

# 15-494/694: Cognitive Robotics

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Lecture 7:

The World Map

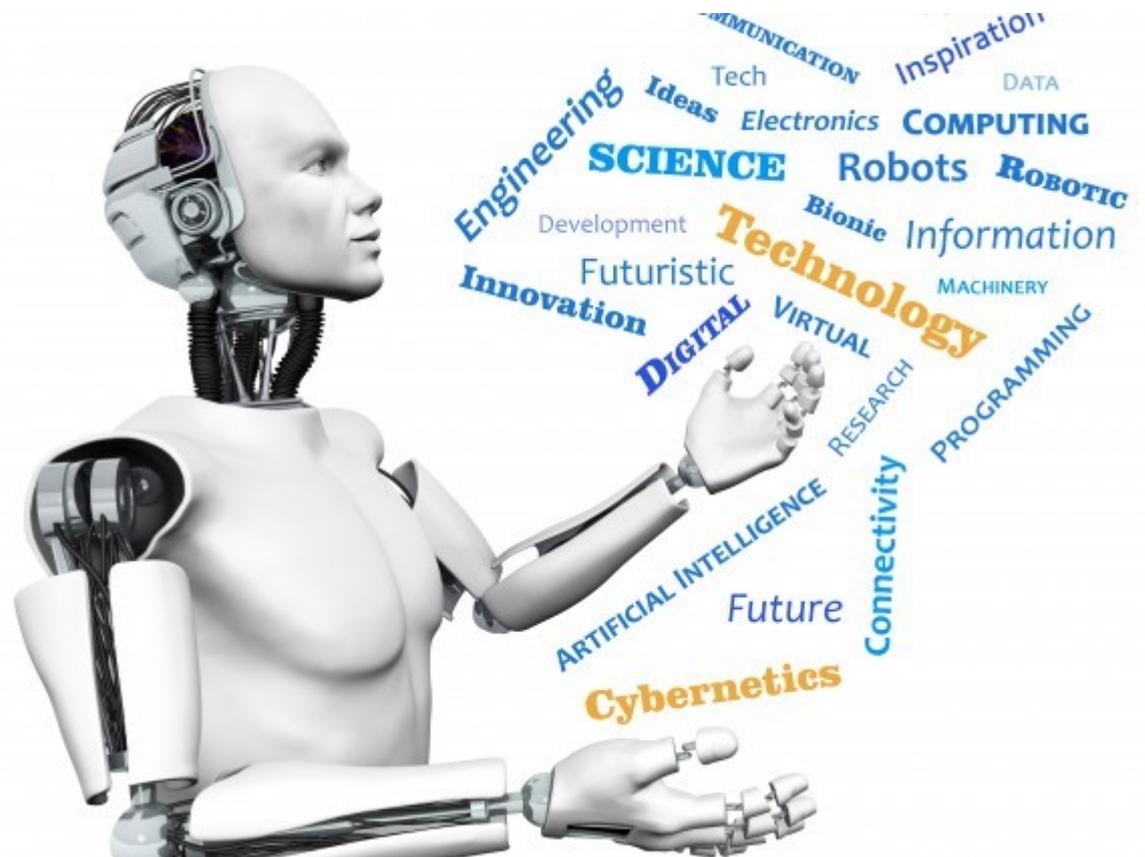


Image from <http://www.futuristgerd.com/2015/09/10>

# Outline

- Why have a world map?
- What's in Cozmo's native world map?
- Cozmo localization
- Object pose: quaternions
- Designing our own world map
- Obstacle detection

# Why Have A World Map?

- Represent objects available to the robot.
- Landmarks to be used for localization.
- Obstacle avoidance during path planning.

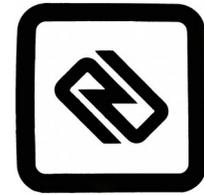
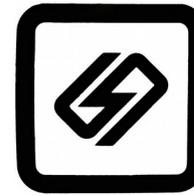
# What's In Cozmo's Native World Map?

