# Lecture 25

# Memory Hierarchy Optimizations & Locality Analysis

# Caches: A Quick Review

- How do they work?
- Why do we care about them?
- What are typical configurations today?
- What are some important cache parameters that will affect performance?

# Optimizing Cache Performance

- Things to enhance:
  - temporal locality
  - spatial locality

- Things to minimize:
  - conflicts (i.e. bad replacement decisions)

What can the *compiler* do to help?

# Two Things We Can Manipulate

- Time:
  - When is an object accessed?
- Space:
  - Where does an object exist in the address space?

*How do we exploit these two levers?*

# Time: Reordering Computation

- What makes it difficult to know *when* an object is accessed?
- How can we predict a better time to access it?
  - What information is needed?
- How do we know that this would be safe?

# Space: Changing Data Layout

- What do we know about an object's location?
  - scalars, structures, pointer-based data structures, arrays, code, etc.
- How can we tell what a better layout would be?
  - how many can we create?
- To what extent can we safely alter the layout?

# Types of Objects to Consider

- Scalars
- Structures & Pointers
- Arrays

# Scalars

- Locals
- Globals
- Procedure arguments
- Is cache performance a concern here?
- If so, what can be done?

```
int x;
double y;
foo(int a){
  int i;
  …
  x = a*i;
  …
}
```

# Structures and Pointers

- What can we do here?
  - within a node
  - across nodes

```
struct {
    int count;
    double velocity;
    double inertia;
    struct node *neighbors[N];
} node;
```

- What limits the compiler's ability to optimize here?

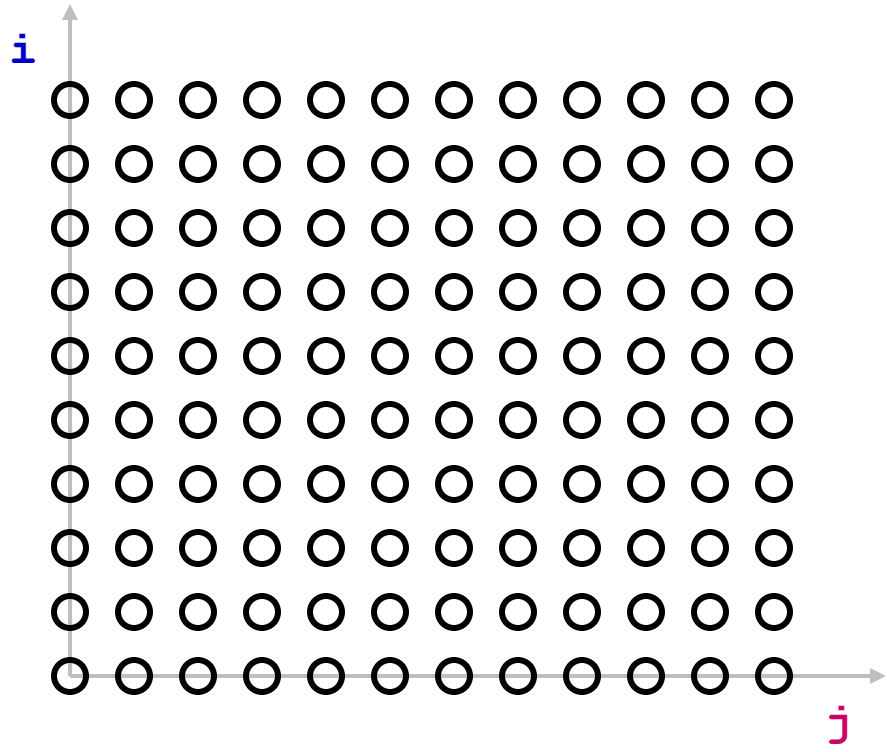
# Arrays

```
double A[N][N], B[N][N];
…
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```

- usually accessed within loops nests
  - makes it easy to understand "time"
- what we know about array element addresses:
  - start of array?
  - relative position within array

# Handy Representation: "Iteration Space"

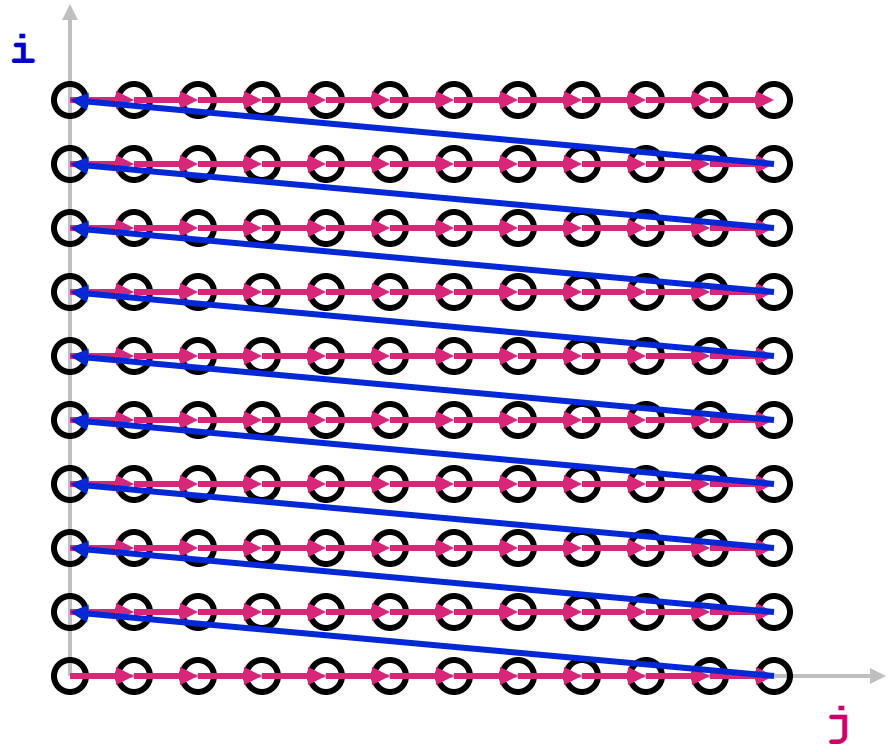
```
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```




