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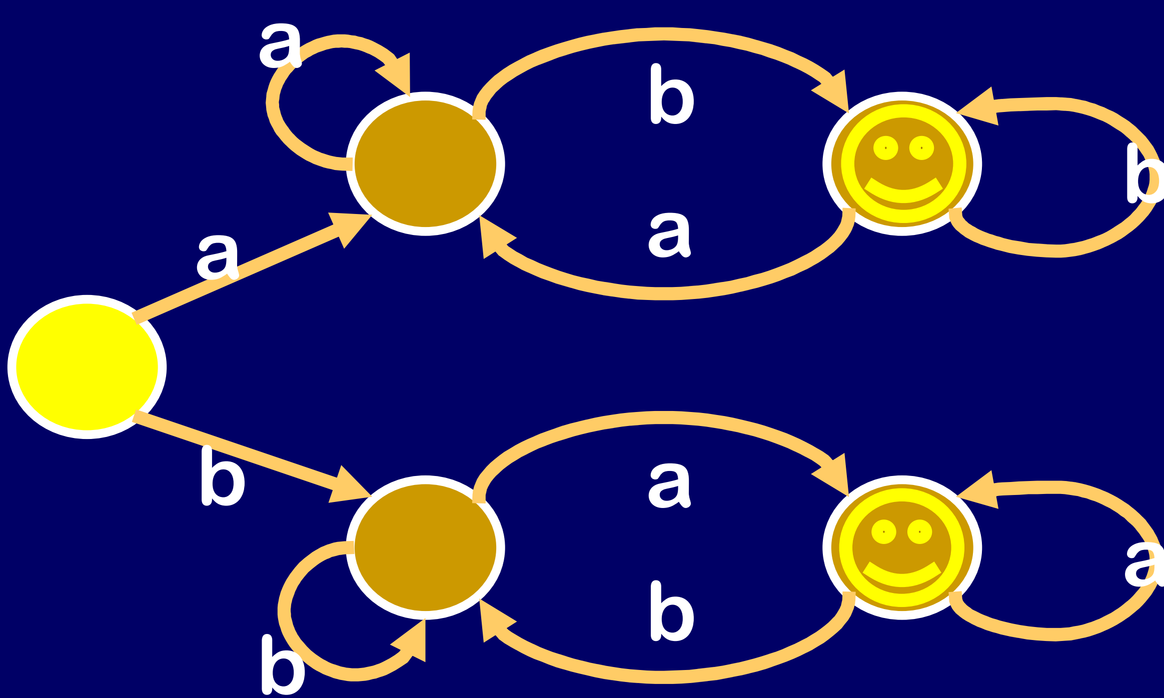
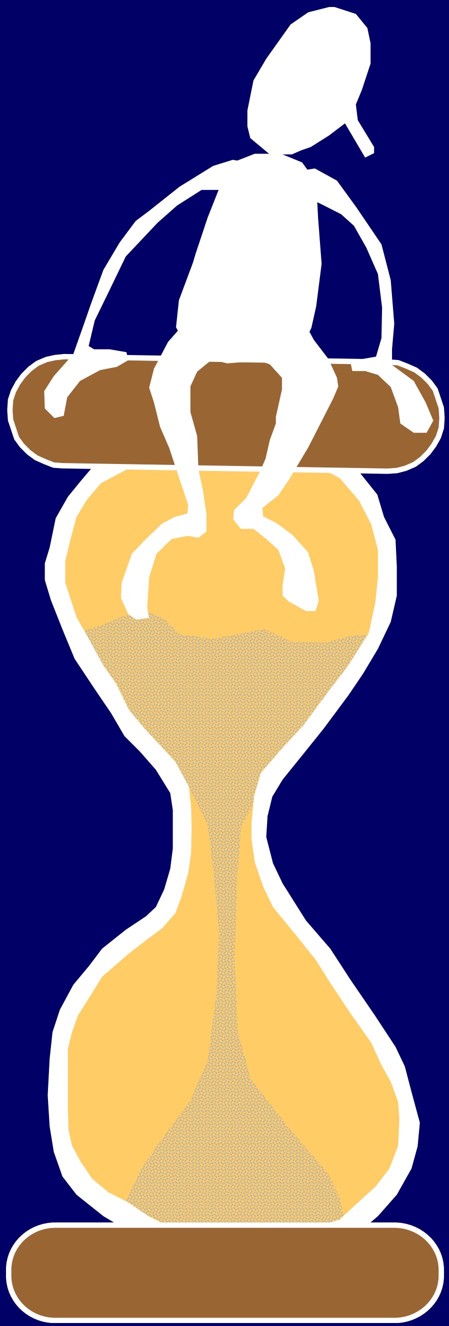
CS 15-251 Fall 2006
Lecture 22 November 9, 2006

