

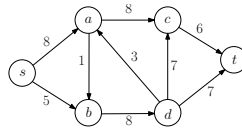
# 15-451 Algorithms, Spring 2017

## Recitation #7 Worksheet

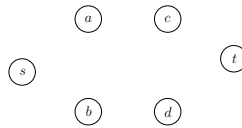
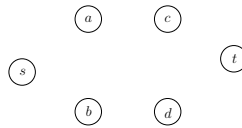
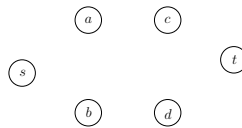
---

### Network Flow

**Example of running Ford-Fulkerson:** Here is a problem from the midterm in 2013. Consider the graph below.



Run Ford-Fulkerson and show the final residual graph. Also what is the maximum flow?



**Network flow for image processing.** Here is an application of network flow to 3-D image processing. To get a 3-D image, you take a stereo camera that produces two pictures, and match the pictures together to produce a single image, where each pixel is labeled with its depth in the image. The problem is that this process can produce noise because of mistakes in matching things up. To fix those mistakes, one approach is to use the fact that in general, most objects have smooth boundaries, which means that most pairs of neighboring pixels should be at the same depth.

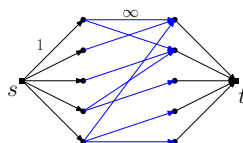
We can formalize the problem like this: Given a pixel image  $I$ , where each pixel is 0 or 1 (indicating close or far — in actual applications, you have many depth levels, but we will just consider 2 levels, so  $I$  is a 0/1 matrix), we want to find the best modification  $I'$  of  $I$  using the following criteria: we pay  $\$a$  for each bit of  $I$  whose depth level we flip, and pay  $\$b$  for each pair of neighboring pixels in  $I'$  at different depths. For instance, if we decide to not change  $I$  at all ( $I' = I$ ) then we pay  $b$  times the number of neighboring pixels in  $I$  that

are at different levels. If we decide to just make all depths equal to 0, we pay  $a$  times the number of ones in  $I$ .

Solve for the best  $I'$  by setting this up as a min-cut problem, and using a max-flow algorithm.

**Hall's Marriage Theorem.** Given a bipartite graph  $G = (L, R, E)$ , we found a maximum matching by finding a  $s$ - $t$  maximum (integer) flow. Here is a slightly different reduction:

(a) Add a new source  $s$  and target  $t$ . Add *unit capacity* directed edges from  $s$  to vertices in  $L$ , and from vertices in  $R$  to  $t$ . Direct edges in  $E$  towards  $R$ , make them *infinite capacity*.



Show a correspondence between integral  $s$ - $t$  flows in this flow network, and matchings in  $G$ . (Hence the maximum matching in  $G$  equals the minimum  $s$ - $t$  cut in this network.)

(b) For any set  $S \subseteq L$ , let  $N(S) = \{v \in R \mid \exists u \in S, (u, v) \in E\}$  be the “neighbors” of  $S$ . Hall's Marriage theorem says: the size of the maximum matching in  $G$  equals  $|L|$  *if and only if* for each subset  $S \subseteq L$ ,  $|N(S)| \geq |S|$ . Deduce Hall's theorem from part (a), and the max-flow min-cut theorem.