- Cozmo himself
- The light cubes, once seen
- The charger, once sensed or seen
- Faces that have been detected
- Custom markers that have been detected
- To access the world map:
  - `robot.world` is a `cozmo.world.World` object
  - components: `robot.world.light_cubes`, etc.

# Light Cube Markers

“Paperclip”

1:



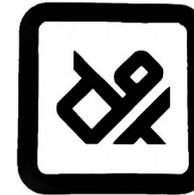
“Anglepoise Lamp”

2:



“Deli Slicer” or  
“car seat”

3:



0°

90°

180°

270°

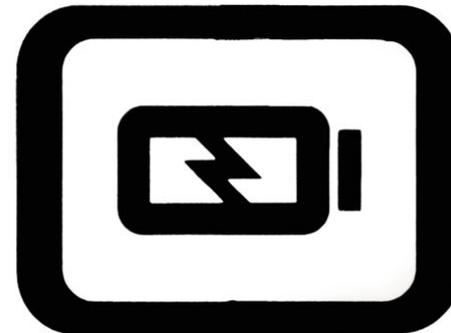
bot

top



# The Charger

- Base frame is front lip; marker in back.
- `robot.is_on_charger` is True or False
- Robot won't move while on the charger

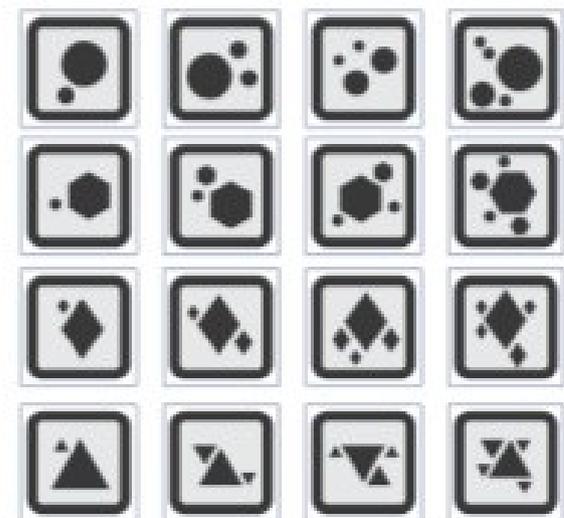


# Custom Markers

Markers: Cubes, Charger, & Custom Objects (SDK)



Custom Markers for SDK



# Origin ID

- The robot's origin\_id starts at 1.
- Every time Cozmo is picked up and put down, he may get a new origin\_id value.
- Landmarks can pull him back to an old id.
  
- Object origin\_id's start at -1 (invalid).
- Every time the robot sees an object, that object's origin\_id is updated to match the robot's.
- Object poses are only valid if their origin\_id matches the robot's.

# Cozmo's Localization

- The cubes serve as visual landmarks that contribute to Cozmo's localization.
- The charger also contributes, if Cozmo has seen the marker.
- When Cozmo is on the charger, he knows exactly where he is.
- If a cube changes position, did the cube move, or did Cozmo move?
  - Cozmo knows when he has moved.

# Object Pose

- The robot, cubes, and charger have a *pose* attribute that is an instance of `cozmo.util.Pose`.
- `robot.pose.position` is (x,y,z) coordinates.
- `robot.pose.rotation` is complicated:
  - a quaternion gives the full 3D pose
  - `angle_z` gives the orientation about the z-axis, which is usually all you care about

# Quaternions

- A quaternion  $q$  is a four-dimensional complex number  $(w, x, y, z)$  or  $(q_0, q_1, q_2, q_3)$ .
- $w$  is a point on the real axis, and  $x, y, z$  are points on the  $i, j, k$  imaginary axes.

$$q = w + x \cdot i + y \cdot j + z \cdot k$$

$$i \cdot i = j \cdot j = k \cdot k = i \cdot j \cdot k = -1$$

$$i \cdot j = k \quad j \cdot k = i \quad k \cdot i = j$$

$$j \cdot i = -k \quad k \cdot j = -i \quad i \cdot k = -j$$

# Quaternions and Rotations

The mathematical properties of quaternions mirror those of 3D rotations: multiplication is not commutative!