Carnegie Mellon University

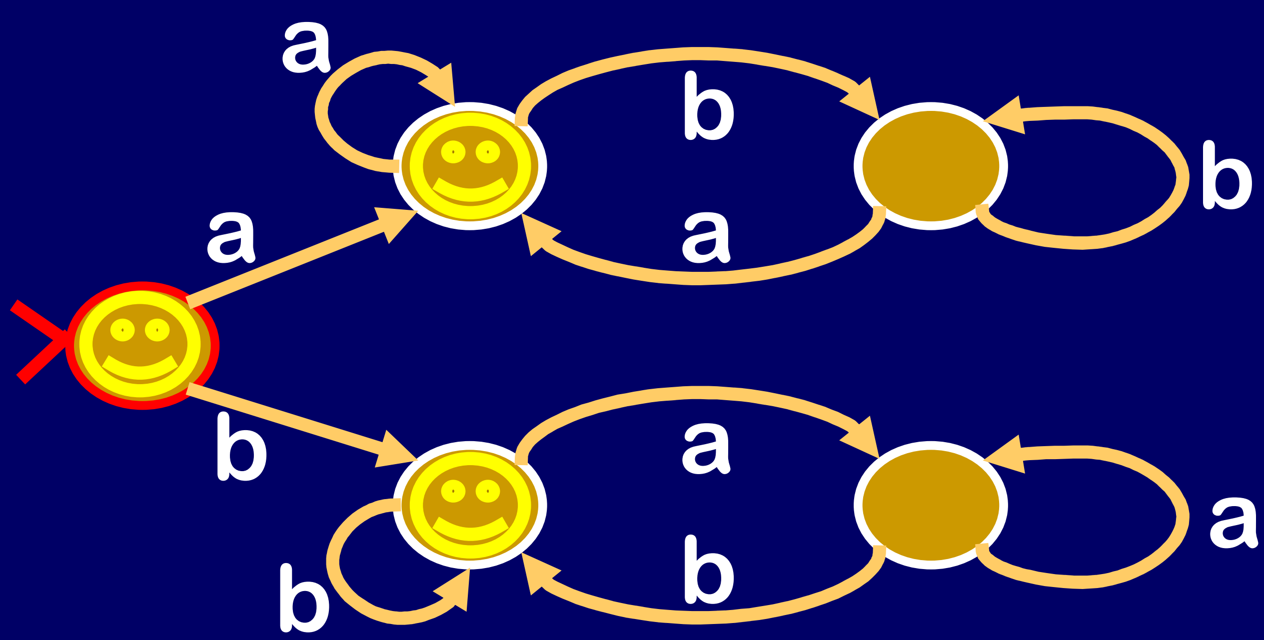
# One Minute To Learn Programming: Finite Automata





Today we'll talk about a programming language so simple that you can learn it in less than a minute.

# Meet "ABA" The Automaton!

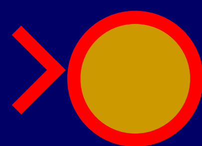
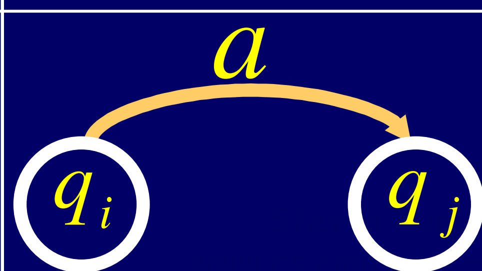



| Input String | Result |
|---|---|
| aba | Accept |
| aabb | Reject |
| aabba | Accept |
| ε | Accept |

# The Simplest Interesting Machine:

## Finite State Machine
## OR
## Finite Automaton

# Finite Automaton

| | Friendly | Formal , "unfriendly" |
|---|---|---|
| Finite set of states |  | $Q = \{q_o, q_1, q_2, \ldots, q_k\}$ |
| A start state |  | $q_o$ |
| A set of accepting states |  | $F = \left\{q_{i_1}, q_{i_2}, \ldots, q_{i_r}\right\}$ |
| A finite alphabet | $a$ $b$ $\#$ <br> $x$ $1$ | $\Sigma$ |
| State transition instructions |   | $\partial : Q \times \Sigma \to Q$ <br> $\partial(q_i, a) = q_j$ |

# How Machine M operates.

M "reads" one letter at a time from the input string (going from left to right)

M starts in state $q_0$.

If M is in state $q_i$ reads the letter $a$ then

If $\delta(q_i. a)$ is undefined then CRASH.

Otherwise M moves to state $\delta(q_i,a)$

Let $M=(Q,\Sigma,F,\delta)$ be a finite automaton.

*M* ***accepts*** the string *x* if when *M* reads *x* it ends in an accepting state.

*M* ***rejects*** the string *x* if when *M* reads *x* it ends in a non-accepting state.

*M* ***crashes*** on *x* if *M* crashes while reading *x*.

# The set (or language) accepted by M is:

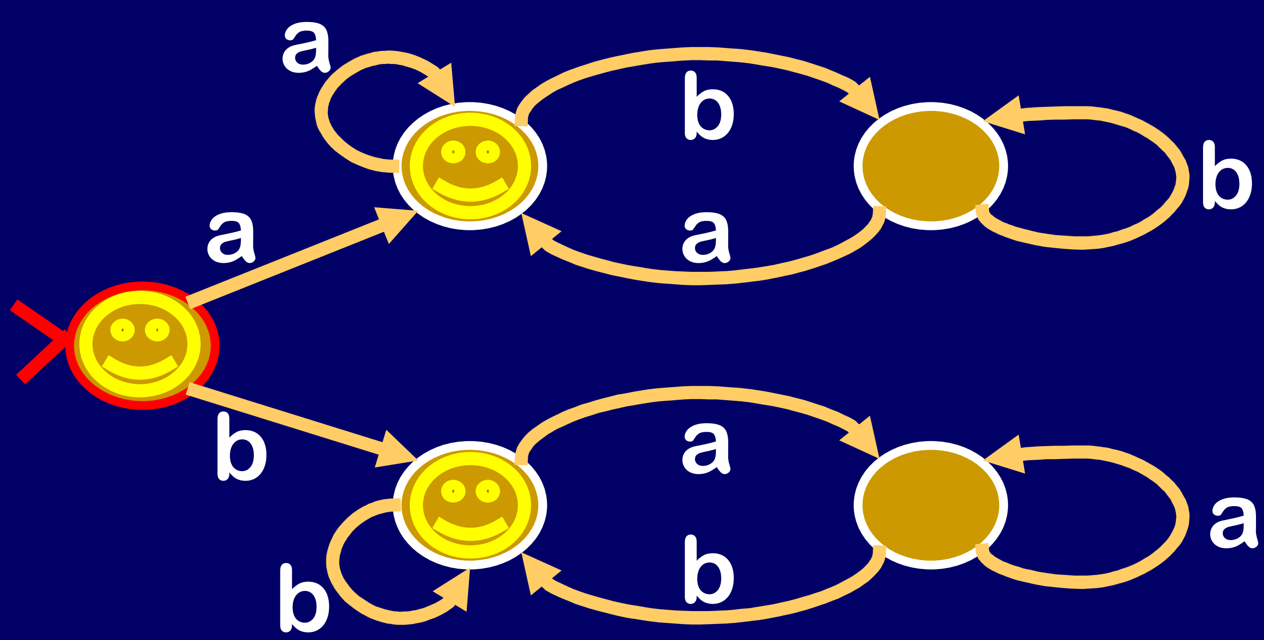
$$L_M = \{x \in \Sigma^* \mid M \text{ accepts } x\}$$

$\Sigma^k \equiv$ All length k strings over the alphabet $\Sigma$

$$\Sigma^* \equiv \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$$

Notice that this is $\{\varepsilon\}$

# Back to "ABA" The Automaton

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# What is the language accepted by this machine?