- each position represents an iteration

# Visitation Order in Iteration Space

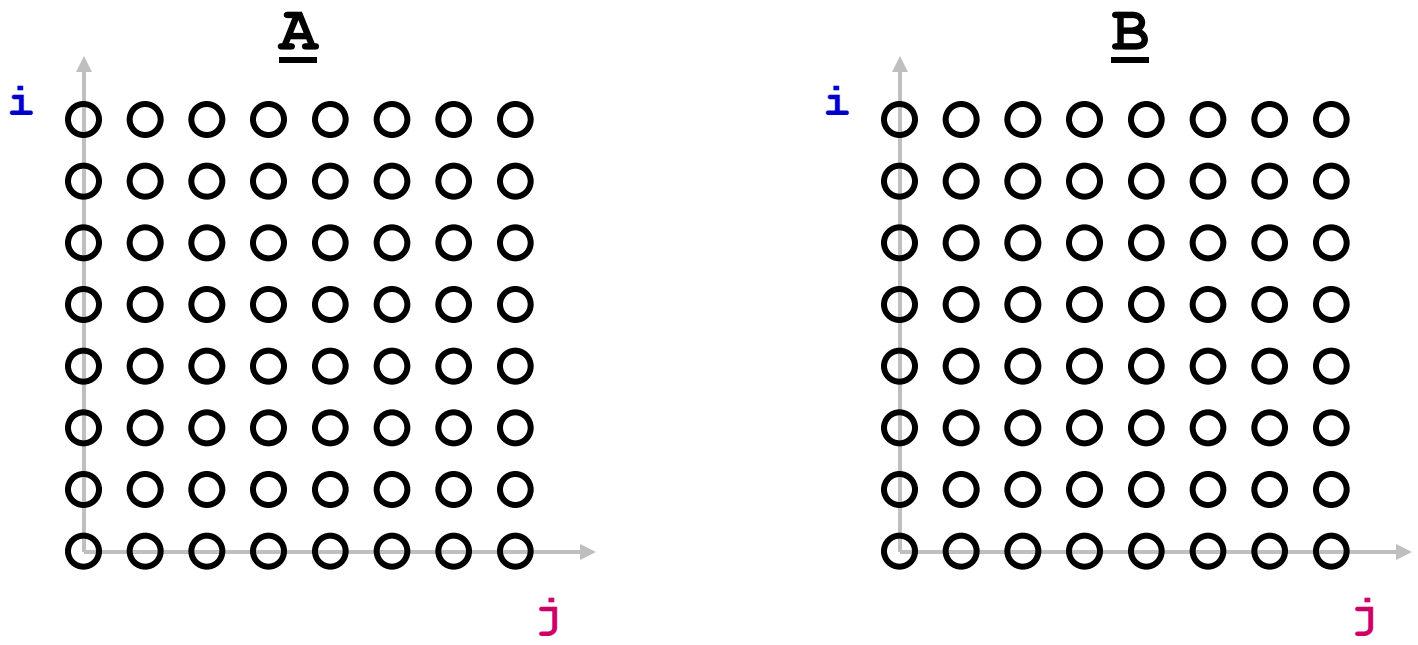
```
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```




- Note: iteration space ≠ data space

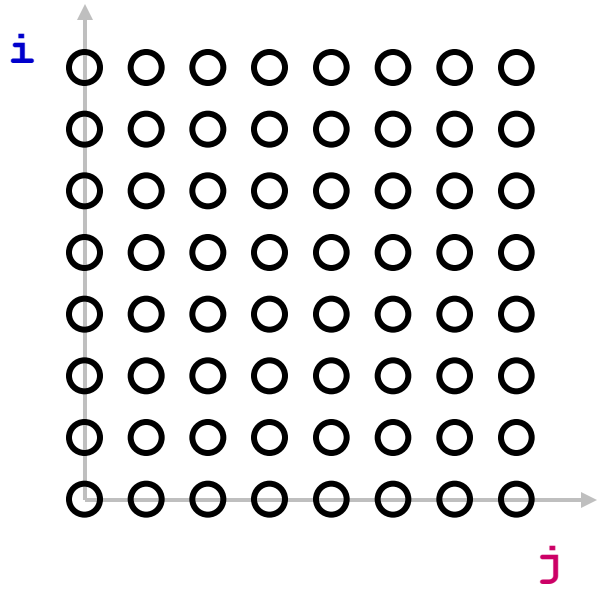
# When Do Cache Misses Occur?

```
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```



# When Do Cache Misses Occur?

```
for i = 0 to N-1
   for j = 0 to N-1
      A[i+j][0] = i*j;
```



# Optimizing the Cache Behavior of Array Accesses

- We need to answer the following questions:
  - when do cache misses occur?
    - use "locality analysis"
  - can we change the order of the iterations (or possibly data layout) to produce better behavior?
    - evaluate the cost of various alternatives
  - does the new ordering/layout still produce correct results?
    - use "dependence analysis"