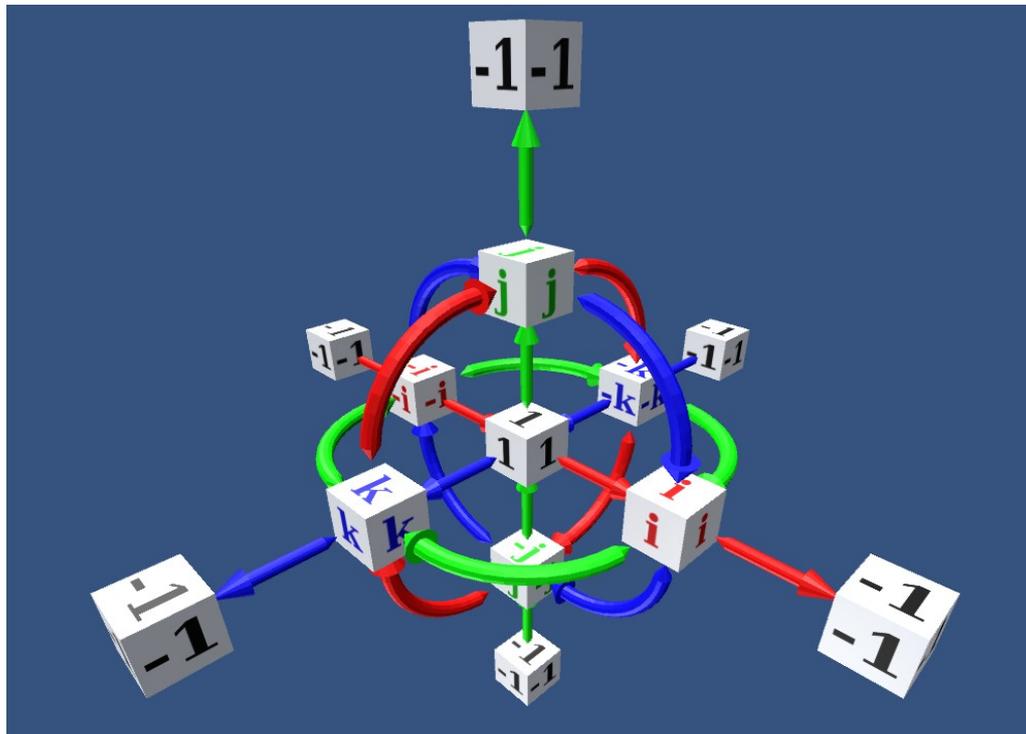


Image source: <https://aha.betterexplained.com>

# Magnitude of a Quaternion

$$q = w + x \cdot i + y \cdot j + z \cdot k$$

$$\|q\| = \sqrt{w^2 + x^2 + y^2 + z^2}$$

For pure rotations we want  $\|q\|=1$ .

# Quaternions as Poses

- Quaternions describe *rotations* in terms of an axis of rotation and an angle  $\theta$ .
- Think of a pose as a rotation from the world reference frame (z up, x forward) to the object's reference frame.
- We can also represent rotations using 4x4 transformation matrices.
- To compose rotations:
  - Multiply the transformation matrices, or
  - Multiply the quaternions

# Simple Cases

- “No rotation”:

$$q = (1, 0, 0, 0)$$

- Rotation by  $\theta$  about the z axis:

$$q = (\cos \theta/2, 0, 0, \sin \theta/2)$$

- General case:

- The magnitude of the rotation is  $\sin \theta/2$ .
- The direction of rotation is indicated by distributing  $\sin \theta/2$  among the i,j,k axes.
- Real  $w = \cos \theta/2$  is a normalization term.