$L = \{a,b\}^* =$ all finite strings of $a$'s and $b$'s

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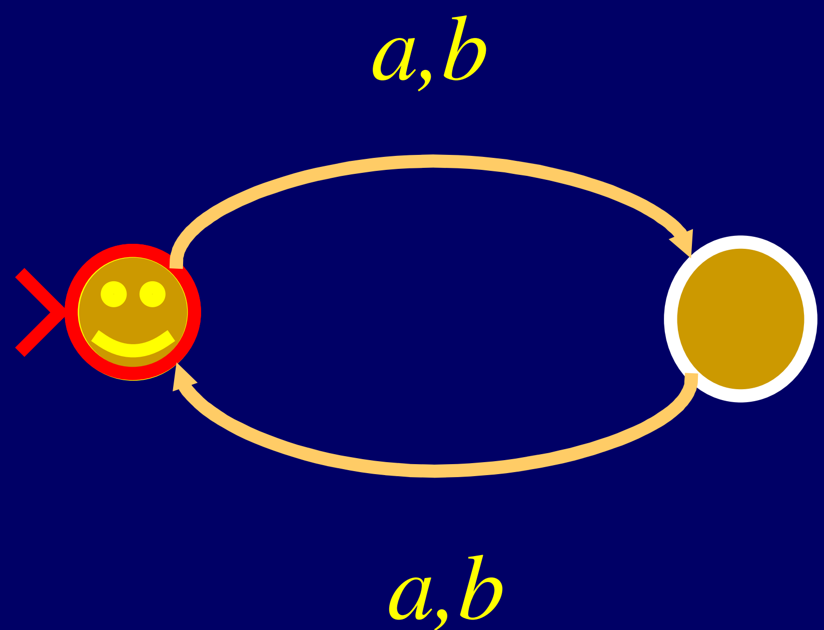
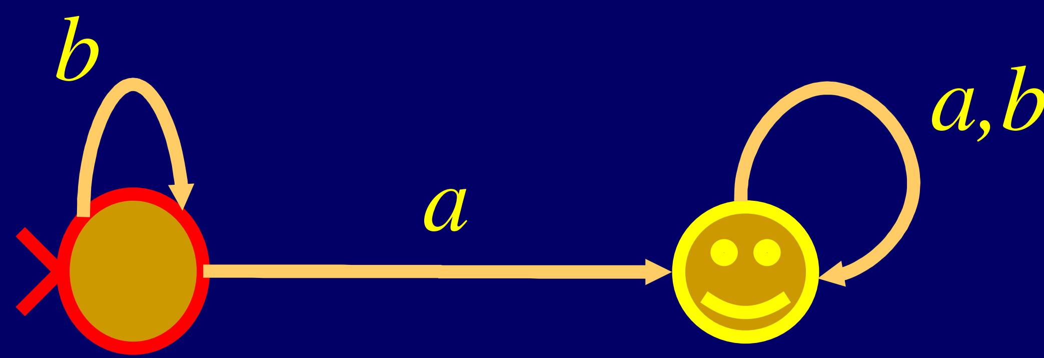


# What is the language accepted by this machine?


$L =$ all even length strings of $a$'s and $b$'s

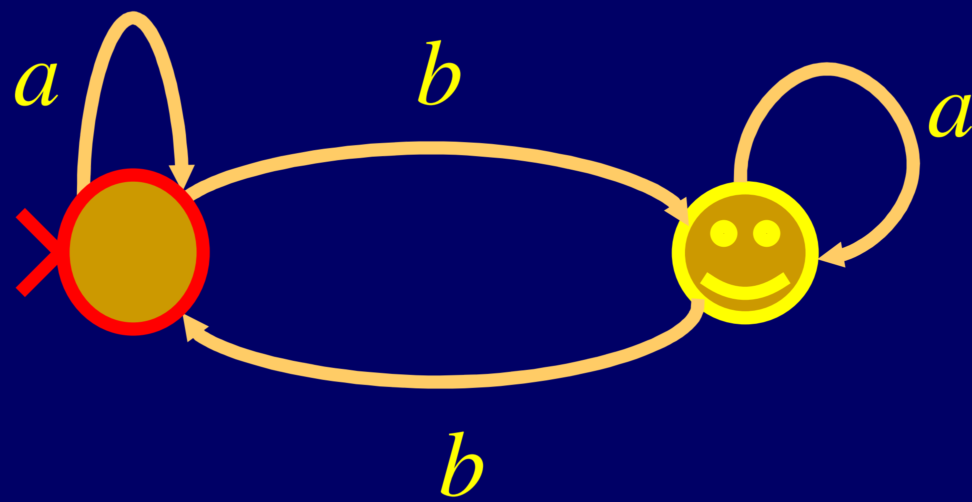
# What machine accepts this language?