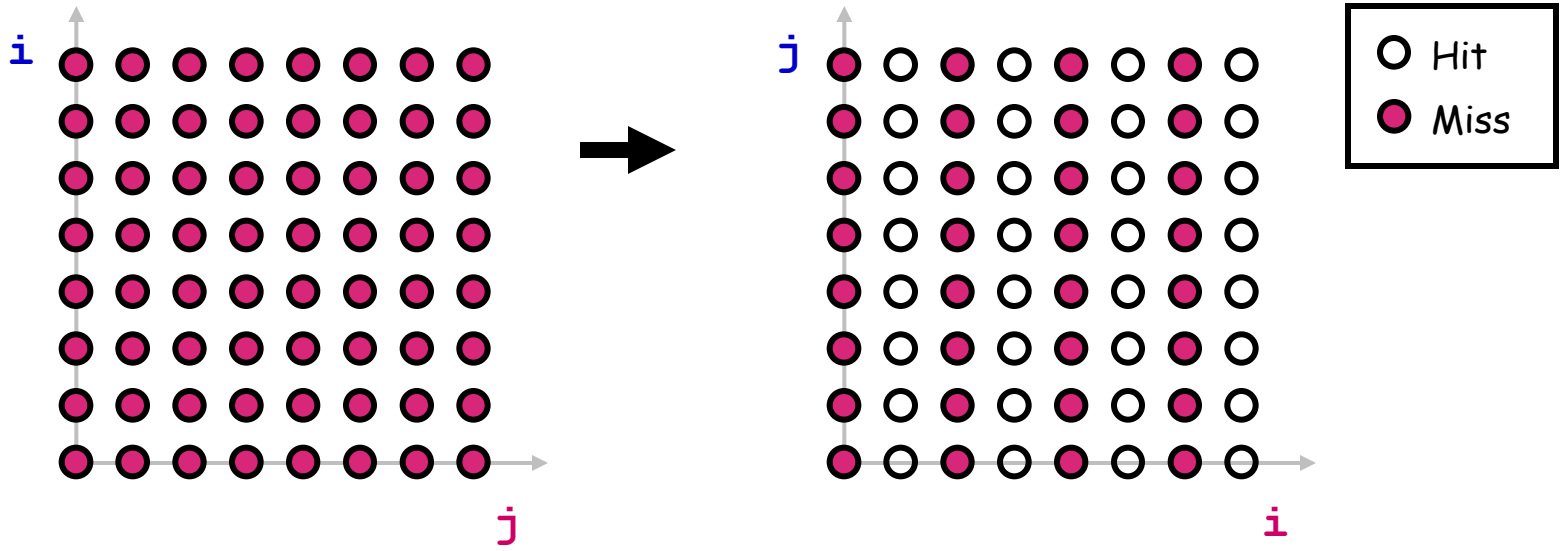
# Examples of Loop Transformations

- Loop Interchange
- Cache Blocking
- Skewing
- Loop Reversal
- …

*(we will briefly discuss the first two)*

# Loop Interchange

```
for i = 0 to N-1                for j = 0 to N-1
   for j = 0 to N-1                for i = 0 to N-1
      A[j][i] = i*j;                  A[j][i] = i*j;
```