# Motion Model

- The cozmo-tools particle filter includes a motion model.
- How does it get motion information?
- Look for changes in robot.pose every few milliseconds.
- If pose changes but origin\_id remains the same, we have valid motion info.
- If origin\_id changed, cannot subtract poses this time. Wait for next time.

# The cozmo-tools World Map

Why make our own world map?

- To support additional object types:
  - ArUco markers
  - Walls
  - Chips
  - Other cameras
  - Other robots (shared world map)
- To allow our own particle filter to maintain the map (SLAM algorithm)

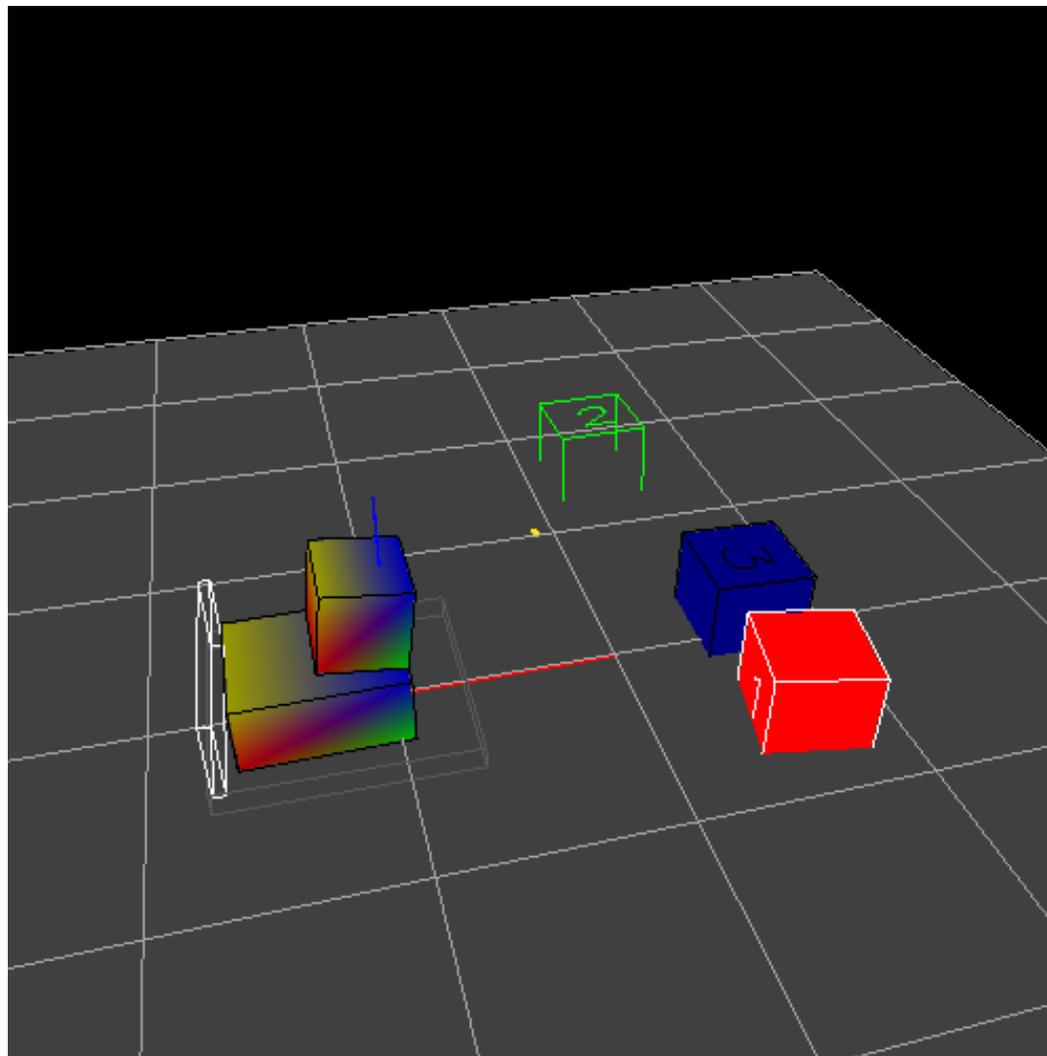
# The cozmo-tools World Map

- Stored in `robot.world.world_map`
- Objects are in a dictionary:  
`robot.world.world_map.objects`
- Subclasses of `WorldObject`:
  - `LightCubeObj`
  - `ChargerObj`
  - `CustomMarkerObj`
  - etc.
- Type “show objects” to display.