$L$ = **all strings in** $\{a,b\}^*$ **that contain at least one** $a$



# What machine accepts this language?

*L* = strings with an odd number of *b*'s and any number of *a*'s



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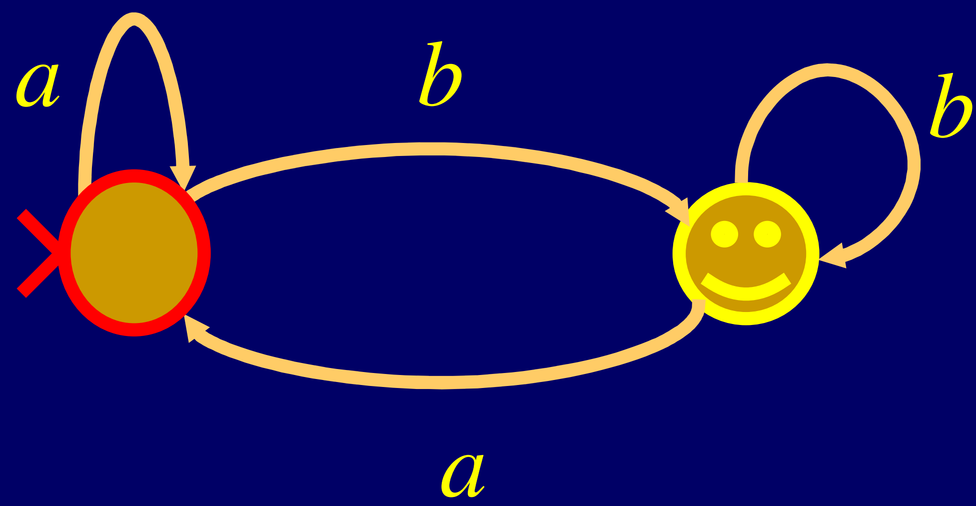


$L$ = any string ending with a $b$

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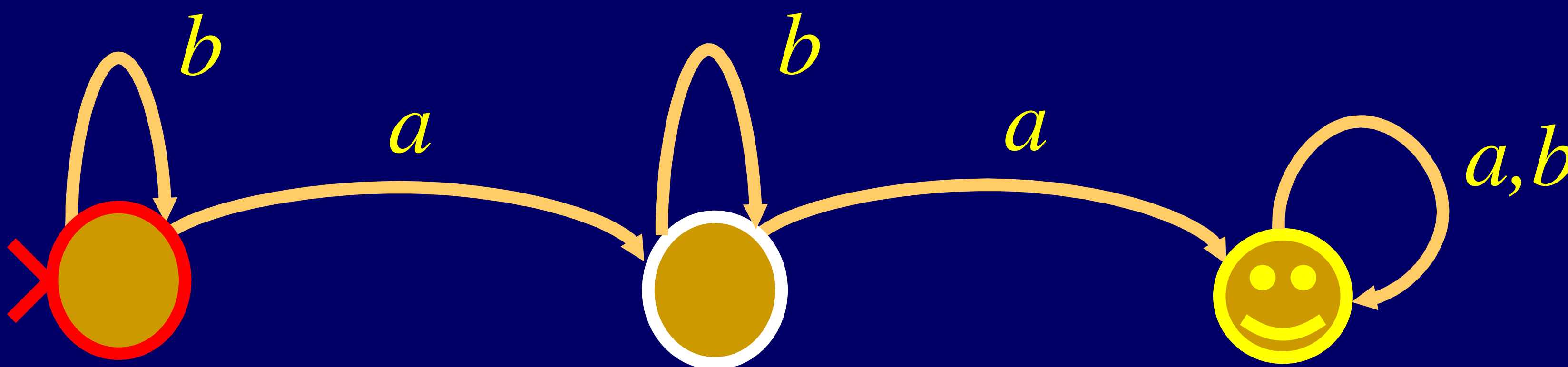

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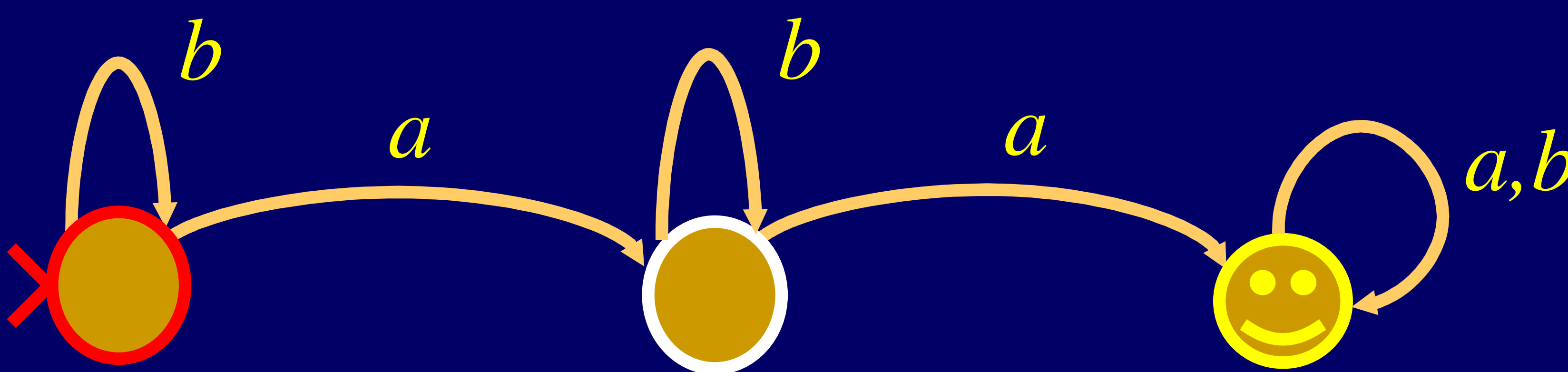