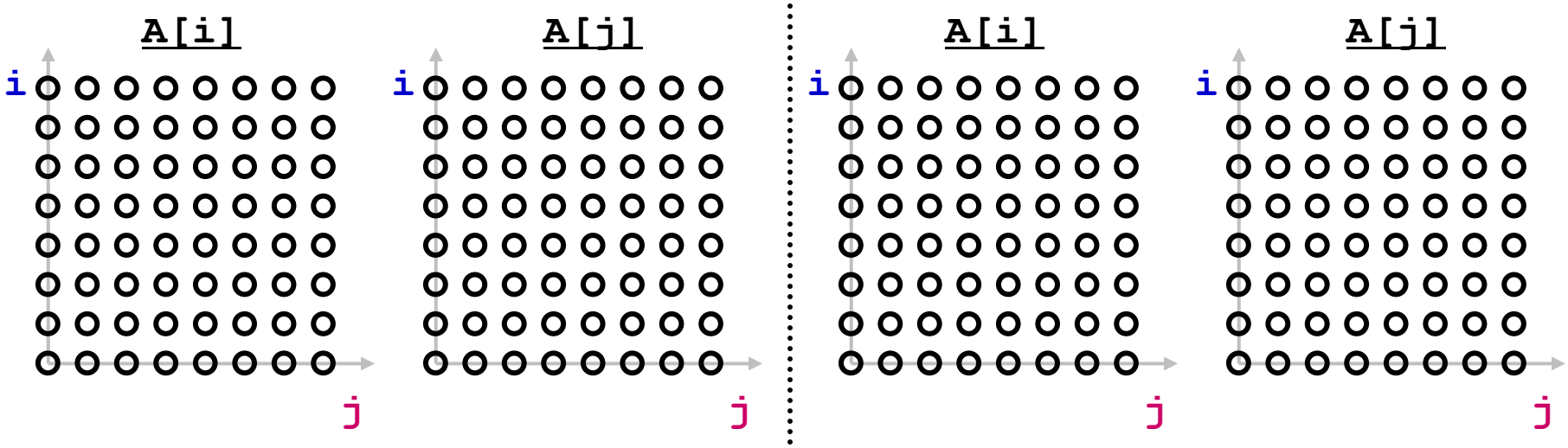

- *(assuming N is large relative to cache size)*

# Cache Blocking (aka "Tiling")

```
for i = 0 to N-1
   for j = 0 to N-1
      f(A[i],A[j]);
```

→

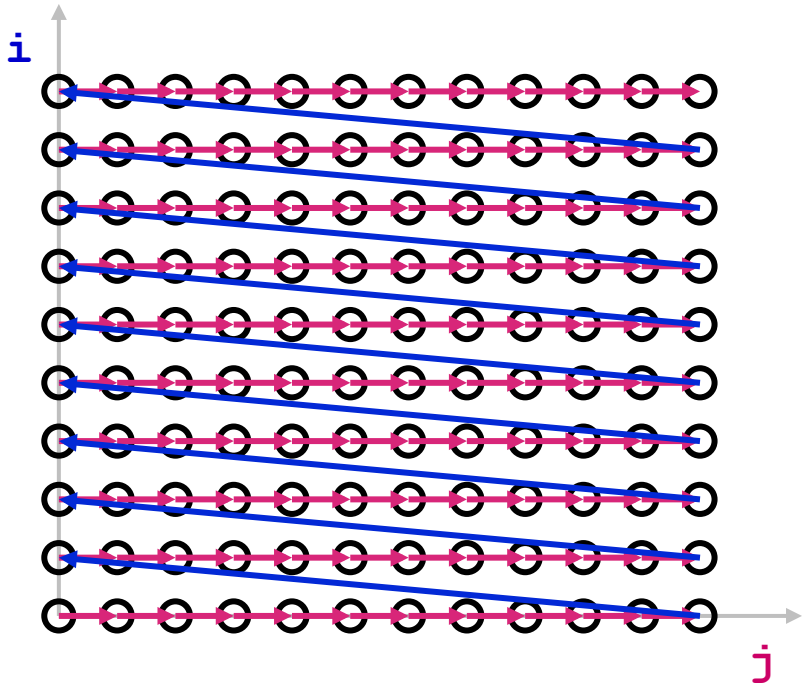
```
for JJ = 0 to N-1 by B
   for i = 0 to N-1
      for j = JJ to max(N-1,JJ+B-1)
         f(A[i],A[j]);
