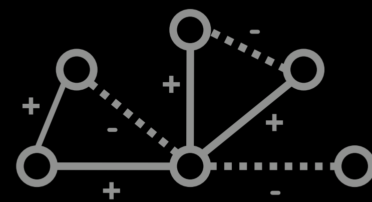
15-251

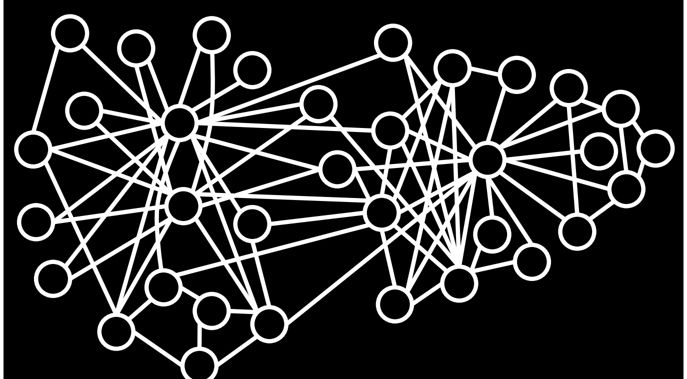
Great Theoretical Ideas in Computer Science

Social Networks

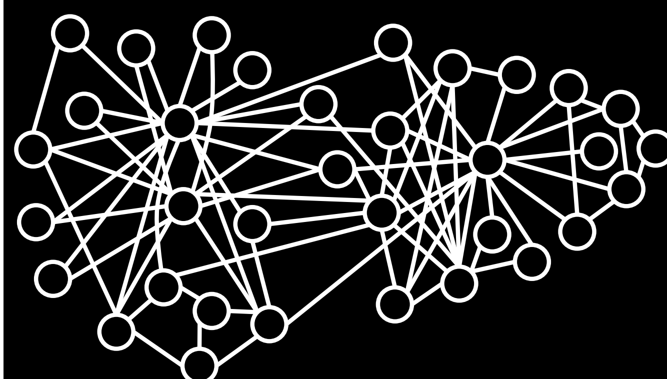


Brendan Meeder
April 6, 2010

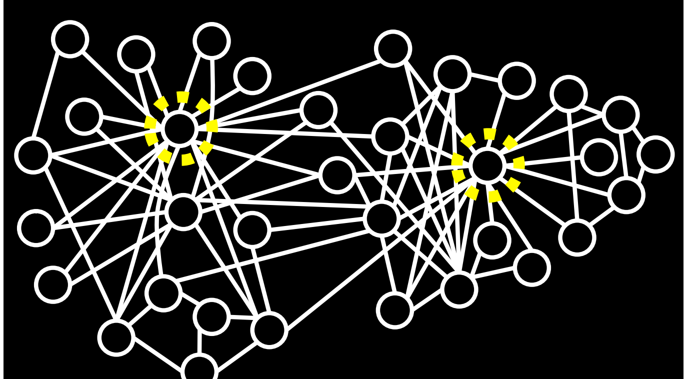
Social Networks (1977)



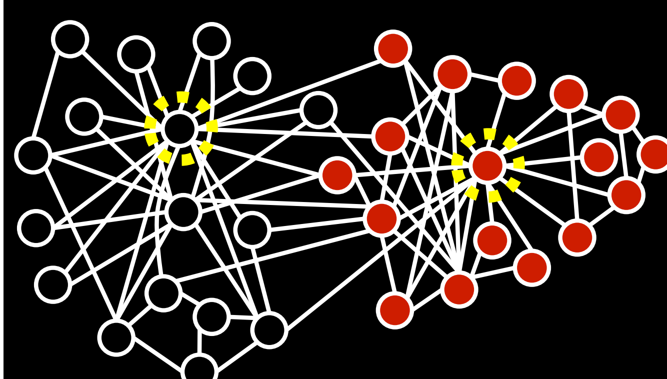
Friendships in Karate Club



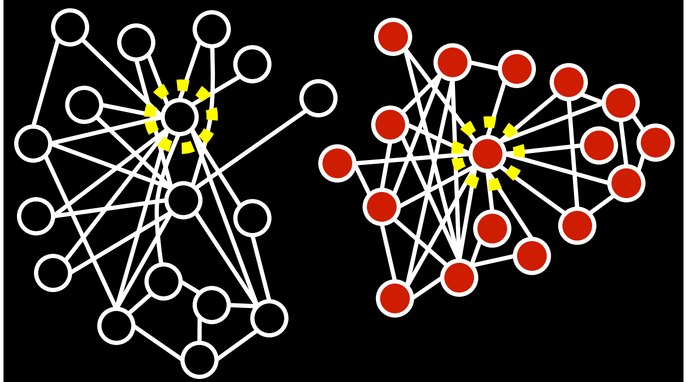
Friendships in Karate Club



Picking Sides



The Breakup



Mathematical "Explanation"