*L* = any string with at least two *a*'s

# What machine accepts this language?

$L$ = any string with an $a$ and a $b$

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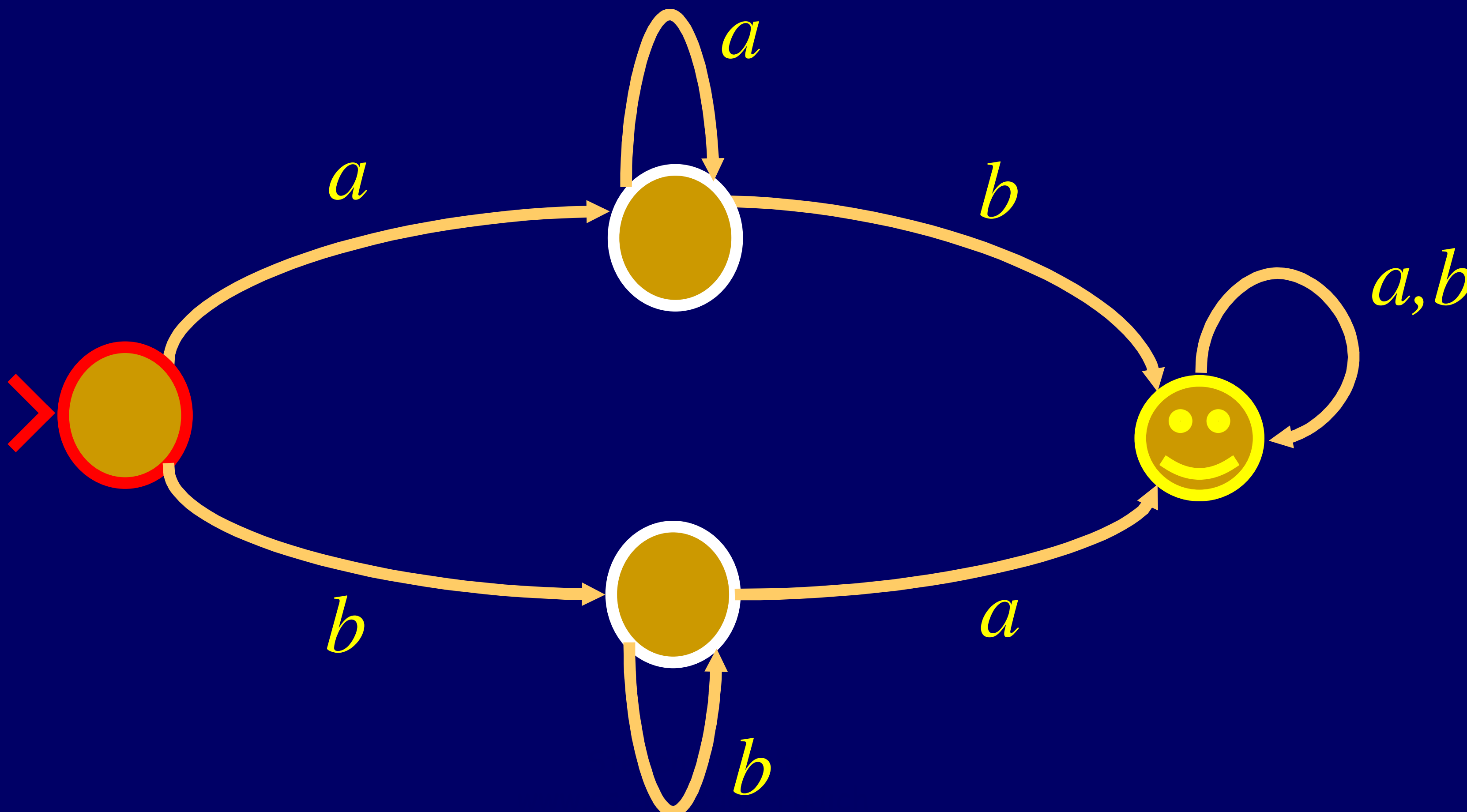
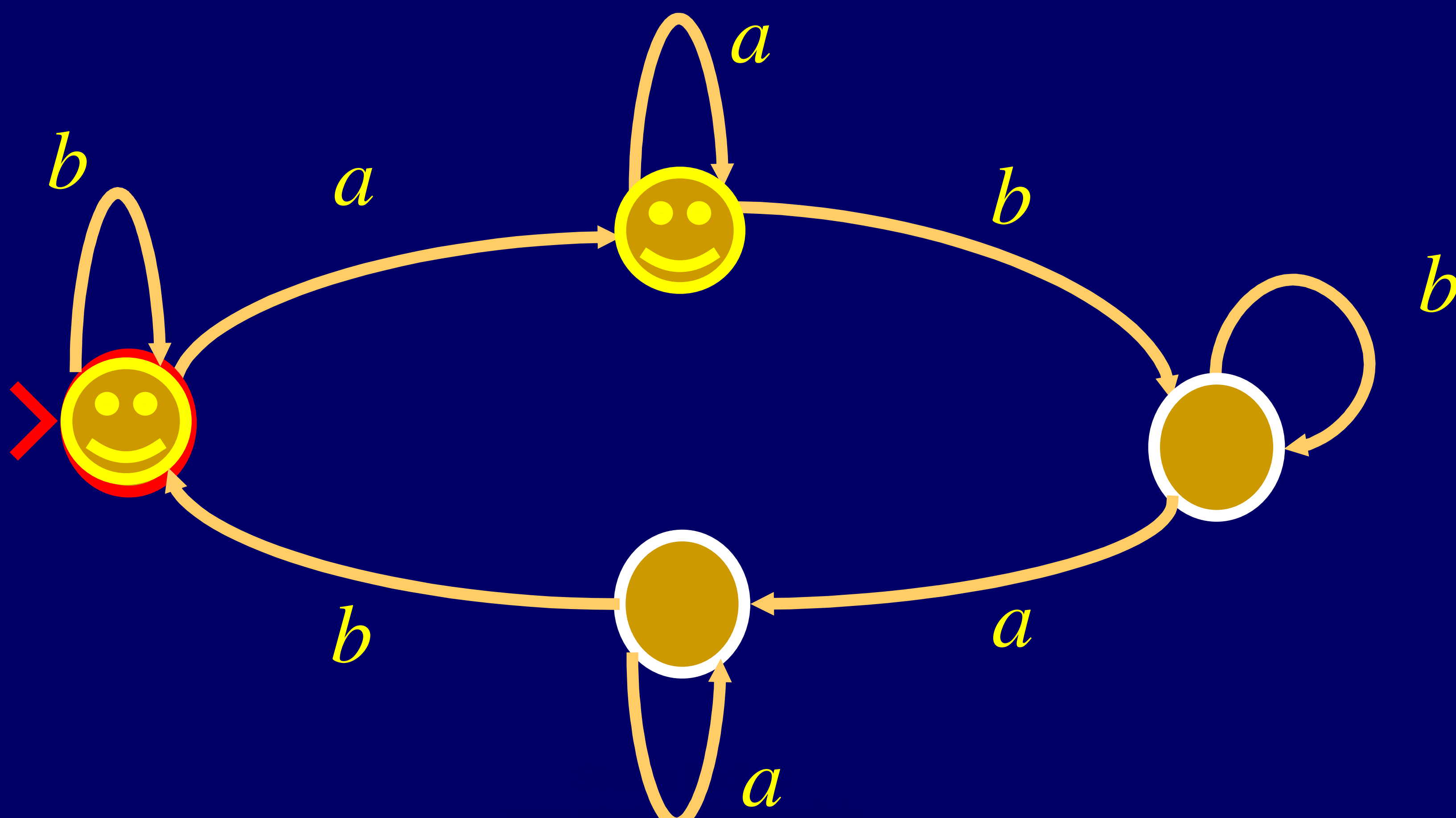


