15-384 Robotic Manipulation
Fall 2008
Final Exam

Time limit: 1 Hour and 30 minutes
Closed book, 1 letter sized paper of notes, calculators not allowed
Please show all work. Intermediate steps must be legible to receive credit.

15 December 2008

Name: ________________
Andrew ID: ________________

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1. (15 points) Consider a 3D mobile robot with a frame attached so that the $x$ axis points forward and the $z$ axis points up. This robot is equipped with a sensor that can detect the distance to the nearest object along the $z$ axis of the sensor. The sensor is rigidly mounted to the robot, and the relationship between the robot and sensor frames is

$$H_s^r = \begin{bmatrix}
0.9063 & 0 & 0.4226 & 1 \\
0 & 1 & 0 & 0 \\
-0.4226 & 0 & 0.9063 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

The robot begins with its frame aligned with the world frame, then it moves 3 meters forward, then it rotates by 90 degrees about its $z$ axis, then it moves backwards by 5 meters. At this position, it receives a sensor measurement of 4 meters. What is the location of the sensed object expressed in the world frame?

You do not need to multiply the matrices, just set them up. However, you do need to write down the full homogeneous matrices explicitly.
2. (15 Points) Attach coordinate frames to the robot in the figure below using the DH convention taught in class. Draw the frames directly on the diagram.
3. For the planar robot in the next figure:

(a) (12 Points) Find the Jacobian of the robot.
(b) (17 Points) For the same robot in the previous page it is given that: \( l_1 = 1 \text{ m}, l_2 = 0.3 \text{ m}, d_3 = 0.5 \text{ m}, \theta_1 = 45^\circ, \theta_2 = 0^\circ \).

Find the joint velocities of the robot for the desired end-effector velocity:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix} 3 \text{ m/s} \\
1 \text{ m/s} \end{bmatrix}
\]

HINT: If there are no solutions find the closest joint velocities. If there are many solutions find the “best” one as discussed in class.
4. (16 Points) For each of the following statements write if True or False?

(a) The angular velocity is the same at all points on a rigid body.

(b) The inverse of the jacobian is its transpose.

(c) The inverse of a homogeneous matrix is its transpose

(d) The dimension of SO(3) is 3

(e) If \( r_1 \) and \( r_2 \) are two rows in a rotation matrix, then \( r_1 r_2^T = 0 \)

(f) If \( c_1 \) and \( c_2 \) are two columns in a rotation matrix, then \( c_1^T c_2 = 1 \)

(g) If \( c_1 \) is a column vector in a rotation matrix, then \( c_1^T c_1 = 1 \)

(h) The \( i \)th row of the Jacobian matrix describes the velocity of the end effector when the velocity of the \( i \)th joint is \( \dot{\theta}_i = 1 \) and the velocity of all other joints are zero
5. (25 Points) A 2 link planar PR robot is given in the figure below. All the links are massless. Two point masses $m_1$ and $m_2$ are at the end of links $l_1$ and $l_2$ respectively as drawn in the figure below.

Use the Lagrangian method learned in class to find the equations of motion of this robot.
extra work space:
extra work space: