15-381 Spring 06 Assignment 3: Robot Motion, Game Theory

Questions to Rong Yan(yanrong@cs.cmu.edu)

Out: 2/21/06   Due: 3/7/06

Name: ________________________   Andrew ID: ________________________

Please turn in your answers on this assignment (extra copies can be obtained from the class web page). This assignment must be turned in at the beginning of class at 1:30pm on March 7. Please write your name and Andrew ID in the space provided on the first page, and write your Andrew ID in the space provided on each subsequent page. This is worth 5 points: if you do not write your name/Andrew ID in every space provided, you will lose 5 points.

This assignment has no coding problems

Late policy. Your assignment is due at 1:30pm on 3/7. Submitting your work late will affect its score as follows:

- If you submit it after 1:30pm on 3/7 but before 1:30pm on 3/8, it will receive 90% of its score.
- If you submit it after 1:30pm on 3/8 but before 1:30pm on 3/9, it will receive 50% of its score.
- If you submit it after 1:30pm on 3/9, it will receive no score.

Collaboration policy. You are to complete this assignment individually. However, you are encouraged to discuss the general algorithms and ideas in the class in order to help each other answer homework questions. You are also welcome to give each other examples that are not on the assignment in order to demonstrate how to solve problems. But we require you to:

- not explicitly tell each other the answers
- not to copy answers
- not to allow your answers to be copied

In those cases where you work with one or more other people on the general discussion of the assignment and surrounding topics, we ask that you specifically record on the assignment the names of the people you were in discussion with (or "none" if you did not talk with anyone else). This is worth five points: for each problem, space has been provided for you to either write people's names or "none". If you leave any of these spaces blank, you will lose five points. This will help resolve the situation where a mistake in general discussion led to a replicated weird error among multiple solutions. This policy has been established in order to be fair to the rest of the students in the class. We will have a grading policy of watching for cheating and we will follow up if it is detected.

1 The Robot Motion Planning Problem (15 points)

References (names of people I talked with regarding this problem or "none"):

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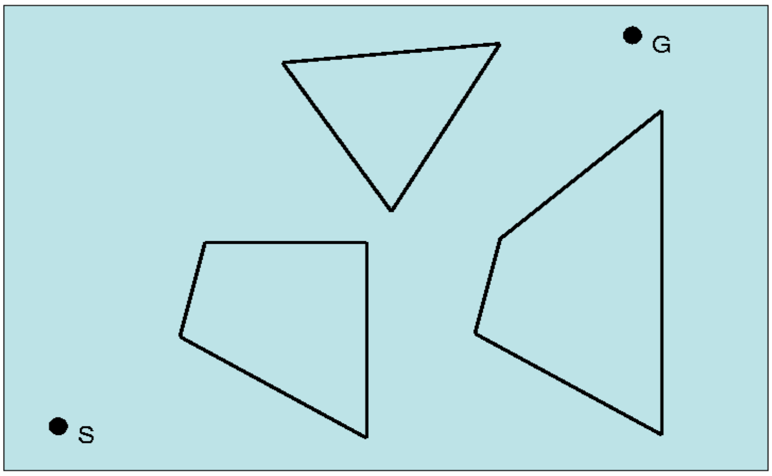
Consider the robot planning problem illustrated in the following figure.

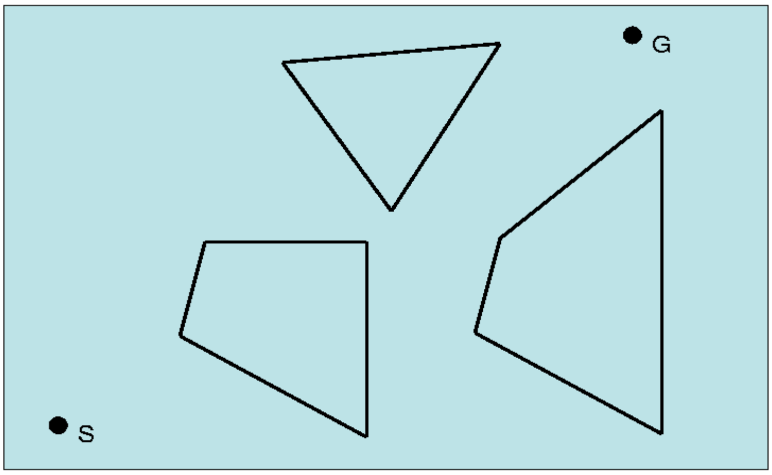
1.1 (5 points)

Draw (approximately) the path found by using potential fields.