# What machine accepts this language?

*L* = strings with an even number of *ab* pairs



## $L =$ all strings containing $ababb$ as a consecutive substring

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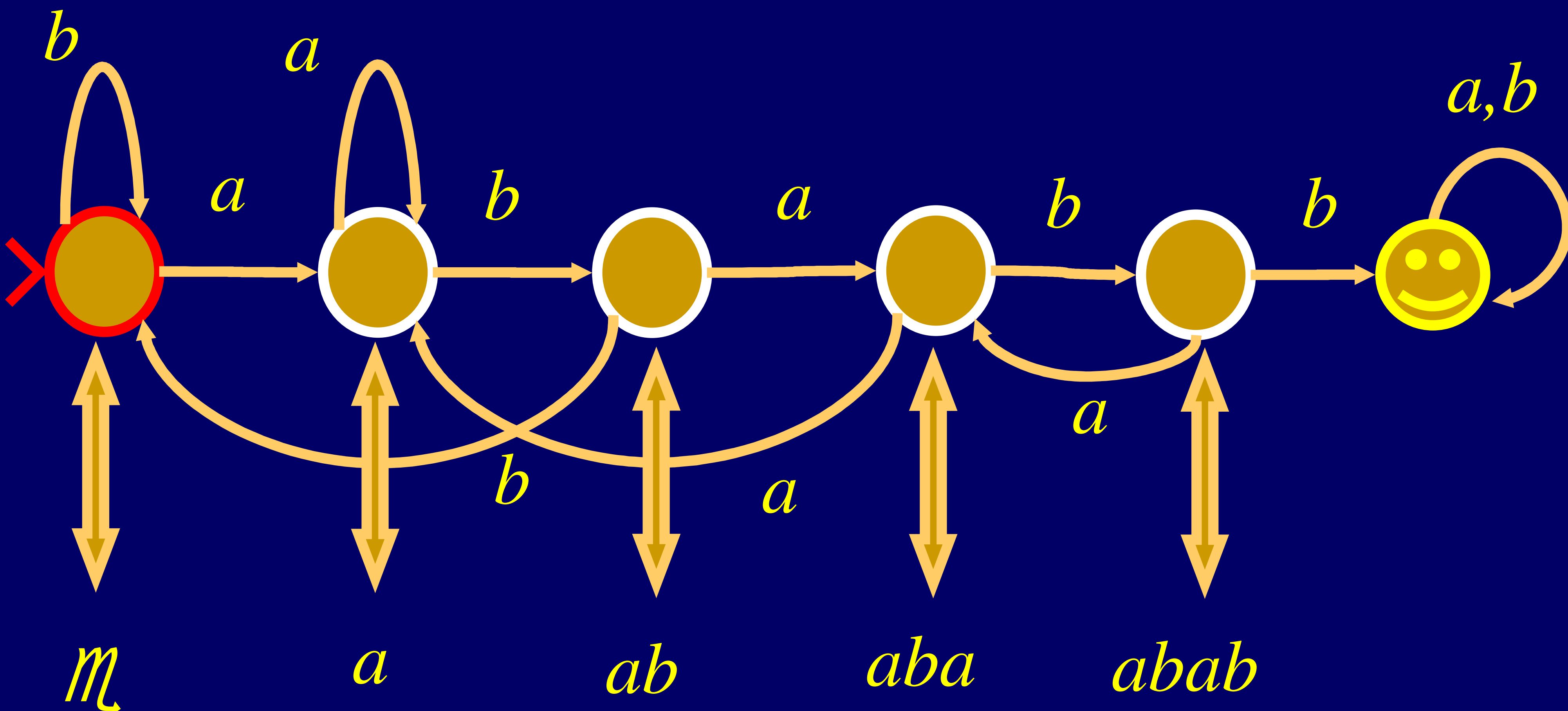


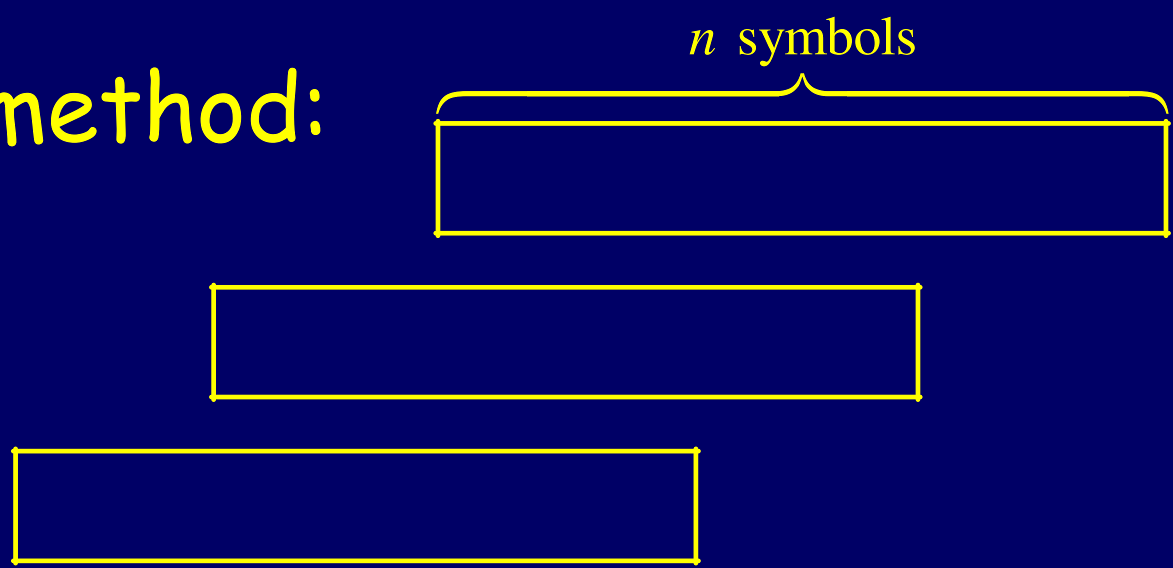

Invariant: I am state $s$ exactly when $s$ is the longest suffix of the input (so far) that forms a prefix of *ababb*.