```



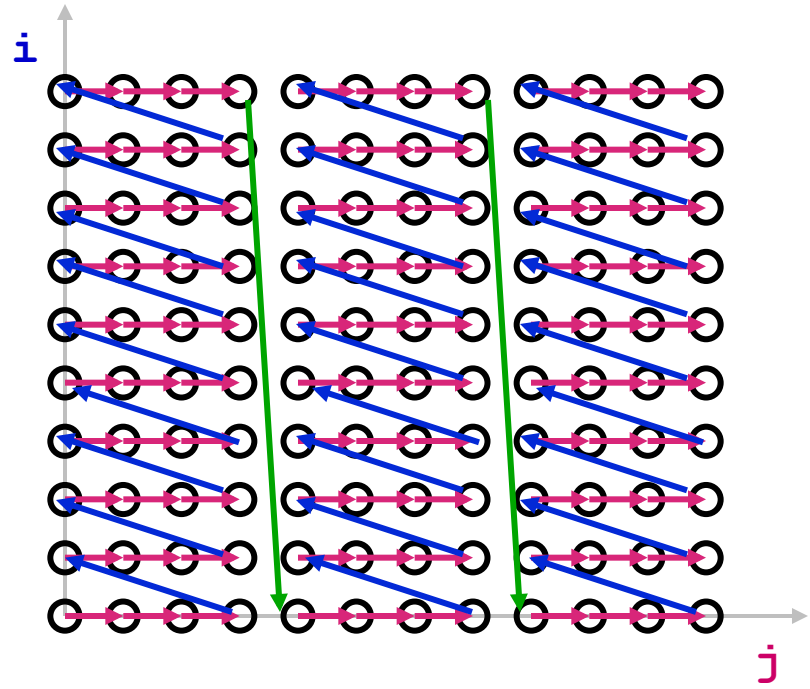

*now we can exploit temporal locality*

# Impact on Visitation Order in Iteration Space

```
for i = 0 to N-1
   for j = 0 to N-1
      f(A[i],A[j]);
```




```
for JJ = 0 to N-1 by B
   for i = 0 to N-1
      for j = JJ to max(N-1,JJ+B-1)
         f(A[i],A[j]);
```



