Playing and Solving Games

Zero-sum games with perfect information

R&N 6

- Definitions
- Game evaluation
- Optimal solutions
  - Minimax
  - Alpha-beta pruning
- Approximations
  - Heuristic evaluation functions
  - Cutoffs
  - Endgames
- Non-deterministic games (first take)

Types of Games (informal)

	Deterministic	Chance
Perfect Information	Chess, Checkers Go	Backgammon, Monopoly
Imperfect Information	Battleship	Bridge, Poker, Scrabble, Nuclear war

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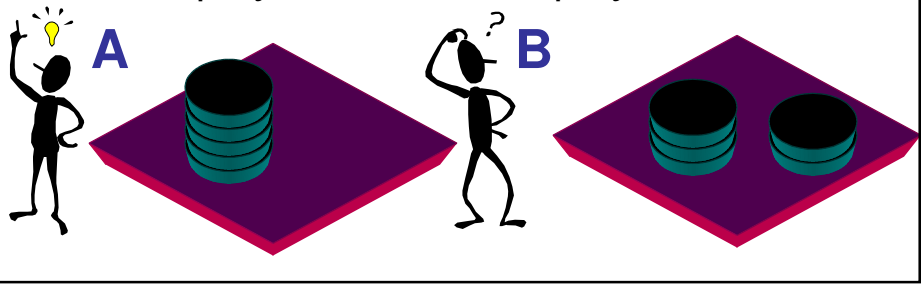
Note: This initial material uses the common definition of what a "game" is. More interesting is the generalization of the theory to scenarios that are far more useful to a wide range of decision making problems. Stay tuned….

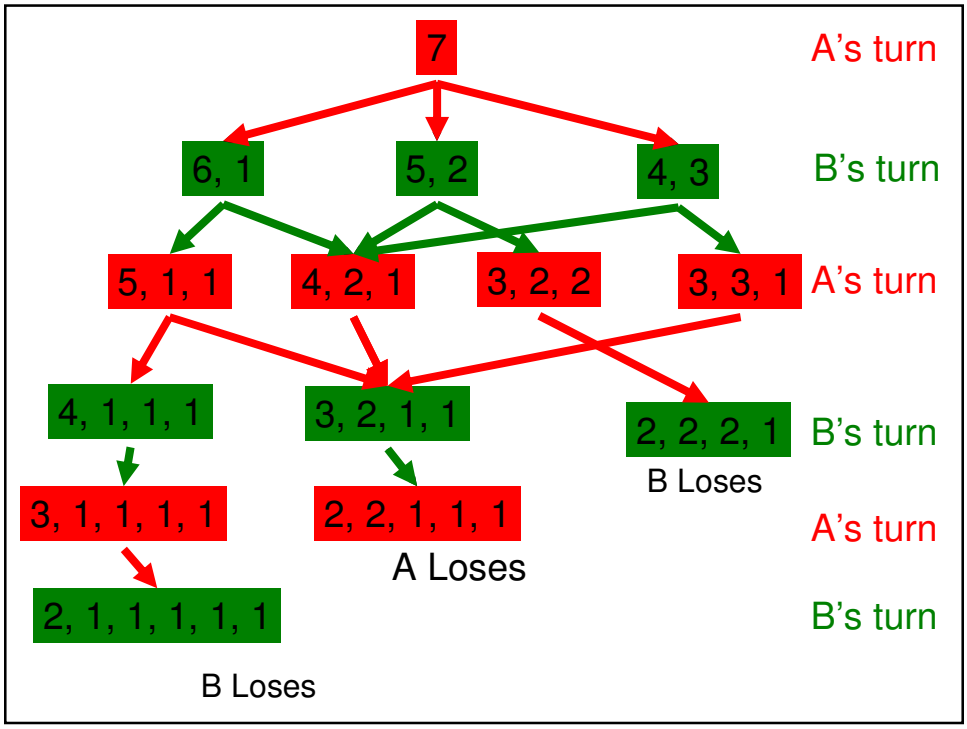
Definitions

- *Two-player game*: Player A and B. Player A starts.
- *Deterministic*: None of the moves/states are subject to chance (no random draws).
- *Perfect information*: Both players see all the states and decisions. Each decision is made *sequentially*.
- *Zero-sum*: Player's A gain is exactly equal to player B's loss. One of the player's must win or there is a draw (both gains are equal).