# The cozmo-tools World Map

- Robot position maintained by our particle filter, not robot.pose
  - Type “show pose” to see both
- Cubes and charger imported from their SDK representations:
  - wcube1, wcube2, wcube3, wcharger
  - wcube1.sdk\_obj is cube1

# show worldmap\_viewer



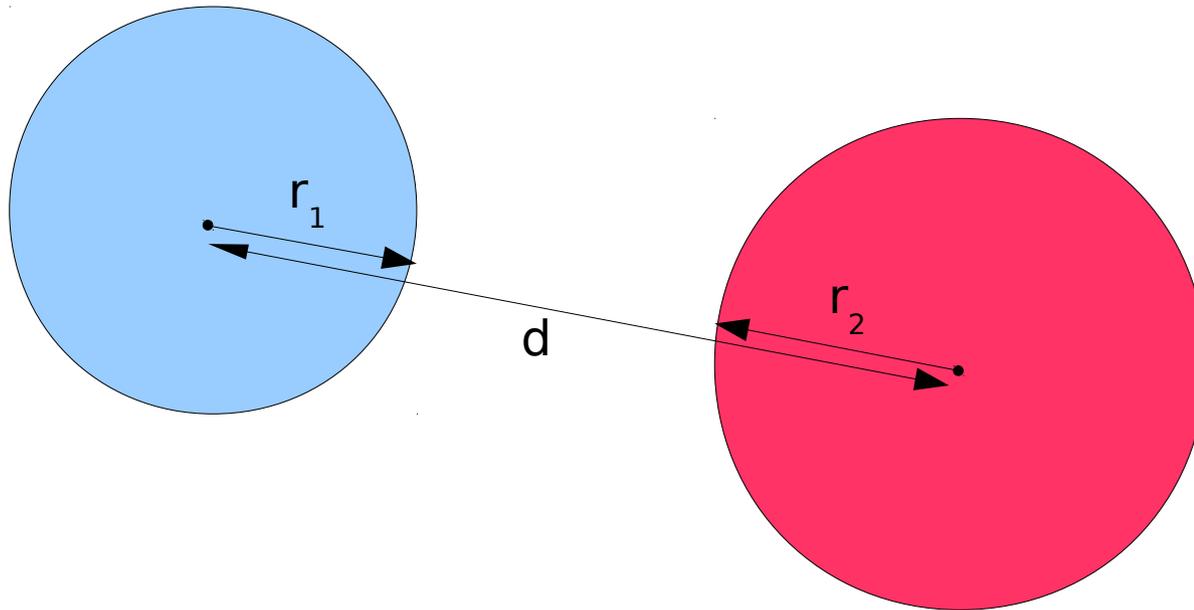
# Obstacle Avoidance

- Having a world map allows us to do path planning to avoid obstacles.
- The path planner needs a way to detect when two objects collide.
- How can we detect collisions?

