CMU 15-671 Models of Software Systems Fall 1997

Abstraction; Algebraic Reasoning

Garlan Homework 8 Due: 20 October 1997

Problem 1.

The following questions refer to the Register problem that you did in HW 6. Consider a concrete implementation of the register state space that represents the class register as two sequences: one that contains the enrolled students, and the other that is a sequence of YES or NO, indicating whether the corresponding student has completed the course.

a. Write a schema to model the concrete state space. Think carefully about the representation invariant.

b. Also write a schema representing the initial state. Argue (informally) that the Initialization Theorem holds for this schema (i.e., there exists a state space satisfying the intial conditions.)

c. Write down (as a schema) the abstraction relation that relates the concrete state space and the previous abstract state space.

d. Explain why your concrete representation is "adequate."

e. Argue that the initial state is a reasonable concrete representation by showing that it corresponds to the initial state of the abstract state space.

f. Produce a concrete version of AddStudent.

g. Argue informally (in the style of Spivey's paper, and the lecture slides) that this concrete operation is correct. (Hint: first show that the operation is "adequate"– defined over an appropriate set of concrete states; then show that it produces results that are consistent with the abstract operation.)

Problem 2.

Refer the Larch trait *Table* defined in Lecture 14.

a. What Bool terms do the following terms reduce to?

i2 ∈ add(add(new, i1, v1), i2, v2) = ?
i2 ∈ add(add(new, i2, v2), i1, v1) = ?
i3 ∈ add(add(new, i1, v1), i2, v2) = ?
i1 ∈ add(add(add(new, i1, v1), i2, v2), i1, v3) = ?

b. What Val terms do the following terms reduce to?

lookup(add(add(new, i1, v1), i2, v2), i2) = ?
lookup(add(add(new, i2, v2), i1, v1), i2) = ?
lookup(add(add(new, i1, v1), i2, v2), i1) = ?
lookup(add(add(add(new, i1, v1), i2, v2), i1, v3), i1) = ?

c. What Int terms (values) do the following terms reduce to?

size(add(add(new, i1, v1), i2, v2)) = ?
size(add(add(new, i2, v2), i1, v1)) = ?
size(add(add(add(new, i1, v1), i2, v2), i1, v3)) = ?

d. Is "add" commutative? (Does it matter in which order I add (i, v) bindings to my table?) Why or why not?

e. Can I tell if multiple copies of the same (i, v) binding are stored in my table? Why or why not?

f. Can multiple values be associated with a single index? Why or why not?

Problem 3.

Write an algebraic specification for a set type. The specification should include the following operations: newSet, addElement, isEmpty, elementOf (to create a set, add an element to a set, check if a set is empty, and check if an element is a member of a set).