The split occurs along a minimum cut separating the two central figures

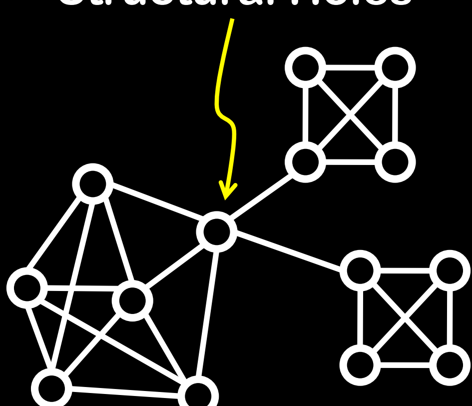
Individuals sided with the central figure with whom they were closer

Social Network Analysis

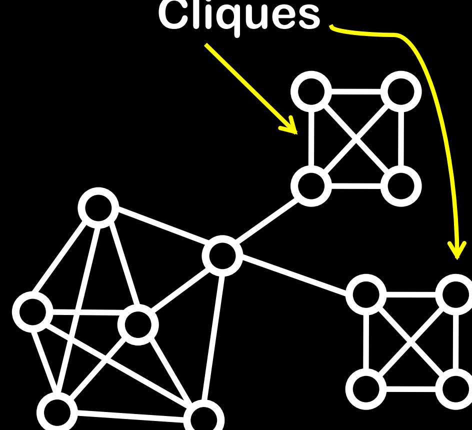
- **Modeling**
  - Network Structure
  - Information Flows
  - User Interactions
- **Predicting**
  - Network Evolution
  - Growth of Fads
  - Outbreak Detection
- **Measuring**
  - Large-scale data collection
  - Detailed, small-scale observation
  - Network Effects, Social Cascades

Network Structures

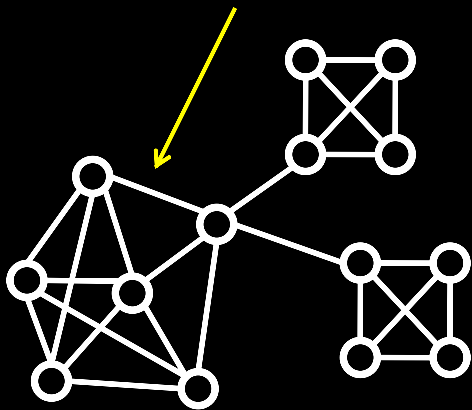
Structural Holes



Cliques

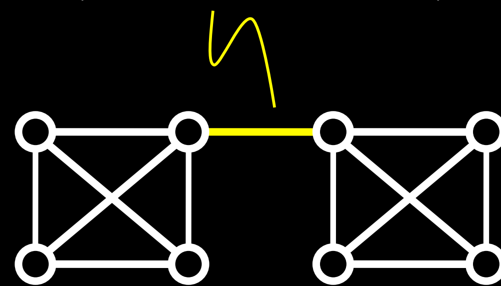


Clusters



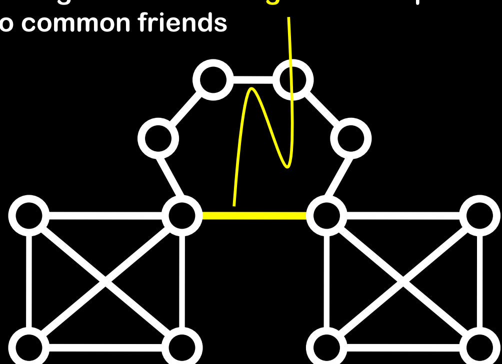
Bridges

An edge is a **bridge** if deleting it would cause its endpoints to lie in different components