Example

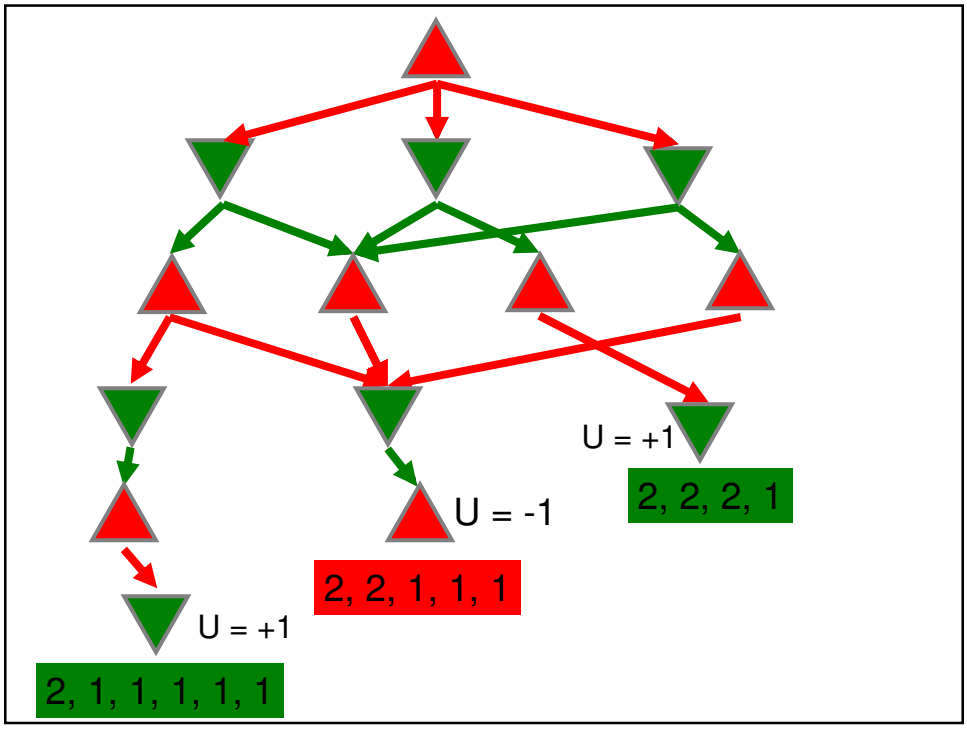
- Initially a stack of pennies stands between two players
- Each player divides one of the current stacks into two unequal stacks.
- The game ends when every stack contains one or two pennies
- The first player who cannot play loses





Search Problem

- *States*: Board configuration + next player to move
- *Successor*: List of states that can be reached from the current state through of legal moves
- *Terminal state*: States at which the games ends
- *Payoff/Utility*: Numerical value assigned to each terminal state. Example:
  - U(s) = +1 for A win, -1 for B win, 0 for draw
- *Game value:* The value of a terminal that will be reached assuming optimal strategies from both players (*minimax* value)
- *Search*: Find move that maximizes game value from current state



Optimal (minimax) Strategies

- Search the game tree such that:
  - A's turn to move → find the move that yields maximum payoff from the corresponding subtree → This is the move most favorable to A
  - B's turn to move → find the move that yields minimum payoff (best for B) from the corresponding subtree → This is the move most favorable to B

Minimax

Minimax (*s*)

If *s* is terminal

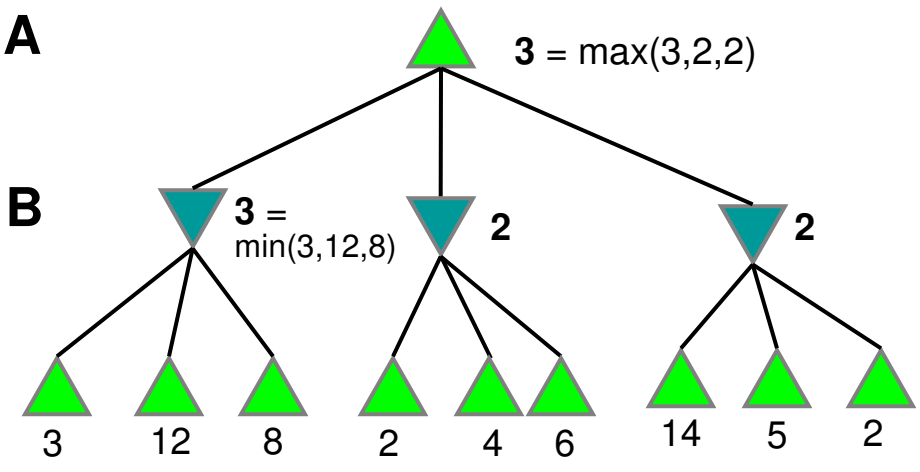
Return *U*(*s*)

If next move is *A*

Return $\max_{s' \in Succs(s)} Minimax(s')$

Else

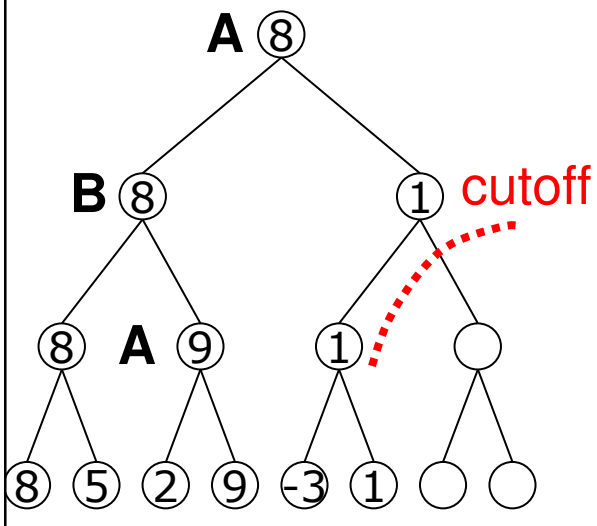
Return $\min_{s' \in Succs(s)} Minimax(s')$



Minimax Properties

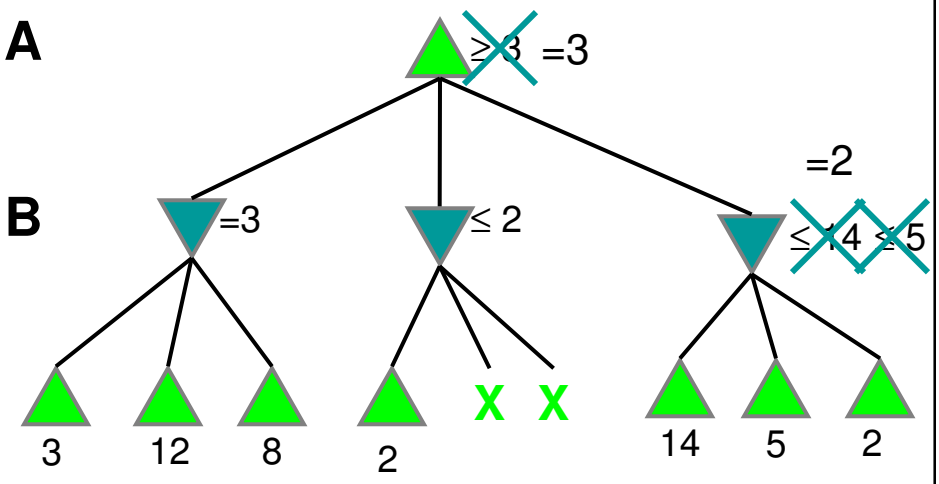
- *Complete:* If finite game
- *Optimal:* If opponent plays optimally
- *Complexity:* Essentially DFS, so:
  - Time: O(B^m)
  - Space: O(Bm)
  - B = number of possible moves from any state (branching factor)
  - m = depth of search (length of game)

Pruning



- The value at A move is 8 (so far)
- If A moves right, the value there is 1 (so far)
- B will *never increase* the value at this node; it will always be less than 8
- B can *ignore* the remaining nodes

Pruning



αβ Pruning

- Maintain:
  - α = Best value found so far at A nodes, including those at current node
  - β = Best value found so far at B nodes, including those at current node
- If at a B node: No need to expand this node any further if α >= β because there is no way that a descendant of the current node can yield a better value

```
Minimax (s,α,β)
    If s is terminal
        Return U(s)
    If A node
        For each s' in Succs(s)
            α = Max(α, Minimax(s',α,β)
            If (α >= β) Return β
        Return α
    If B node
        For each s' in Succs(s)
            β = Min(β, Minimax(s',α,β)
            If (α >= β) Return α
        Return β
```

```
Minimax (s,α,β)
    If s is terminal
        Return U(s)
    If A node
        For each s' in Succs(s)
            α = Max(α, Minimax(s',α,β)
            If (α >= β) Return β
        Return α
    If B node
        For each s' in Succs(s)
            β = Min(β, Minimax(s',α,β)
            If (α >= β) Return α
        Return β
```

