15-251: Great Theoretical Ideas	**Guru:** Brendan Meeder
Assignment 2 Common Mistakes	Due: Yesterday

The purpose of this document is for us to communicate to you what we feel common mistakes were, and what you can learn from them. It lets the TA's express what they were looking for when grading a particular question. If you feel like your grade is severely out of line from the comments in this document, please contact your grader.

1 Hungry 15-251 Students (25 points)

Homework 2, Problem 1 Graded by Brian Thompson

This is a clear, concise, and complete solution, and would get you full credit on the problem:

Claim: The optimal distribution for student n entails giving 1 pancake to each other student of the same parity as n, and keeping the rest for himself. Base Case: $n = 1$. There is only one student who keeps all the pancakes and votes for himself, achieving more than 50% of the votes and passing the class. Thus the distribution works. Inductive Hypothesis: Assume the Claim is true for student k. Inductive Step: Suppose student $k + 1$ proposes giving a pancake to each student of the same parity as $k + 1$. Since those students are of different parity than k, they would receive no pancakes under k's optimal distribution by IH, and so they would prefer to receive one pancake and will therefore vote for student $k + 1$'s proposal. There are $\lceil (k + 1)/2 \rceil$ students of the same parity as $k+1$ (including himself), so he is achieving the minimum number of votes possible to pass his distribution with the fewest number of pancakes per vote, thus it is optimal. Conclusion: The claim has been proven by induction for all $n \geq 1$. Thus the optimal distribution for student #100 is to give 1 pancake to each of the 49 even-parity students, and keep the remaining 1951 pancakes for himself.

Here is a guideline to my grading criteria:

To receive 0 points:

- you propose a distribution with no reasoning or explanation
- you seem to disregard the basic premise of the problem

To receive 5 points:

- You explore a few small cases
- You do not try to generalize or recognize a pattern

To receive 10 points:

- You explore a few small cases
- You observe it is easiest to bribe those who would get none otherwise
- You recognize the pattern, and conclude that it must work

To receive 15 points:

- You explore a few small cases
- You recognize the pattern
- You generalize the distribution (give a pancake to students of same parity)

To receive 20 points:

- You generalize the distribution
- You use some form of inductive reasoning
- There is a flaw in your inductive argument (no base case, unclear or flawed IH, did not prove optimality)

To receive 25 points:

- You get the correct generalized distribution
- You give a clear and complete inductive argument

2 Induction proofs and misproofs (20 points)

Serious errors

- In part 2, students frequently did not explain why the case where $n + 1$ is prime breaks the proof. Specifically, one of the factors of $n+1$ in that case is $n+1$, and the inductive hypothesis does not cover $n + 1$.
- Also for part 2, some students claimed that strong induction automatically requires exactly two or at least two base cases. This is wrong; the number of base cases depends on the specific claim and the associated definitions (such as recurrences that depend on two previous values).
- In their answers for the same part, a fair number of students claimed that using 1 as a base case for the proof was incorrect on algebraic grounds, since the statement shown there obviously holds for logarithms of any base, and using another value in the base case would have revealed the error. There is nothing wrong with the base case, even if it is a trivial algebraic identity. Those students should have directed their attention to the inductive case, since the claim is incorrect, but the base case is quite convincing.
- Many students answered part 3 with a simple statement such as "true." Worse, some students reprinted the claim, and then wrote "true" under it, or explicitly stated that they thought the claim was true. The question, however, was about the proof and not the claim. Of course, the proof can be incorrect even if the statement is true. Students need to take care to make sure they answer the correct question, leaving no ambiguity as to which question they have actually answered. A decent answer here was "the proof is correct."

- Some students objected to the base case for $n = 1$ being proven by assumption in part 3. There is nothing wrong with concluding the base case holds under that assumption, since the statement is an implication: "*if* the assumption holds, then the conclusion holds." Using the assumption to prove the conclusion is fine. In fact, if the assumption was not necessary, then the statement and proof would be in rather bad style.

Pedantic comments

- Some students claimed that there was a circular argument in the proof in part 2. While this was not an error, there was in fact no circular argument, but an incorrect argument: there would be a "circular" argument if the inductive hypothesis had actually covered $n + 1$.

3 Foosball Tournament (10 points)

1. This is not really a mistake, but for virtually all of the solutions strong induction was unnecessary; ordinary induction would have sufficed.
2. If you're starting with n people and inserting another, you must explain why any tournament can be formed by taking n people and adding another, even if it's obvious. An easier way is to take $n + 1$ people, and ignore one for a while as we apply the induction hypothesis.
3. Ordering by number of wins doesn't work: imagine that A beat B beat C beat A. Each person will have won once and lost once. (You may say that in the case of two people having the same record it doesn't matter, but it does: in this case, for example, the ordering ACB does not work).

4 Induction and dot proofs (10 points)

There were almost no errors on the second part of this question. Just be careful that your inductive hypothesis doesn't say 'for all n...' as you are then assuming in your IH what you are trying to prove.

For the dot proof, the biggest problem that people had was that they didn't show all three regions had the same area. In order to make this argument work, you have to show that each of the three regions has the same area, then you can correctly conclude that the area of each regions is one third of the total area.

5 Function running times (15 points)

People generally did the first question very well. Again, be careful that you don't write 'for all n, ...' since this is an incorrect formulation of an induction hypothesis.

In the second part, it is incorrect to add inequalities in the way presented in the problem. In general, if you have that $A \leq B$, $C \leq D$ it follows that $A + C \leq B + D$. We cannot subtract inequalities in the same way. As an example, $1 \leq 10$ and $3 \leq 4$ but $3 - 1 \not\leq 4 - 10$.

6 Circles dividing the plane (20 points)

The way in which this question was graded is as follows:

- 5 points for figuring out how the regions were created (more on this below).
- 5 points for finding some recurrence related to region creation.
- 5 points for reaching a closed form for the recurrence.
- 5 points for proving your recurrence is correct.

In the first part, people usually said something to the effect of “for each intersection, a new region is created.” Why should we believe this? If we look at the instance of the lines variant, we see that in fact there isn’t a 1-1 correspondence between new regions being created and intersection of the $n + 1$st line with the other n lines. What *really* matters here is the arcs that are formed on the circle. It’s the *arcs* that are creating the new region. An intersection is not a geometric object which can do that. The above lack of explanation cost 1-2 points at most.

A more serious error is using flawed reasoning about how the regions are created. Claiming that a circle can divide all existing regions is a serious flaw; if you try doing that with 4 circles, you will see it is impossible.

People either verbally or equationally expressed a recurrence describing the number of regions formed. Note that it’s not acceptable to give a recurrence as an answer: it isn’t telling me what the answer is, but rather how to recursively compute what the answer is. This is almost like saying “draw the circles in such a way, and then count them for yourself.”

Finally, you need to prove your answer is correct. Unrolling leaves open the possiblity for mistakes. Proving by induction that your solution works is a good way of making sure that you are 100% correct.