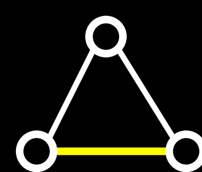


Local Bridges

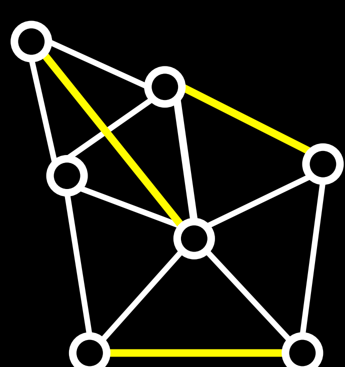
An edge is a **local bridge** if its endpoints have no common friends



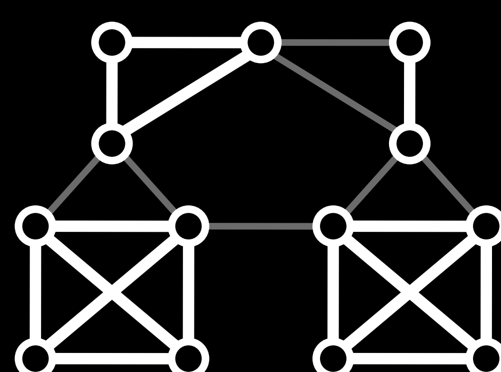
Triadic Closure: If two nodes have common neighbor, there is an increased likelihood that an edge between them forms



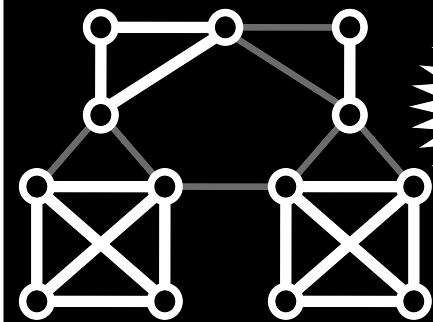
How will this network evolve?



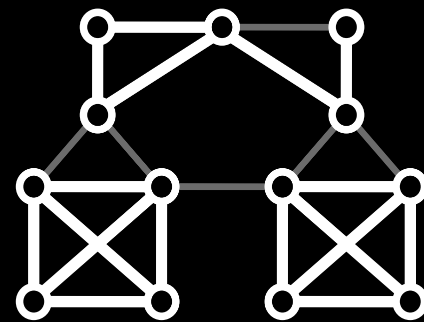
Weak Versus Strong Ties



Definition: Node v satisfies the **Strong Triadic Closure** if, for any two nodes u and w to which it has strong ties, there is an edge between u and w (which can be either weak or strong)



This graph satisfies the strong triadic closure



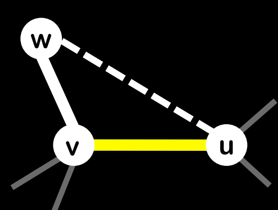
Theorem: If node v satisfies the Strong Triadic Closure and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie

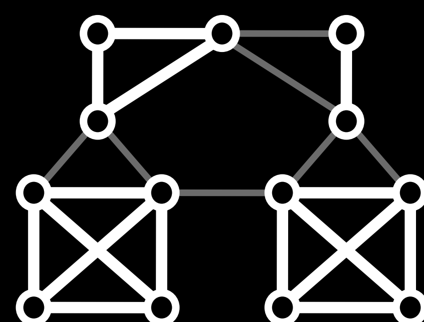
Proof (by contradiction):