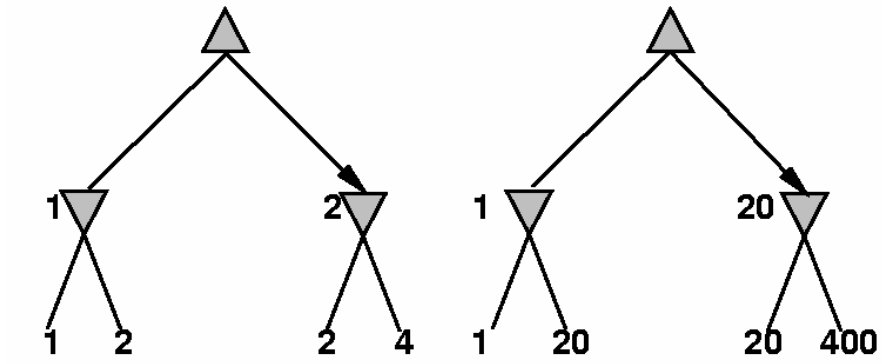
Same as default minimax

Prune if no better solution can be found

Properties

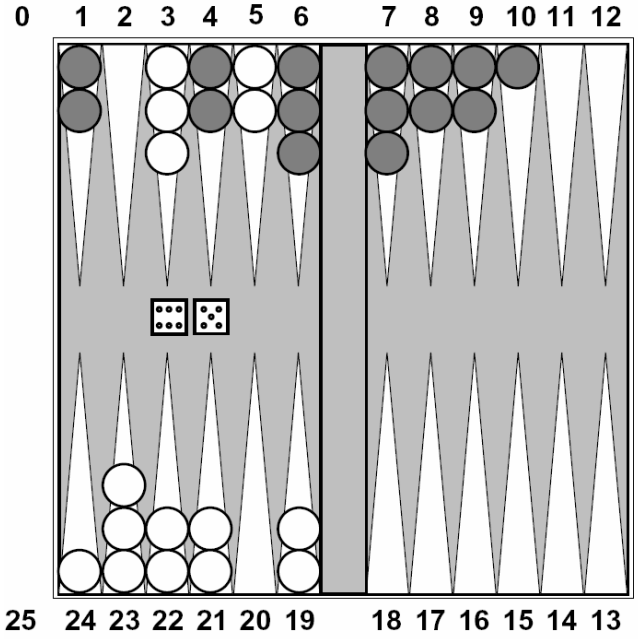
- Guaranteed to find same solution
- O($B^{m/2}$) with proper ordering of the nodes → At "A" node, the successor are in order from high to low score
- Use heuristic evaluation functions to cut off search early
- Example: Weighted sum of number of pieces (material value of state)
- Stop search based on cutoff test (e.g., maximum depth)
- Iterative deepening search to limit DFS
- Solve by brute-force dynamic programming when the number of states is small

Choice of Value?

