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Exercise 1 SOLUTION

Problem 1: Problem Scaling

1. Fill in the table below showing how the amounts of computation and communication would scale *per processor* and *per iteration*.

	Computation	Communication
(a): $N' = 2N$, $P' = P$	8×	4×
(b): $N' = N$, $P' = 8P$	1/8×	1/4×

2. Based on these specific cases, give formulas for how the computation, communication, and arithmetic intensity would scale (per processor and per iteration) as functions of N and P.

Computation	Communication	Arithmetic Intensity
N^3/P	$N^2/P^{2/3}$	$N/P^{1/3}$

3. Fill in the following table with formulas indicating the problem size N', the per-processor memory requirement M', the ideal total time T', and the change in arithmetic intensity.

Scaling Type	N'	M'	T'	Arith. Intensity
Problem	N	$M/8$	$T/8$	$1/2\times$
Memory	$2N$	M	$2T$	$1\times$
Time	$2^{3/4}N \approx 1.68N$	$2^{-3/4}M \approx 0.59M$	T	$2^{-1/4}\times \approx 0.84\times$

Problem 2: Interconnection Networks

1. Give a formula for $n(k, l)$.

   We can write a recurrence as follows:

$$\begin{aligned} n(k,1) &= 2k \\ n(k,l) &= k \cdot n(k, l-1) \end{aligned}$$

   The solution to this recurrence is $n(k, l) = 2k^l$.

2. What is $n(18, 3)$?

   $2 \cdot 18^3 =$11,664.

3. Give a formula for the number of switches required to construct network $N(k, l)$.

   Let $s(k, l)$ be the number of switches in network $N(k, l)$. We can write the following recurrence:

$$\begin{aligned} s(k,1) &= 1 \\ s(k,l) &= k \cdot s(k, l-1) + n(k, l-1) \end{aligned}$$

   The solution for this recurrence is: $s(k, l) = (2l - 1)k^{l-1}$

4. How many switches are in network $N(18, 3)$?

   $5 \cdot 18^2 =$1,620