# The "grep" Problem

**Input:**
- text $T$ of length $t$
- string $S$ of length $n$

**Problem:**
Does the string $S$ appear inside the text $T$?

**Naïve method:**




$$a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots, a_t$$

**Cost:** $O(nt)$ comparisons

# Automata Solution

- Build a machine $M$ that accepts any string with $S$ as a consecutive substring.
- Feed the text to $M$.
- Cost: $t$ comparisons + time to build $M$.
- As luck would have it, the Knuth, ***Morris***, Pratt algorithm builds $M$ quickly.

# Real-life uses of finite state machines

- grep
- coke machines
- thermostats (fridge)
- elevators
- train track switches
- lexical analyzers for parsers

Any $L \subseteq \Sigma^*$ is defined to be a <u>language</u>.

L is just a set of strings. It is called a language for historical reasons.

Let $L \subseteq \Sigma^*$ be a language.

$L$ is called a regular language if there is some finite automaton that accepts $L$.

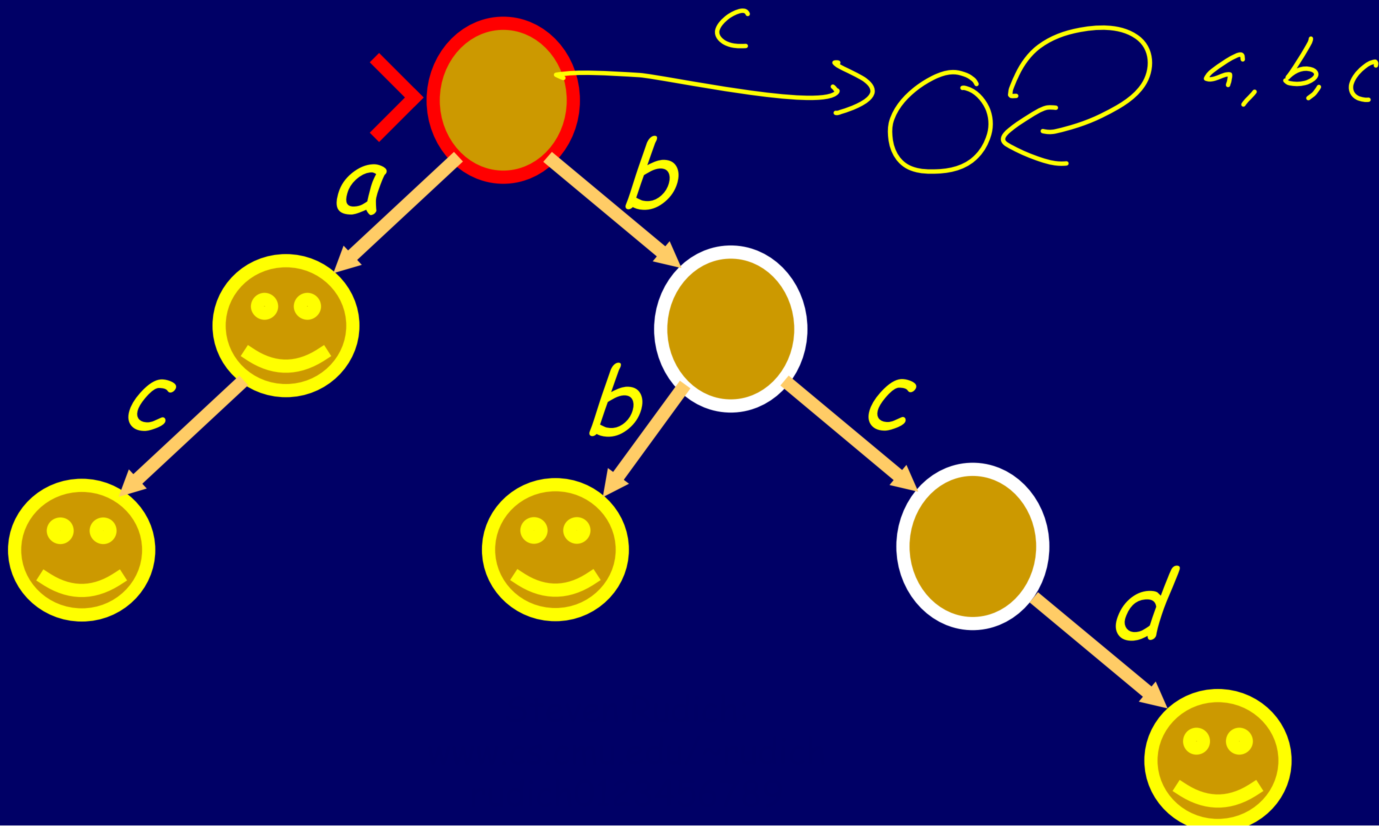
In this lecture we have seen many regular languages.

- $\Sigma^*$
- even length strings
- strings containing *ababb*

Theorem: Any finite langage is regular.

Proof: Make a machine with a "path" for each string in the language, sharing prefixes

Example: $L = \{a, bcd, ac, bb\}$



Are all languages regular?

