15-451 Algorithms, Fall 2006

Homework # 7 due: Thursday December 7, 2006

Please hand in each problem on a separate sheet and put your name and recitation (time or letter) at the top of each sheet. You will be handing each problem into a separate box, and we will then give homeworks back in recitation. Remember: written homeworks are to be done individually. Group work is only for the oral-presentation assignments.

Problems:

(25 pts) 1. [NP-completeness and approximation algorithms]

Let \( A \) be the set of pairs \((G, k)\) such that \( G \) is a graph with a vertex cover of size \( k \) or less. Let \( C \) be the set of pairs \((G, k)\) such that \( G \) has a vertex cover of size \( k/2 \) or less. Notice that if \((G, k) \in C\) then clearly \((G, k) \in A\) also, so \( A \supseteq C \). Determining whether a given input \((G, k)\) belongs to \( A \) is NP-Complete (this is the Vertex-Cover problem), and also determining whether a given input \((G, k)\) belongs to \( C \) is NP-complete (since this is really the same problem). Describe a set \( B \) such that \( A \supseteq B \supseteq C \) but membership in \( B \) can be decided in polynomial time. So this is just like the situation on Mini 5. Hint: think approximation algorithms.

(25 pts) 2. [Random-access\(^1\) long division].

Give a polynomial time algorithm to find the \( N \)th digit of the fraction \( A/B \), where \( A \), \( B \) and \( N \) are all given in binary.

Input: integers \((A, B, N)\) in binary notation, where \( A < B \).

Let \( 0.d_1d_2d_3 \ldots \) be the decimal expansion of the fraction \( A/B \).

Output: \( d_N \).

Note: the key thing here is that your algorithm’s running time should be polynomial in \( \log N \) (and \( \log A \) and \( \log B \)). The standard way of doing long division would instead be polynomial in \( N \). In particular, the standard long division would look like this:

\[
\text{for } i = 1 \text{ to } N \text{ do:} \\
\quad d_i = 10A \text{ div } B; \\
\quad A = 10A \text{ mod } B;
\]

where “div” is integer division.

(50 pts) 3. [Review] Last year’s final is attached to this assignment. We recommend that you complete the entire final for practice. For this homework, for 50 points, choose 5 problems out of \( \{1, 3, 4, 6, 8, 9, 10\} \) and turn in solutions to them. For the purpose of this assignment, they will be graded at 10 points apiece. (Problems 2, 5, and 7 have already appeared on previous homeworks, minis, or tests).

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\(^1\)“Random access” as in random-access memory, i.e., as opposed to sequential-access. Not “random” as in probability.