15-451 Recitation 11

Computational Geometry

1 Setup and Sweep

You are given a set S of n points with integer coordinates in the plane. You are also given m axis-alligned rectangles, the ith one specified by a pair of points $((x_i, y_i), (x'_i, y'_i))$. (x_i, y_i) is the lower left corner and (x'_i, y'_i) is the upper right corner.

The goal is, to compute for each rectangle the number of points of S that are in it. Give an algorithm whose running time is $O((n+m)\log m)$. For simplicity, assume that all coordinates are bounded by m.

2 Circle with Most Points

Given a set of points $S = \{p_1, \ldots, p_n\}$, and a radius $r > 0$, the goal is to find a circle of radius r that contains the maximum number of points from S.

(a) Give an $O(n^3)$ algorithm for this problem.

(b) Give an $O(n^2 \log n)$ algorithm. (Hint: sweep-*angle*.)

3 The Width of a Set of Points

You're given a set $S = \{p_1, \ldots, p_n\}$ of n points in the plane. A strip of width w is the region between two parallel lines, where the distance between the two lines is w. The goals is to find the strip of minimum width that contains all the points. Given an $O(n \log n)$ algorithm for this problem.

Here's a bit of useful background. The equation $Ax + By = C$ defines a line. Any line (including vertical or horizontal) can be represented this way, where at least one of A or B is non-zero. We can normalize it by dividing the whole equation by $\sqrt{A^2 + B^2}$.

So let's assume that the line has been normalized so that $A^2 + B^2 = 1$. The result of this is a normalized vector (A, B) that is perpendicular to the line $Ax + By = C$.

Consider the function of x and y given by $d(x, y) = Ax + By - C$. Now this function is 0 on the line. In fact, its value at any point (x, y) in the plane is just the (signed) distance between the point and the line.

So the problem of finding the minimum width of a strip containing the set S becomes that of finding a unit vector (A, B) such that the following quantity is minimized:

$$(\max_{(x,y)\in S} Ax + By) - (\min_{(x,y)\in S} Ax + By)$$

If we think of $Ax + By$ as the objective function, we want the difference between the maximum value of it over the set of points minus the minimum of it.