# Collision Detection

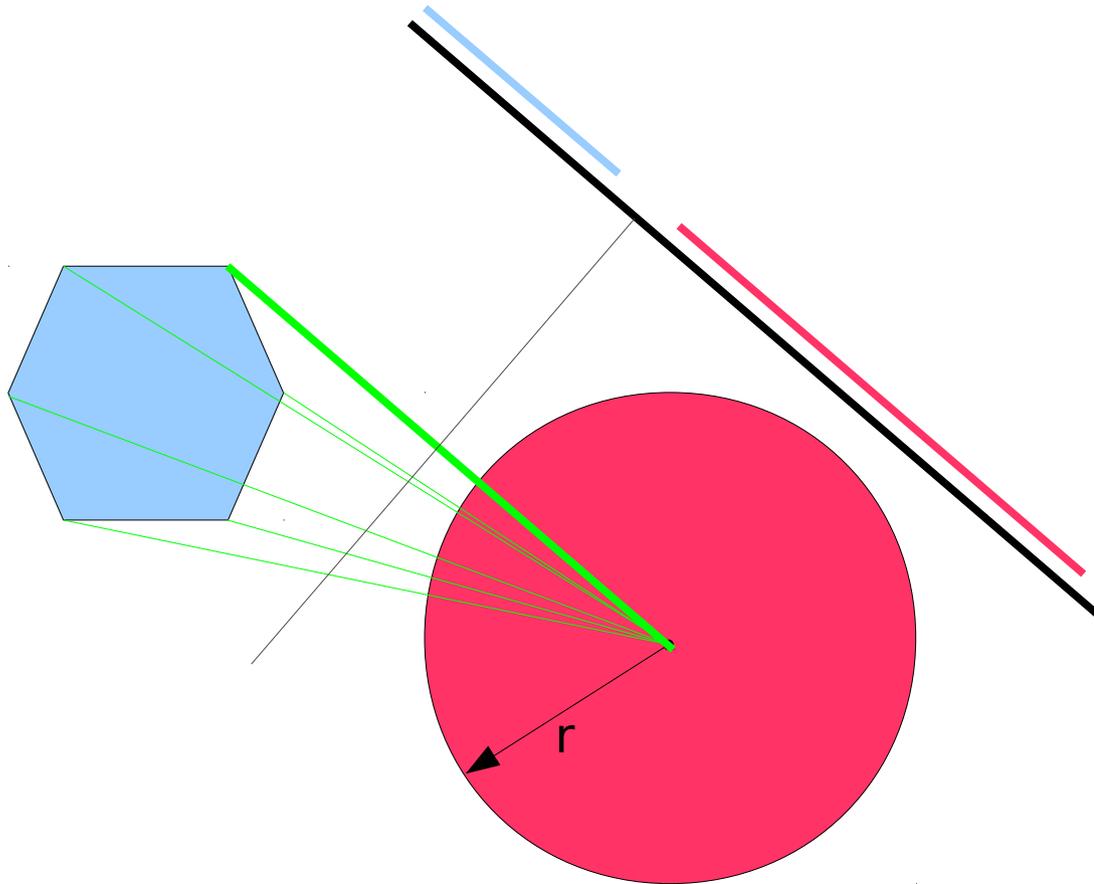
- Represent the robot and the obstacles as **convex polygons**.
- In 2D, use the Separating Axis Theorem to check for collisions.
  - Easy to code
  - Fast to compute
- In 3D, things get more complex.
  - The GJK (Gilbert-Johnson-Keerthi) algorithm is used in many physics engines for video games.

# Collision Detection: Circles



- Let  $d$  = distance between centers
- Let  $r_1, r_2$  be the radii
- No collision if  $d > r_1 + r_2$

