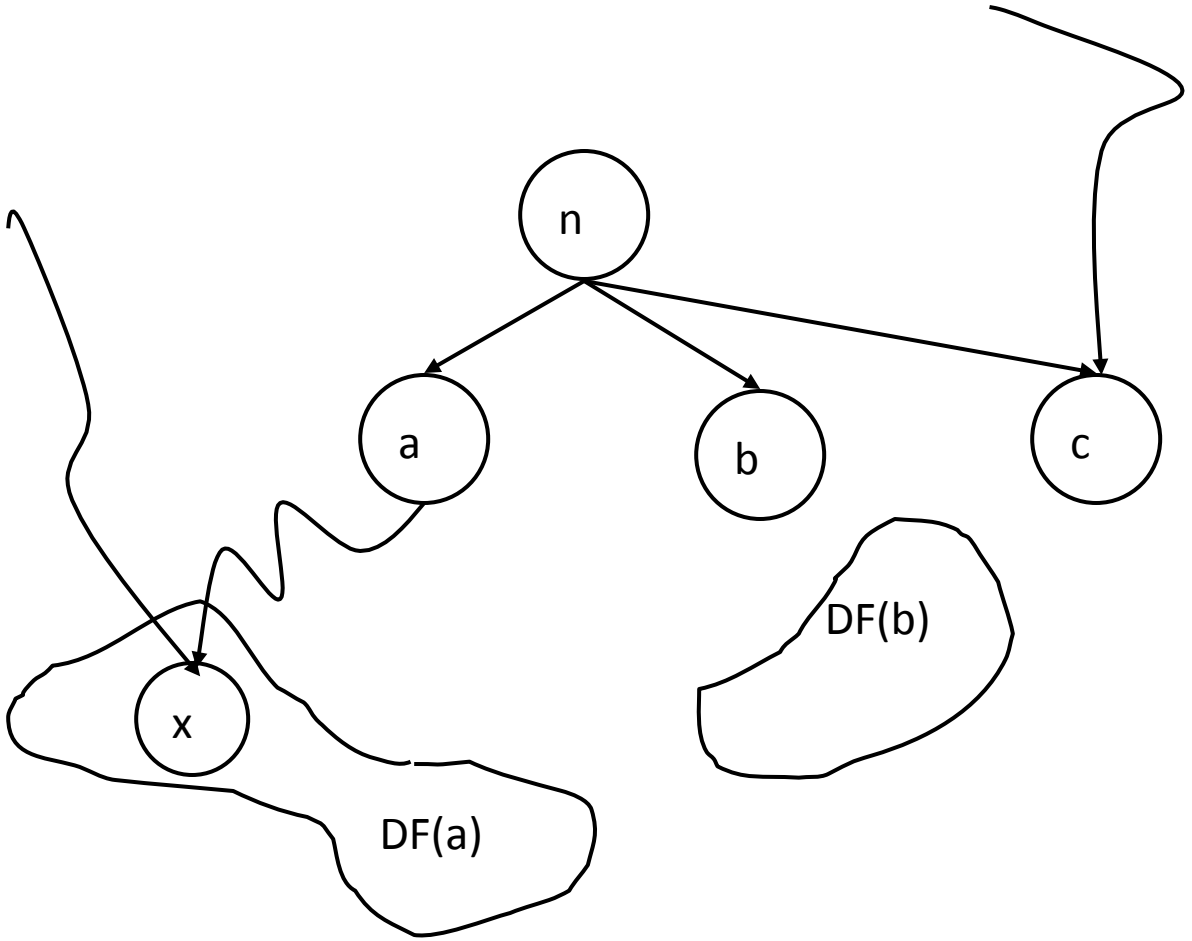

1
2
3
4
5
6
7

Computing DF(n)



n dom a
n dom b
!n dom c

Computing DF(n)



n dom a
n dom b
!n dom c

Computing the Dominance Frontier

The Dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

```
compute-DF(n)
        S = {}
        foreach node y in succ[n]
                if idom(y) ≠ n
                        S = S ∪ { y }
        foreach child of n, c, in D-tree
                compute-DF(c)
                foreach w in DF[c]
                        if !n dom w
                                S = S ∪ { w }
        DF[n] = S
```

SSA Properties

- Only 1 assignment per variable
- Definitions dominate uses