Consider the language

$$a^n b^n = \{\varepsilon, ab, aabb, aaabbb, \ldots\}$$

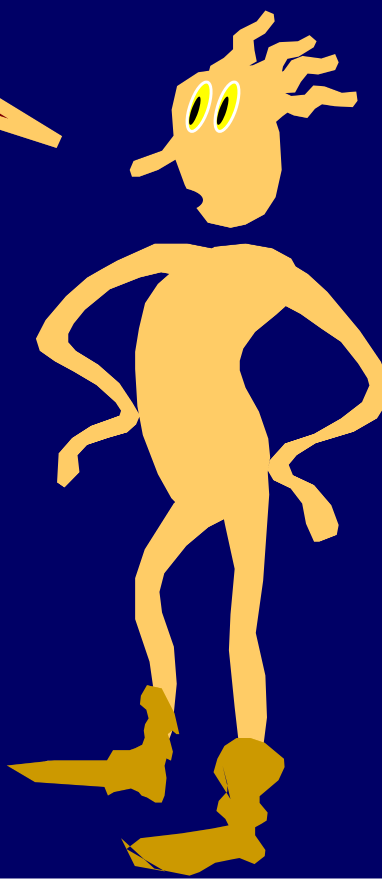
i.e., a bunch of $a$'s followed by an equal number of $b$'s

No finite automaton accepts this language.

Can you prove this?

$a^n b^n$ is not regular. No machine has enough states to keep track of the number of $a$'s it might encounter.

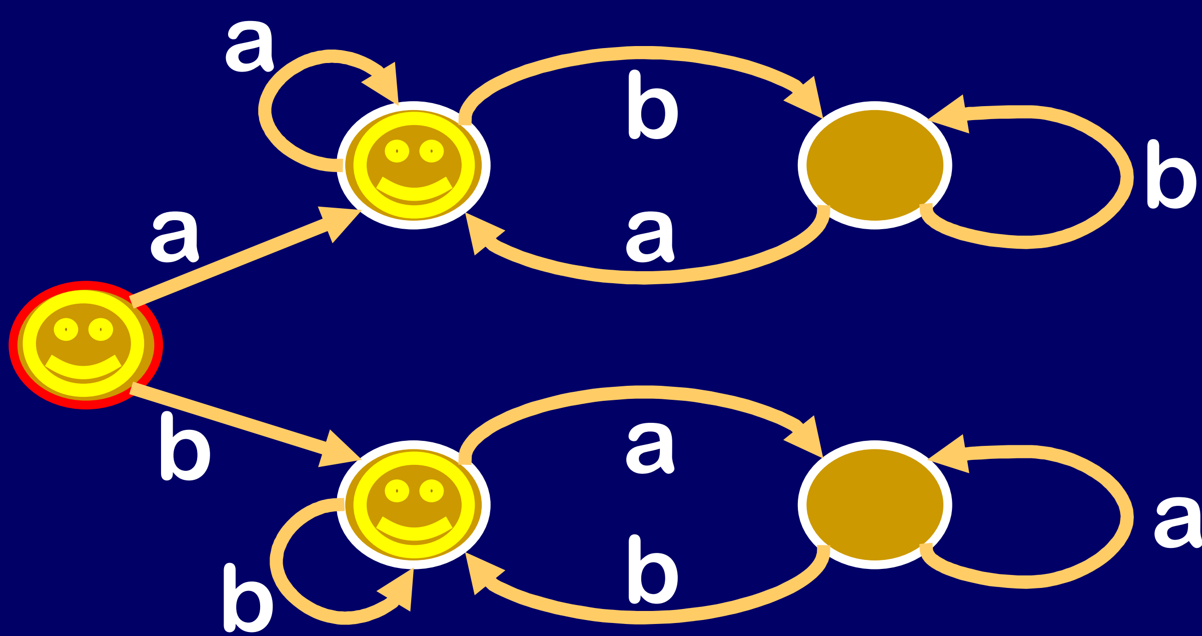
That is a fairly weak argument.  Consider the following example…



*L* = strings where the # of occurrences of the pattern *ab* is equal to the number of occurrences of the pattern *ba*

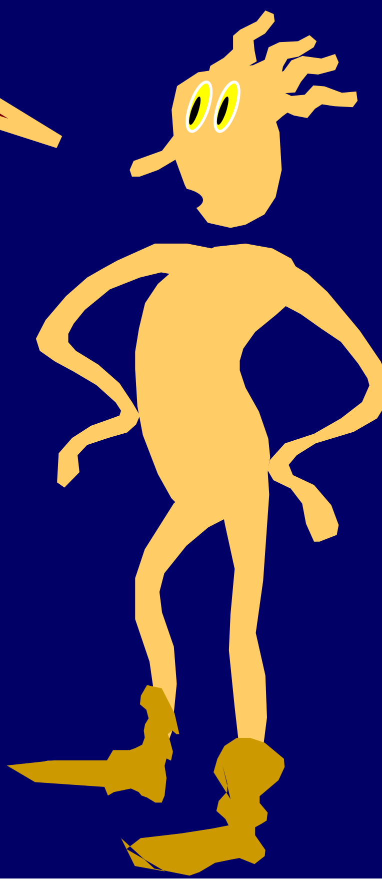
Can't be regular. No machine has enough states to keep track of the number of occurrences of *ab*.

# Remember "ABA"?




ABA accepts only the strings with an equal number of *ab*'s and *ba*'s!

Let me show you a professional strength proof that $a^n b^n$ is not regular....



Pigeonhole principle: Given $n$ boxes and $m > n$ objects, at least one box must contain more than one object.

Letterbox principle: If the average number of letters per box is $a$, then some box will have at least $a$ letters. (Similarly, some box has at most $a$.)

Theorem: $a^n b^n$ is not regular.

Proof: Assume that it is. Then $\exists$ $M$ with $k$ states that accepts it.

For each $0 \leq i \leq k$, let $S_i$ be the state $M$ is in after reading $a^i$.

$\exists i,j \leq k$ s.t. $S_i = S_j$, but $i \neq j$

$M$ will do the same thing on $a^i b^i$ and $a^j b^i$.

But a valid $M$ must reject $a^j b^i$ and accept $a^i b^i$.

$\Rightarrow\Leftarrow$

# MORAL:

Finite automata can't count.

# Advertisement

**You can learn much more about these creatures in the FLAC course.**

**Formal Languages, Automata, and Computation**

- **There is a unique smallest automaton for any regular language**

- **It can be found by a fast algorithm.**