# Collision Detection: Circle and Convex Polygon

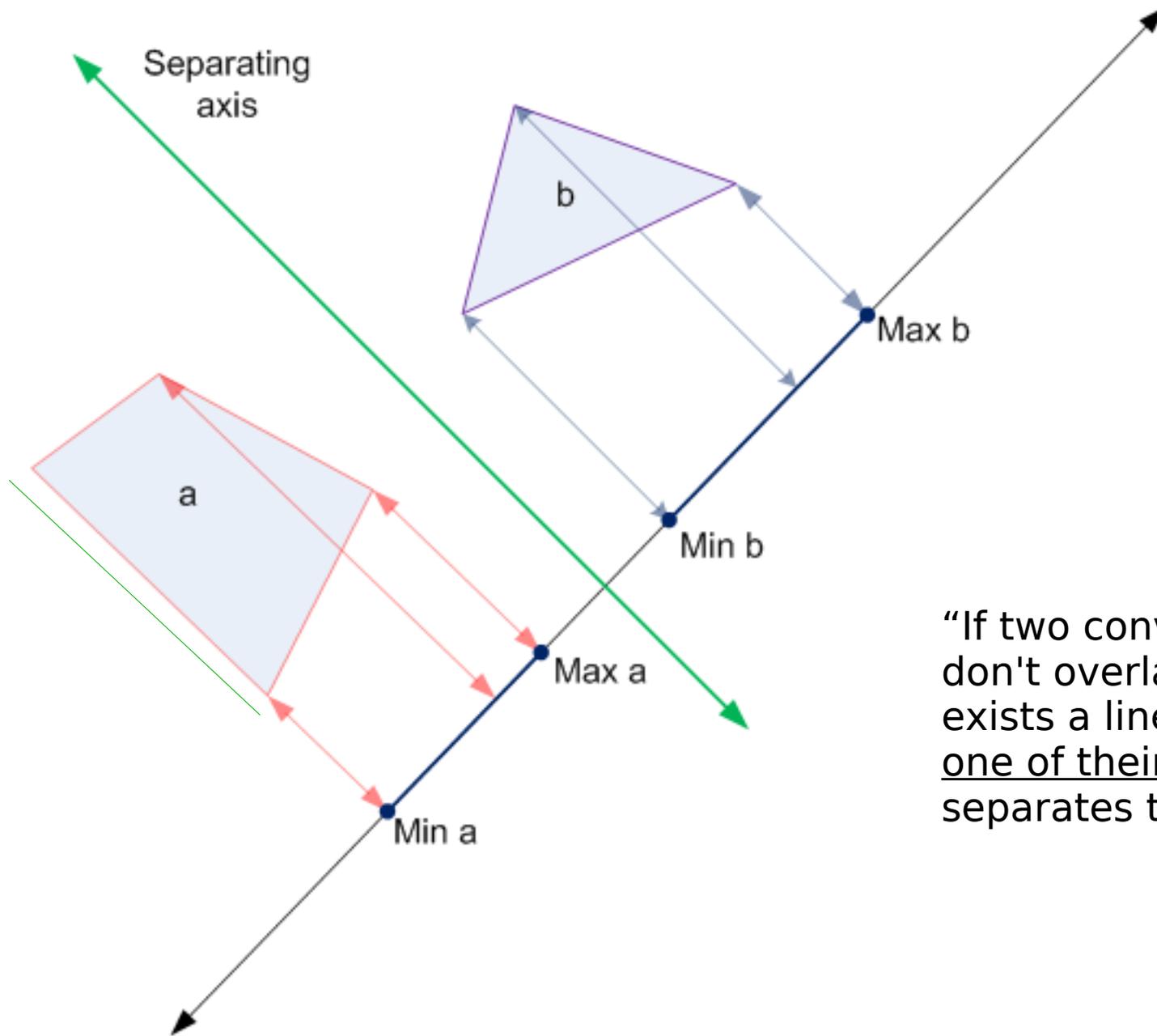


- Separating axes to check are orthogonal to the lines joining the center of the circle to the vertices of the polygon.