Suppose edge v-u is a local bridge and it is a strong tie

Then u-w must exist because of Strong Triadic Closure

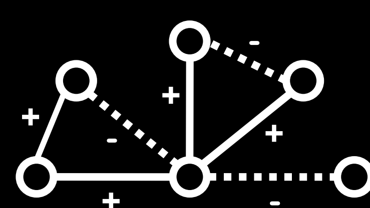
But then v-u is not a bridge



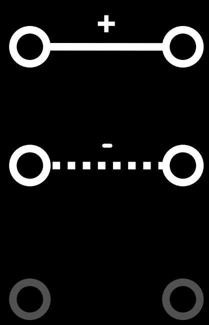


Networks with Extra Structure

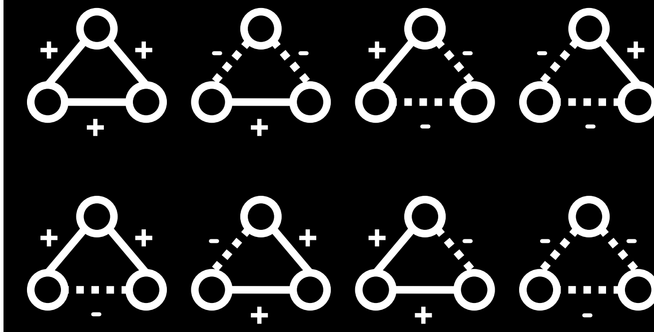
Signed Graphs

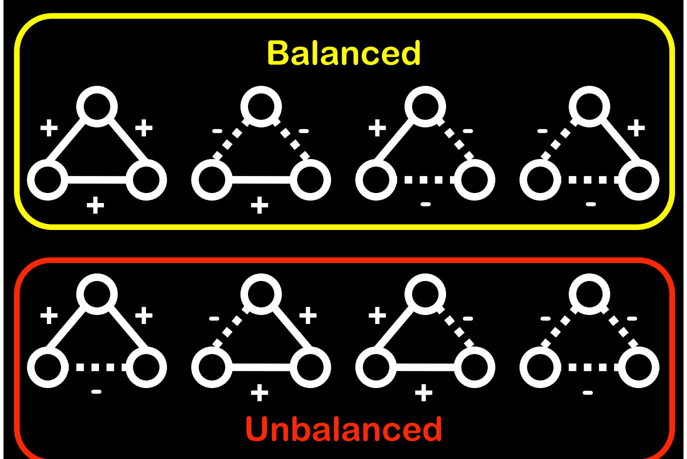


Two-Node Signed Graphs

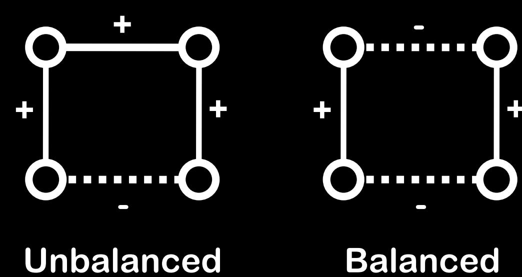


Complete Three-Node Signed Graphs





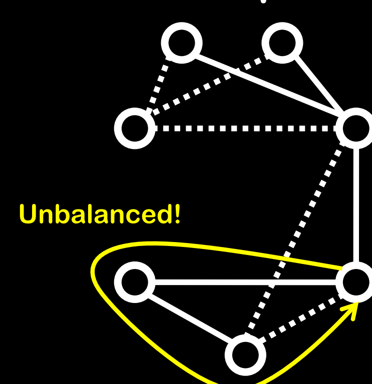
Four Node Cycles



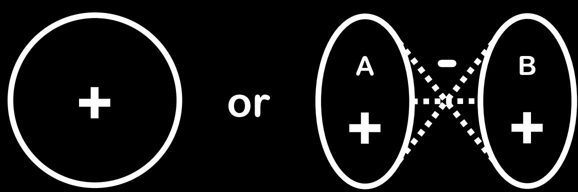
Definition: A cycle is balanced if the product of its signs is positive

Definition: A graph is balanced if all its cycles are balanced

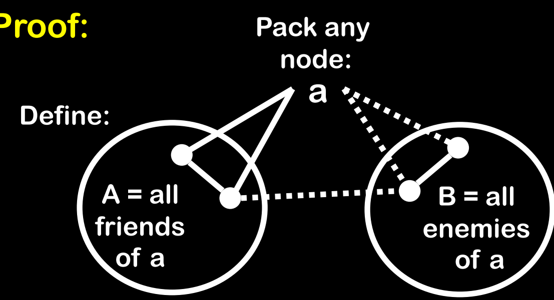
Example



Theorem: If every 3-cycle in a signed complete graph is balanced, then either (1) all nodes are friends, or (2) the nodes can be divided into two groups, A and B, such that every pair of people in A like each other, every pair of people in B like each other, and everyone in A is the enemy of everyone in B.

