

15-451 Algorithms, Fall 2006

Homework # 7

due: Thursday December 7, 2006

Please hand in each problem on a separate sheet and put your **name** and **recitation** (time or letter) at the top of each sheet. You will be handing each problem into a separate box, and we will then give homeworks back in recitation. Remember: written homeworks are to be done **individually**. Group work is only for the oral-presentation assignments.

Problems:

(25 pts) 1. [NP-completeness and approximation algorithms]

Let \mathcal{A} be the set of pairs (G, k) such that G is a graph with a vertex cover of size k or less. Let \mathcal{C} be the set of pairs (G, k) such that G has a vertex cover of size $k/2$ or less. Notice that if $(G, k) \in \mathcal{C}$ then clearly $(G, k) \in \mathcal{A}$ also, so $\mathcal{A} \supseteq \mathcal{C}$. Determining whether a given input (G, k) belongs to \mathcal{A} is NP-Complete (this is the Vertex-Cover problem), and also determining whether a given input (G, k) belongs to \mathcal{C} is NP-complete (since this is really the same problem). Describe a set \mathcal{B} such that $\mathcal{A} \supseteq \mathcal{B} \supseteq \mathcal{C}$ but membership in \mathcal{B} can be decided in polynomial time. So this is just like the situation on Mini 5. Hint: think approximation algorithms.

(25 pts) 2. [Random-access¹ long division].

Give a polynomial time algorithm to find the N th digit of the fraction A/B , where A , B and N are all given in binary.

Input: integers (A, B, N) in binary notation, where $A < B$.

Let $0.d_1d_2d_3\cdots$ be the decimal expansion of the fraction $\frac{A}{B}$.

Output: d_N .

Note: the key thing here is that your algorithm's running time should be polynomial in $\log N$ (and $\log A$ and $\log B$). The standard way of doing long division would instead be polynomial in N . In particular, the standard long division would look like this:

for $i = 1$ to N do:
 $d_i = 10A \text{ div } B$;
 $A = 10A \text{ mod } B$;

where "div" is integer division.

(50 pts) 3. [Review] Last year's final is attached to this assignment. We recommend that you complete the entire final for practice. For this homework, for 50 points, choose 5 problems out of $\{1, 3, 4, 6, 8, 9, 10\}$ and turn in solutions to them. For the purpose of this assignment, they will be graded at 10 points apiece. (Problems 2, 5, and 7 have already appeared on previous homeworks, minis, or tests).

¹"Random access" as in random-access memory, i.e., as opposed to sequential-access. Not "random" as in probability.