15-451 Algorithms, Spring 2017
Recitation #10 Worksheet

Counting Axis-Aligned Segment Intersections

You're given n vertical line segments in the plane and m horizontal ones. The problem is to count the number of intersections. The brute-force algorithm is to try intersecting all n verticals with all m horizontals, which is $O(nm)$. Devise a solution based on SegTrees that is $O((n+m)\log(n+m))$.

Hint 1: Take advantage of the fact that there are only $2n+m$ relevant y coordinates where anything interesting happens.

Hint 2: Sweep from left to right, processing each event as it comes. The events are: (1) a vertical segment appears, (2) a left end of a horizontal segment appears and (3) the right end of a horizontal segment appears.

First-Fit

You are packing up n items into boxes, and want to use as few boxes as possible. Each box can fit a total of 10 pounds of stuff, and the weight of the i^{th} item is $w_i \leq 10$. Your algorithm is this: initially, all the boxes are lined up, empty. You pick the next unpacked item (say item i), and put it in the *first* box that can hold the item (i.e., whose current weight is at most $10 - w_i$).

1. Argue that if OPT is the optimal number of boxes into which you can pack all the items, then your algorithm uses at most $2 \cdot OPT + 1$ bins.

2. How would you implement the algorithm in time $O(n \log n)$.

VCG and Pricing Advertisements

We saw the VCG mechanism for incentive-compatible auctions in Lecture. Let's use this for pricing online advertising slots. There are 2 ad slots that ElGogo wants to sell on a page, the first slot has a clickthru rate of 0.5, the second has a clickthru rate of 0.3. *Each bidder can get at most one slot.* There are 4 bidders, with the following valuations:

- A: \$10 per click (so, e.g., this bidder values the first slot at 10*0.5 = 5, and the second slot at 10*0.3 = 3.)
- B: \$8 per click
- C: \$7 per click
- D: \$2 per click

1. What is the social-welfare maximizing allocation?

2. What are the VCG payments?

Combinatorial Auctions

VCG can be used even with complicated preferences. Suppose we have two identical hotel rooms in Las Vegas, a flight ticket f from PIT to LAS, and a concert ticket c in Vegas to auction off. In the following, a generic hotel room is denoted by h, and none of the people want two rooms.

- Buyer A: values $\{h\}$ at \$100, $\{f\}$ at \$200, $\{h, f\}$ at \$450, $\{h, f, c\}$ at \$440. (He hates the band in question so much, he gets *negative value* from getting c along with h, f.) All other sets are valued at \$0.
- Buyer B (doesn't care for the concert): values $\{h\}$ at \$50, $\{f\}$ at \$400, $\{h, f\}$ at \$500, and $\{h, f, c\}$ at \$501. All other sets are valued at \$0.
- Buyer C (lives in Vegas): values $\{c\}$ (and all sets containing c) at \$200.

What is the social-welfare maximizing allocation, and what are the VCG payments?