# Cache Blocking in Two Dimensions

```
for i = 0 to N-1
  for j = 0 to N-1
    for k = 0 to N-1
      c[i,k] += a[i,j]*b[j,k];
```

```
for JJ = 0 to N-1 by B
  for KK = 0 to N-1 by B
    for i = 0 to N-1
      for j = JJ to max(N-1,JJ+B-1)
        for k = KK to max(N-1,KK+B-1)
          c[i,k] += a[i,j]*b[j,k];
```

- brings square sub-blocks of matrix "b" into the cache
- completely uses them up before moving on

# Predicting Cache Behavior through "Locality Analysis"

- Definitions:
  - Reuse:
    - accessing a location that has been accessed in the past
  - Locality:
    - accessing a location that is now found in the cache

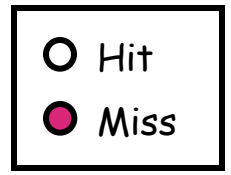
- Key Insights
  - Locality only occurs when there is reuse!
  - BUT, reuse does not necessarily result in locality.
    - why not?

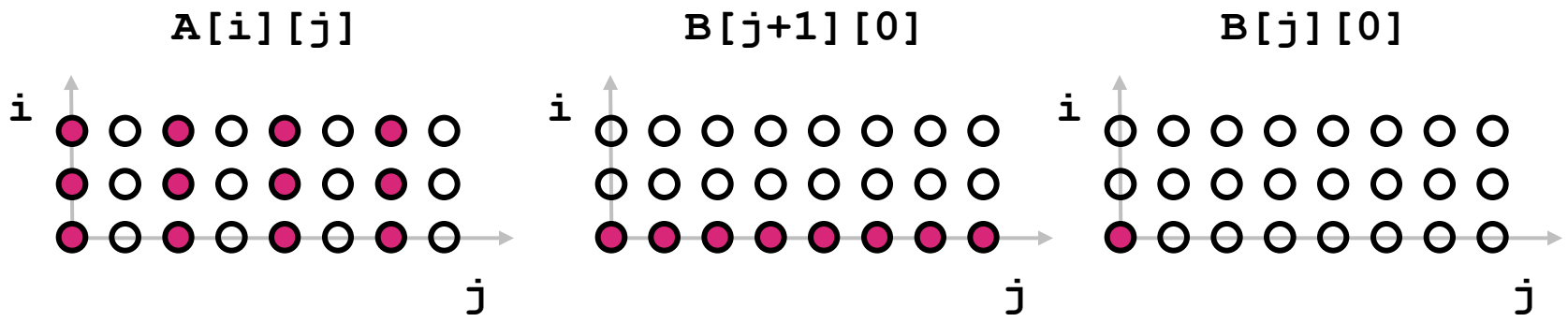
# Steps in Locality Analysis

1. Find data reuse
   - if caches were infinitely large, we would be finished
2. Determine "localized iteration space"
   - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
   - reuse ∩ localized iteration space ⇒ locality

# Types of Data Reuse/Locality

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```




Spatial Temporal Group

# Reuse Analysis: Representation

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

- Map *n* loop indices into *d* array indices via array indexing function:

$$\vec{f}(\vec{\imath}) = H\vec{\imath} + \vec{c}$$

$$\texttt{A[i][j]} = \texttt{A}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\texttt{B[j][0]} = \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\texttt{B[j+1][0]} = \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

# Finding Temporal Reuse

- Temporal reuse occurs between iterations $\vec{i}_1$ and $\vec{i}_2$ whenever:

$$H\vec{i}_1 + \vec{c} = H\vec{i}_2 + \vec{c}$$

$$H(\vec{i}_1 - \vec{i}_2) = \vec{0}$$

- Rather than worrying about individual values of $\vec{i}_1$ and $\vec{i}_2$, we say that reuse occurs along direction vector $\vec{r}$ when:

$$H(\vec{r}) = \vec{0}$$

- Solution: compute the *nullspace* of *H*

# Temporal Reuse Example

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

- Reuse between iterations $(i_1,j_1)$ and $(i_2,j_2)$ whenever:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ j_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ j_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