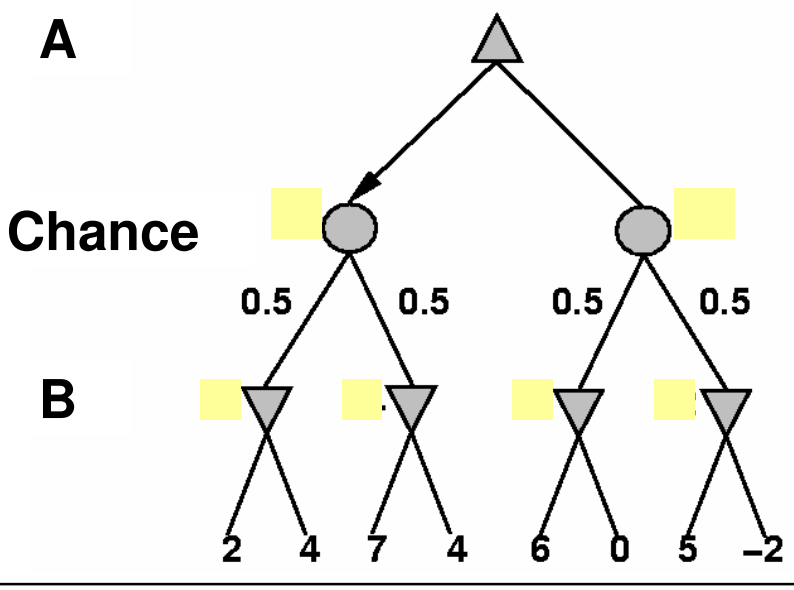


- Absolute game value is different in the two cases
- Minimax solution is the same
- Only the relative ordering of values matters, not the absolute values → *ordinal utility values*
- True only for *deterministic* games
- Evaluation functions can be any function that preserves the ordering of the utility values

Non-Deterministic Games



Non-Deterministic Games

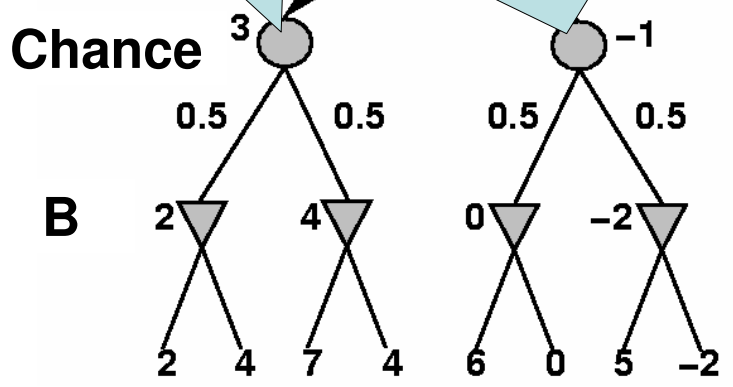


Non-Deterministic Games

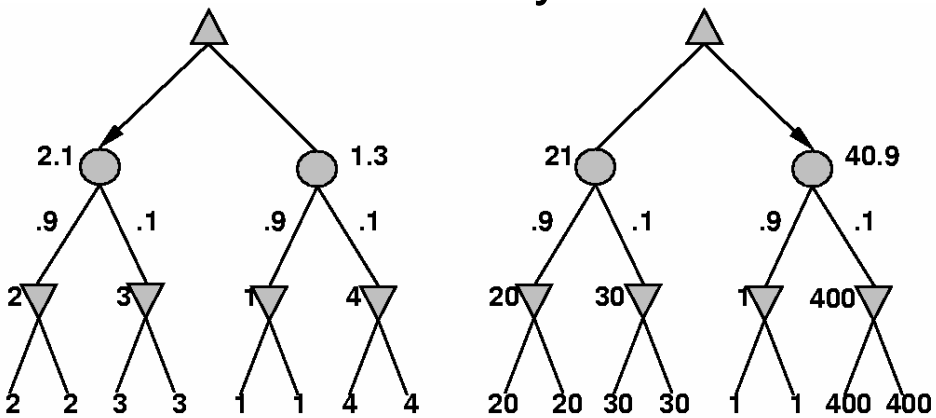
Includes states where neither player makes a choice. A random decision is made (e.g., rolling dice)

Use expected value of successors at chance nodes:

$$\sum_{s' \in Succs(s)} p(s')MiniMax(s')$$



Choice of Utility Values



- Different utility values may yield radically different result even though the order is the same → Absolute utility values do matter
- Utility should be proportional to actual payoff, it is not sufficient to follow the same order
- Think of choosing between 2 lotteries with same odds but radically different payoff distributions
- Implication: Evaluation functions must be *linear positive* functions of utility
- Kind of obvious but important consideration for later developments

Non-Deterministic Minimax

Minimax (*s*)

If *s* is terminal

Return *U*(*s*)

If next move is *A:* Return $\max_{s' \in Succs(s)} Minimax(s')$

If next move is *B* Return $\min_{s' \in Succs(s)} Minimax(s')$

If chance node Return $\sum_{s' \in Succs(s)} p(s') Minimax(s')$

Properties

- α-β pruning can be extended provided that the utility values are bounded → We don't need to evaluate all the children of a chance node to bound the average
- Less effective
- Different outcomes depending on exact values of utility, not just ordering

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