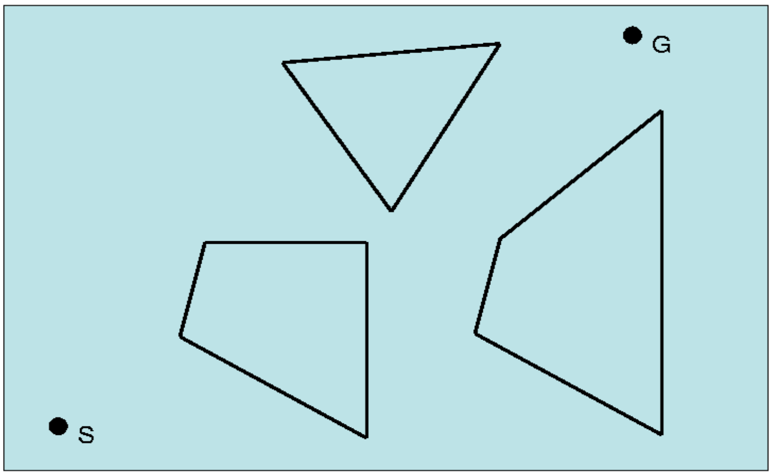
1.2 (5 points)

Draw (approximately) the path found by using approximate cell decomposition (there are several possibilities). Briefly discuss what are the advantages and disadvantages of the approximate cell decomposition technique.



1.3 (5 points)

Draw (exactly) the path found by using the visibility graph technique. Draw the initial visibility graph. Briefly discuss what are the advantages and disadvantages of the visibility graph technique.



2 The Game Play Problem(40 points)

References (names of people I talked with regarding this problem or "none"):

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Gomoku, go-moku, or gobang is an abstract strategy board game with two players. It is traditionally played with black and white stones on a 19 * 19 go board. One player has black stones and the other has white stones. To simplify, we assume the game are played on a 10*10 board with crosses(x) representing black stones and circles(o) representing white stones. In this game, black plays first and players alternate in placing a stone of their color on an empty space. The winner is the first player to get an unbroken row of five stones horizontally, vertically, or diagonally. Some examples of the winning situations for black stones are shown as follows. Please see http://en.wikipedia.org/wiki/Gomoku for more details about this game. This game is an example of 2-player zero-sum deterministic game of perfect information.

		x						x	
		o	x					x	
			o	x	o			x	
				o	x			x	
						x		x	
			o	o			o		
		x	x	x	x	x			
	o	o			o				

2.1 (5 points)

Any two-player zero-sum deterministic game can be represented by the following quintuple: (S, I, Succ, T, V), where S is the entire space of game states, I is the initial state, Succ is the successor function, T are the terminal states, V maps from terminal states to its payoff/utility. Please describe what is each element of the quintuple in the Gomoku game.

2.2 (10 points)

Design two different heuristic evaluation functions for this game given the current state of the board. In this problem, the evaluation function should be able to handle the intermediate states of the game. For example, you cannot define "+1 for black wins, -1 for white wins, 0 otherwise" because it cannot provide useful information for the intermediate states. (Hints: you can consider the number of connected stones on the board.)

In the following discussions, let us assume we have arrived to the following state of board. Player x is going to move next.

					o				
				o		x			
			o			x			
		o		x	x	x		o	
	x					o			

2.3 (10 points)

Solve the whole game and find the next move for x with the mini-max algorithm. Rather than using heuristic evaluation functions, we will evaluate the board state based on the simple win/loss between x and o (i.e., +1 for x wins, -1 for x loses and 0 otherwise). For the purpose of grading, you only need to consider the nodes labeled in the following figure (from 1 to 4) to expand and must use the ascending order for expansion, i.e., on each level of expansion, you must expand the lowest remaining numbered node before expanding the higher numbered nodes. Any node where a player have 5 stones connected is the leave node and has no children.