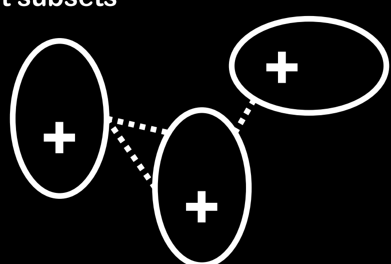


Proof:



1. Every two nodes in A are friends
2. Every two nodes in B are friends
3. Every node in A is an enemy of every node in B

Definition: A signed graph is clusterable if the nodes can be partitioned into a finite number of subsets such every positive edge is between nodes of the same subset, and every negative edge is between nodes of different subsets

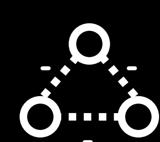


Theorem: A signed graph has a clustering if and only if the graph contains no cycles which have exactly one negative edge

Completing Signed Graphs

Given a signed graph G, when can it be completed?

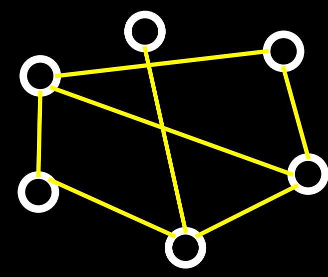
Weakly Unbalanced Triads



Three enemies are only weakly unbalanced

Models of Network Structure

Random Graphs



Random Graphs

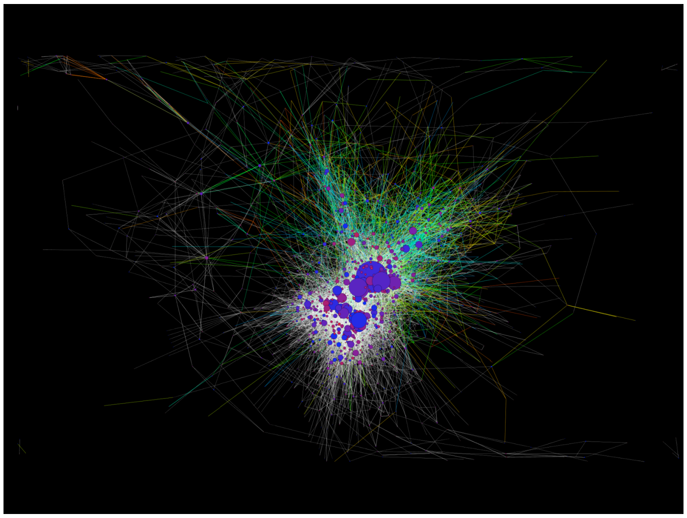
- Graph with N people
- For every pair (i,j) of people in the graph, add the edge (i,j) with probability p
- Called the Erdos-Renyi model G(n,p): n vertices, each possible edge occurs with probability p

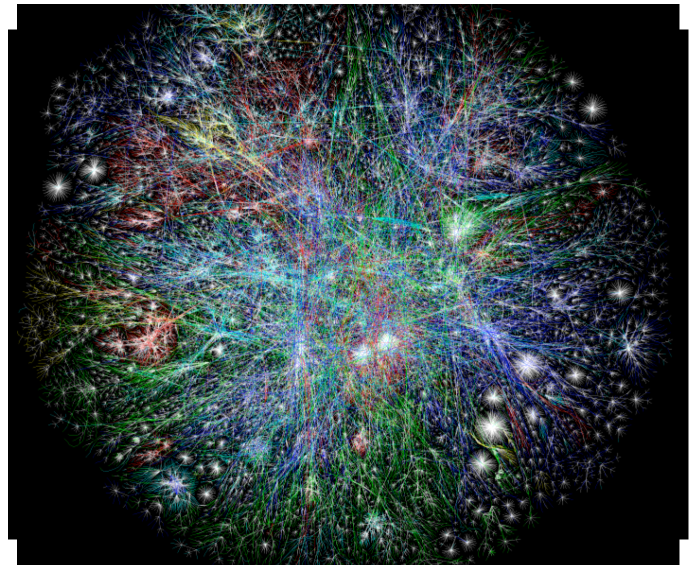
Math, old school style

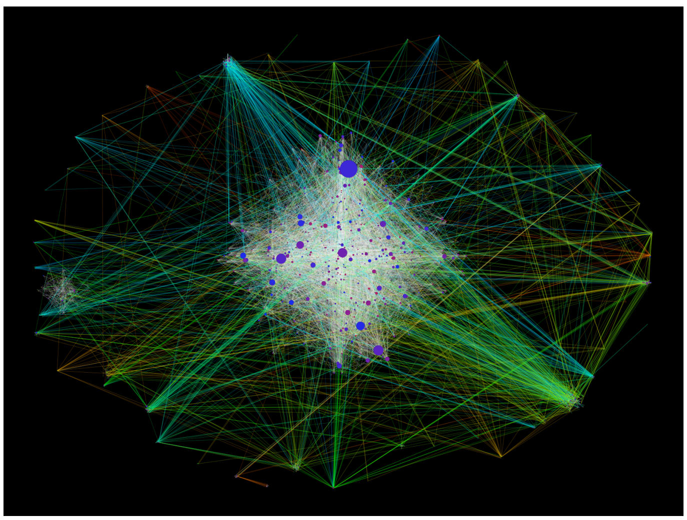
Research Topics

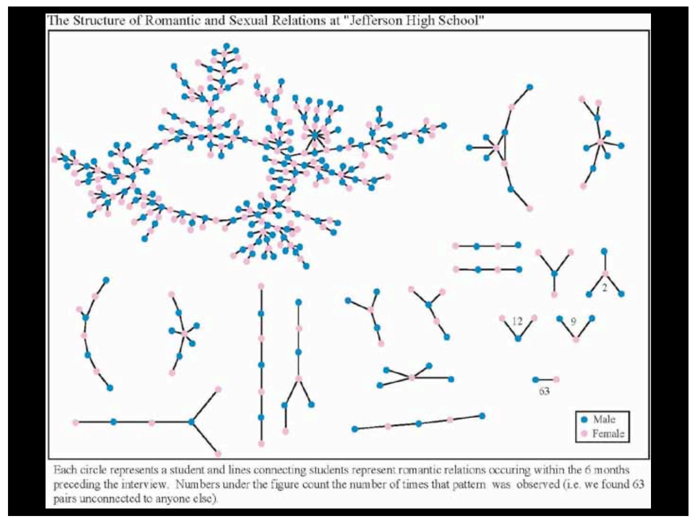
- Lots of data: Twitter data (billions of messages, 100M node graph)
- How do fads become popular?
- Mathematical models for network evolution
- Algorithmic game theory

The Beauty in Networks

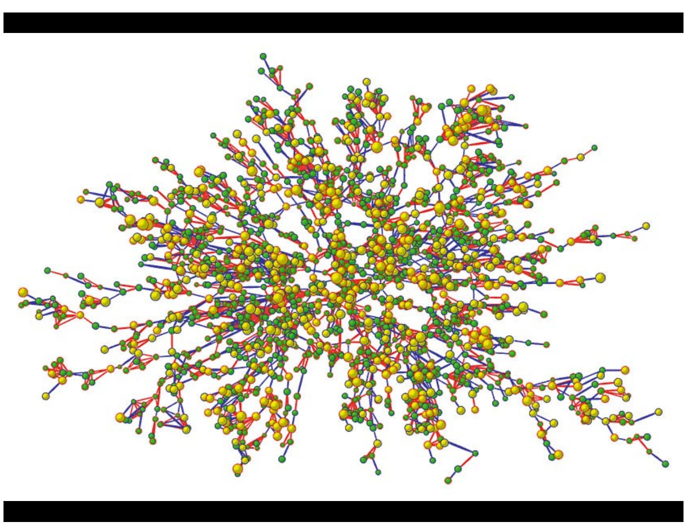


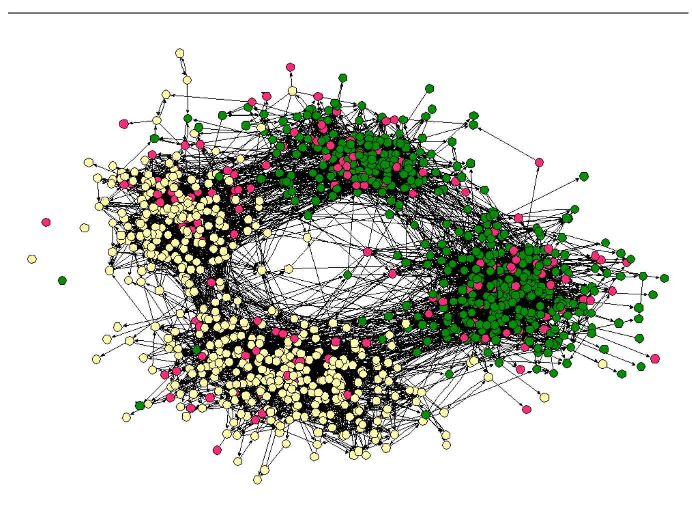


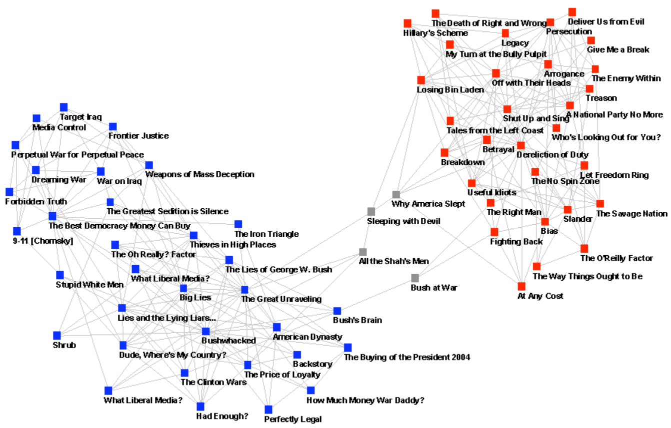


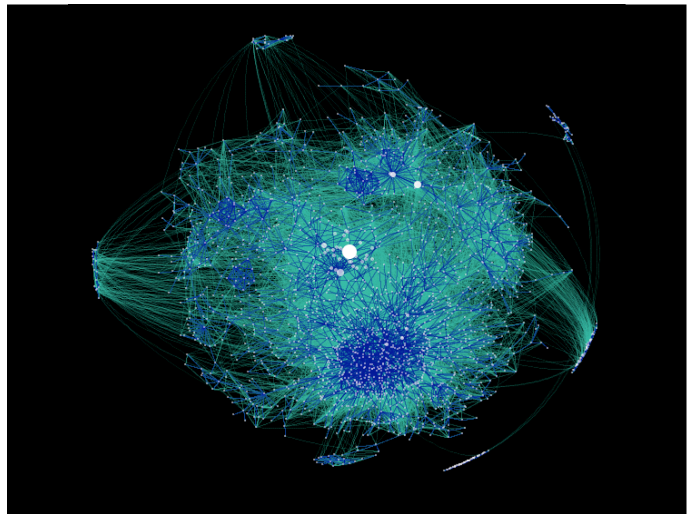


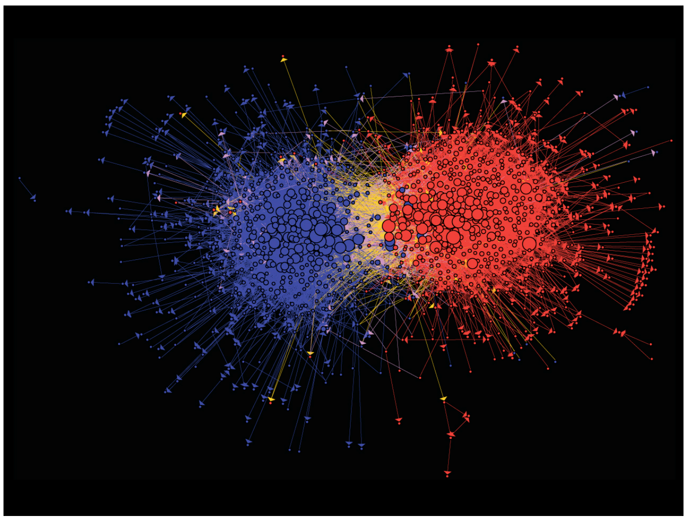
The Structure of Romantic and Sexual Relations at "Jefferson High School"
2
12
9
63
Male
Female
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

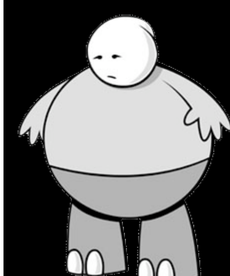












Here's What You Need to Know…

Network Structures

- Structural holes, cliques, clusters, bridges, local bridges
- Balanced cycles and graphs
- Signed graphs
- Strong triadic closure

Random Graphs

- Erdos-Reyni model
- Union bound, tail inequalities
- Preferential Attachment Model