# Collision Detection: Two Convex Polygons

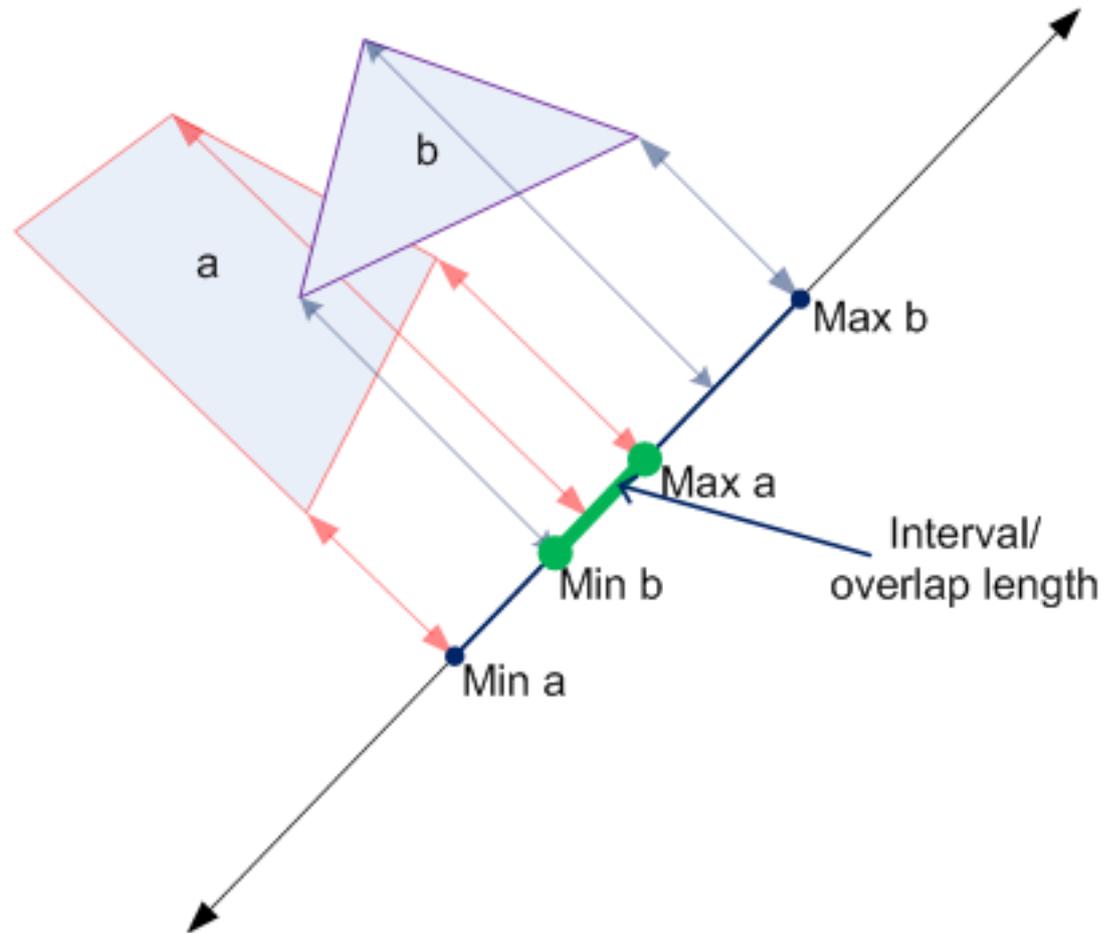
- The Separating Axis Theorem can be used to detect collisions between two convex polygons.
- Time is proportional to the number of vertices.
- To handle non-convex polygons, decompose them into sets of convex polygons and check for collisions between any two components.

# Separating Axis Theorem



“If two convex polygons don't overlap, then there exists a line, parallel to one of their edges that separates them.”

# Separating Axis Theorem



# Collision Detection Algorithm

We only need to find one separating axis to be assured of no collision.

```
def collision_check(poly1, poly2):  
    for axis in Edges(poly1) ∪ Edges(poly2):  
        base = perpendicular_to(axis)  
        proj1 = project_verts(poly1, base)  
        proj2 = project_verts(poly2, base)  
        if not overlap(proj1, proj2):  
            return False  
    return True
```

# How To Build A World Map

- SLAM: Simultaneous Localization and Mapping algorithm.
- Each particle stores:
  - a hypothesis about the robot's location
  - a hypothesis about the map, e.g., a set of landmark identities and locations.
- Particles score well if:
  - Landmark locations match the sensor values predicted by the robot's location.