You do *not* need to draw the entire game tree in this problem, but you need to draw the best path found by the mini-max algorithm (if there are several best paths, you should show the first one expanded). For each node in the path, indicate the index of the node expanded and the evaluated value computed/propagated by the algorithm. For each node, also indicate if it is from a max level or a min level. Moreover, please answer: what should be x's first move? how many nodes does the minimax algorithm need to expand in this case?

						1			
					o	3			
				o		x			
			o			x			
		o	2	x	x	x	4	o	
	x					o			

2.4 (10 points)

Solve the whole game with the alpha-beta pruning. The problem settings are the same as the last problem. But you *need* to draw the entire game tree, except for those parts that you can prune (you must leave them out). Indicate which level of the trees are max levels or min levels. For each node in the game tree, indicate the index of the node expanded and the alpha-beta values discovered. In this case, how many nodes does the algorithm need to expand?

2.5 (5 points)

Now let us consider a different situation. The state of the board is the same as before. But player o need to leave town for his next move (right after the first x's move). So that move will be handled by his representative r. But it is well known that r will just make a random move on the board (from 1 to 4)! Good news is that o will return for his last move. Therefore, the entire moving sequence is x, r, x, o.

Briefly explain how you can modify the mini-max algorithm accordingly to deal with this change. Under this condition, does player x's first move change? Please explain why.

3 Optimal Mixed Strategy (10 points)

Consider the game with the following matrix: (if you look carefully, this is a generalization of the children's game "scissors, rock, paper")

0	1	-2
-1	0	3
2	-3	0

3.1 (5 points)

Show that the value of that game (i.e., the expected payoff that will be obtained by either player using the optimal mixed strategy) is 0. In fact, this is true for any game for which $m_{ij} = -m_{ji}$, but you don't need to prove the general result.

3.2 (5 points)

Derive the optimal mixed strategy, which is the same for both players. If you have trouble with question 1, just assume that the value of the game is 0 and proceed with this question.

4 Widget Companies (10 points)

Consider two widget companies, Acme and U.S. Widgets, competing for the same market and each firm must choose a high price ($2 per widget) or a low price ($1 per widget). Here are the rules of the game:

1. At a price of $2, 5000 widgets can be sold for a total revenue of $10000;
2. At a price of $1, 10000 widgets can be sold for a total revenue of $10000;
3. If both companies charge the same price, they split the sales evenly between them;
4. If one company charges a higher price, the company with the lower price sells the whole amount and the company with the higher price sells nothing;
5. Payoffs are profits – revenue minus the $5000 fixed cost.

(Note: This is a very special case of something called the Cournot's game that models price competition. Unfortunately, it is an overly simplified and unrealistic example because we force it to be a zero-sum game.)

4.1 (5 points)

Write down the game matrix.

4.2 (5 points)

Is there a pure strategy solution? Briefly explain.

5 The Nim Game(10 points)

A very simple version of the game of "Nim" is played as follows. There are 2 players and, at the start, two piles on the table in front of them, each containing 2 matches. In turn, the players take any positive number of matches from one of the piles. The player taking the last match loses.

Now consider a more elaborate version of the game of "Nim" is played as follows. There are 2 players and, at the start, three piles on the table in front of them, each containing 2 matches. In turn, the players take any positive number of matches from one of the piles. The player taking the last match loses. Sketch a game tree. Show that the first player has a sure win.

6 The Pin Game(10 points)

A game is played as follows: The two players, i.e., A and B, simultaneously hold up either one or two pins. A wins if there is a match (the number of pins are the same for both players) and B wins otherwise. The amount won is the number of pins held up by the winner. It is paid to the winner by the loser.

6.1 (5 points)

Write down the matrix form of the game and specify the 2 players' possible pure strategies.

6.2 (2 points)

Explain why there is no pure strategy solution in this game.

6.3 (3 points)

Compute the mixed strategy solution and the value of the game.

7 Mixed strategy(5 points)

Consider a two-player game in which each player has 4 possible strategies (denoted 1 to 4) with the matrix form shown below. Compute a mixed strategy solution, assuming that each player assigns the same probability to strategies 1, 2, and 3.

	1	2	3	4
1	-1	1	1	-1
2	1	-1	1	-1
3	1	1	-1	-1
4	-1	-1	-1	1