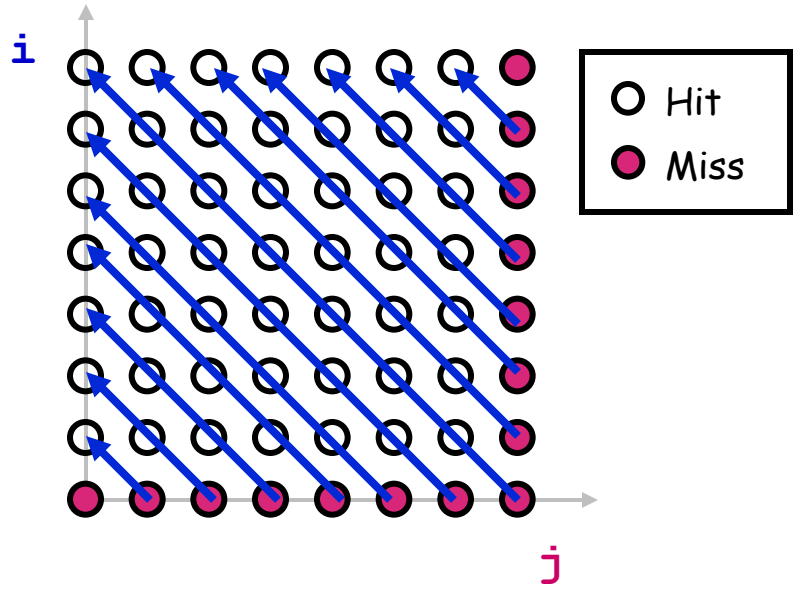
$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- True whenever $j_1 = j_2$, and regardless of the difference between $i_1$ and $i_2$.
  - i.e. whenever the difference lies along the nullspace of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, which is span{(1,0)} (i.e. the outer loop).

# More Complicated Example

```
for i = 0 to N-1
   for j = 0 to N-1
      A[i+j][0] = i*j;
```

$$\texttt{A[i+j][0]} = \texttt{A}\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$




- Nullspace of $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ = span{(1,-1)}.

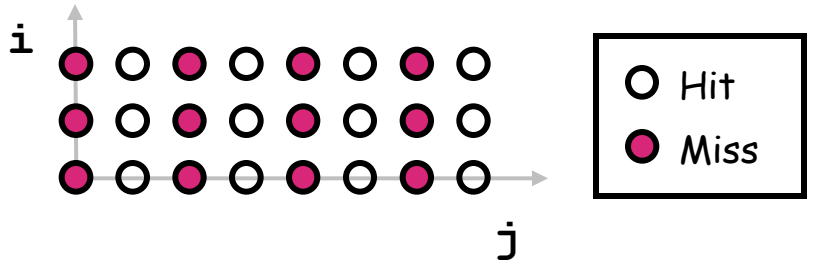
# Computing Spatial Reuse

- Replace last row of $H$ with zeros, creating $H_s$
- Find the nullspace of $H_s$

- Result: vector along which we access the same row

# Computing Spatial Reuse: Example

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```



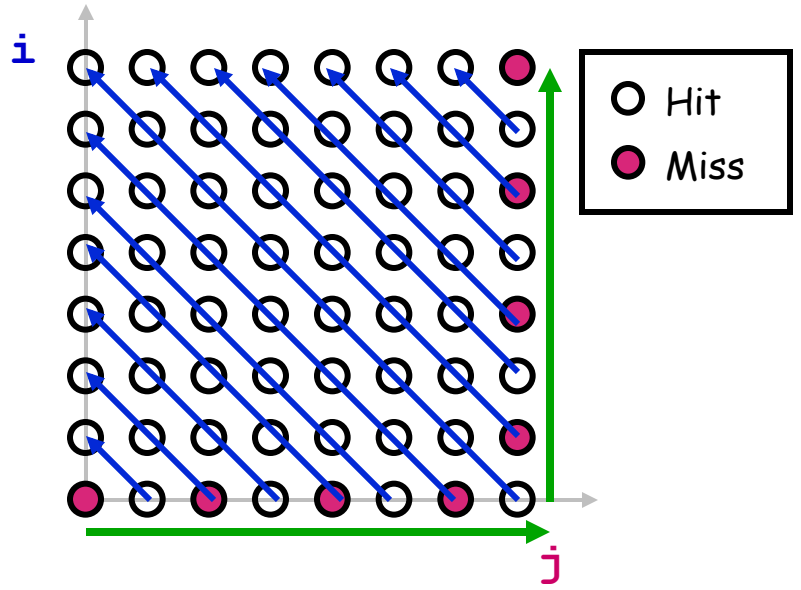

$$\texttt{A[i][j]} = \texttt{A}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

- $H_s = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
- Nullspace of $H_s$ = span{(0,1)}
  - i.e. access same row of **A[i][j]** along inner loop

# Computing Spatial Reuse: More Complicated Example

```
for i = 0 to N-1
   for j = 0 to N-1
      A[i+j] = i*j;
```

$$\texttt{A[i+j]} = \texttt{A}\left(\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}\right)$$




- $H_s = \begin{bmatrix} 0 & 0 \end{bmatrix}$
- Nullspace of $H$ = span{(1,-1)}
- Nullspace of $H_s$ = span{(1,0),(0,1)}

# Group Reuse

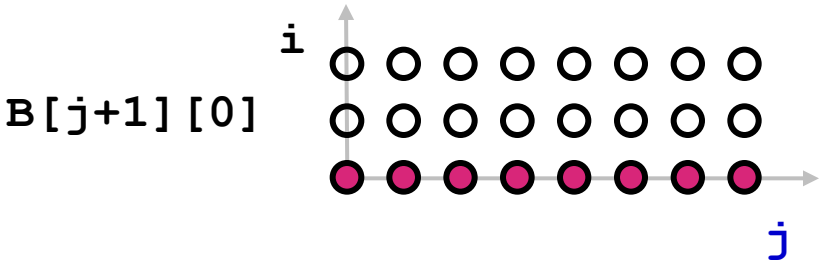
```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

- Only consider "uniformly generated sets"
  - index expressions differ only by constant terms
- Check whether they actually do access the same cache line
- Only the "leading reference" suffers the bulk of the cache misses

# Localized Iteration Space

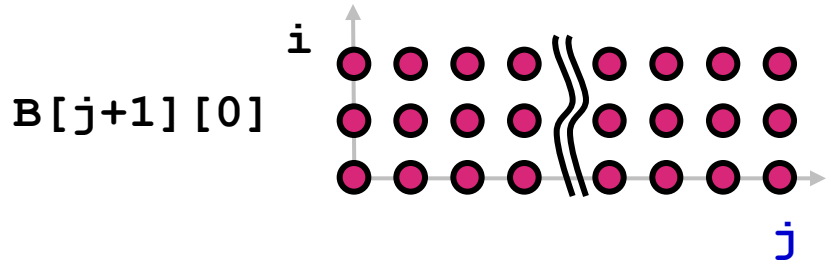
- Given finite cache, when does reuse result in locality?

```
for i = 0 to 2
  for j = 0 to 8
    A[i][j] = B[j][0] + B[j+1][0];
```




Localized: both i and j loops
(i.e. span{(1,0),(0,1)})

```
for i = 0 to 2
  for j = 0 to 1000000
    A[i][j] = B[j][0] + B[j+1][0];
```




Localized: j loop only
(i.e. span{(0,1)})

- Localized if accesses less data than *effective cache size*

# Computing Locality

- Reuse Vector Space ∩ Localized Vector Space ⇒ Locality Vector Space
- Example:

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

- If both loops are localized:
  - span{(1,0)} ∩ span{(1,0),(0,1)} ⇒ span{(1,0)}
  - i.e. temporal reuse *does* result in temporal locality
- If only the innermost loop is localized:
  - span{(1,0)} ∩ span{(0,1)} ⇒ span{}
  - i.e. no temporal locality