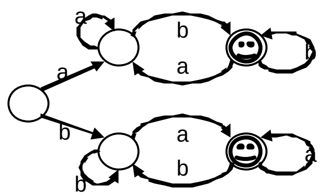
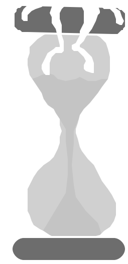
Great Theoretical Ideas In Computer Science		
Steven Rudich		CS 15-251 Spring 2005
Lecture 9	Feb 8 2005	Carnegie Mellon University

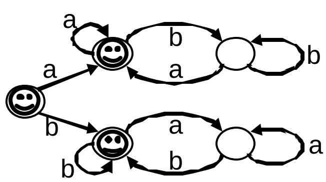
One Minute To Learn Programming: Finite Automata





Let me teach you a programming language so simple that you can learn it in less than a minute.

Meet "ABA" The Automaton!



Input String	Result
aba	Accept
aabb	Reject
aabba	Accept
ε	Accept

The Simplest Interesting Machine:

Finite State Machine
OR
Finite Automaton

Finite Automaton

Finite set of states		$Q = \{q_o, q_1, q_2, \ldots, q_k\}$
A start state		q_o
A set of accepting states		$F = \left\{q_{i_1}, q_{i_2}, \ldots, q_{i_r}\right\}$
A finite alphabet	a b $\#$ x 1	Σ
State transition instructions	a	$\partial : Q \times \Sigma \rightarrow Q$ $\partial(q_i, a) = q_j$

How Machine M operates.

M "reads" one letter at a time from the input string (going from left to right)

M starts in state q_0.
If M is in state q_i reads the letter a then

If $\delta(q_i, a)$ is undefined then CRASH.

Otherwise M moves to state $\delta(q_i,a)$

Let $M=(Q,\Sigma,F,\delta)$ be a finite automaton.

M ***accepts*** the string *x* if when *M* reads *x* it ends in an accepting state.

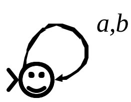
M ***rejects*** the string *x* if when *M* reads *x* it ends in a non-accepting state.

M ***crashes*** on *x* if *M* crashes while reading *x*.

The set (or language) accepted by M is:

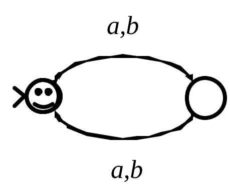
Notice that this is $\{\varepsilon\}$

What is the language accepted by this machine?



$L = \{a,b\}^*$ = all finite strings of *a*'s and *b*'s

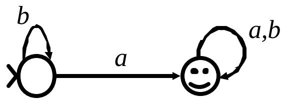
What is the language accepted by this machine?



L = all even length strings of *a*'s and *b*'s

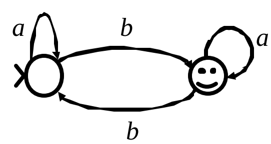
What machine accepts this language?

L = all strings in $\{a,b\}^*$ that contain at least one *a*

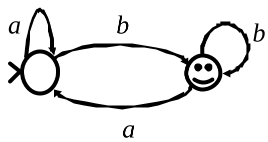


What machine accepts this language?

L = strings with an odd number of *b*'s and any number of *a*'s

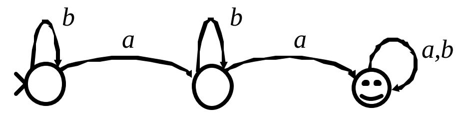


What is the language accepted by this machine?



L = any string ending with a *b*

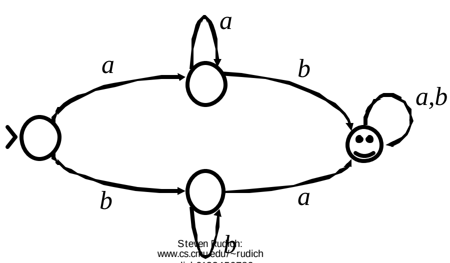
What is the language accepted by this machine?



L = any string with at least two *a*'s

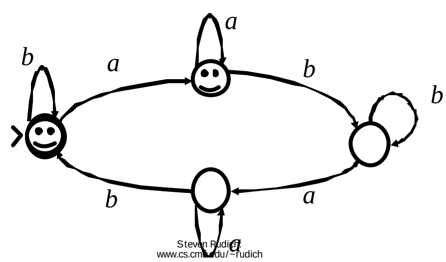
What machine accepts this language?

L = any string with an *a* and a *b*

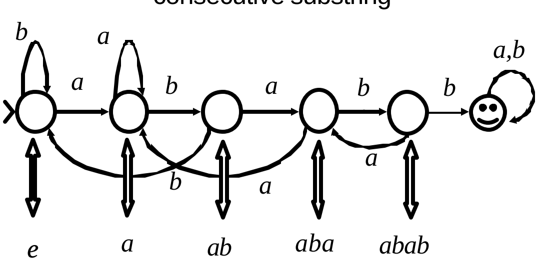


What machine accepts this language?

L = strings with an even number of *ab* pairs



L = all strings containing *ababb* as a consecutive substring



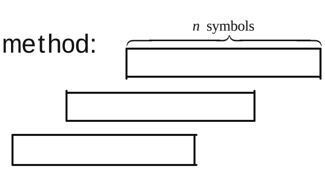
Invariant: I am state s exactly when s is the longest suffix of the input (so far) that forms a prefix of *ababb*.

The "grep" Problem

Input:
- text *T* of length *t*
- string *S* of length *n*

Problem:
Does the string *S* appear inside the text *T* ?

Naïve method:



$$a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots, a_t$$

Cost: *O(nt)* comparisons

Automata Solution

- Build a machine *M* that accepts any string with *S* as a consecutive substring.
- Feed the text to *M*.
- Cost: *t* comparisons + time to build *M*.
- As luck would have it, the Knuth, ***Morris***, Pratt algorithm builds *M* quickly.
- By the way, it can be done with fewer than *t* comparisons in the worst case!

Real-life uses of finite state machines

- grep
- coke machines
- thermostats (fridge)
- elevators
- train track switches
- lexical analyzers for parsers

Any $L \subseteq \Sigma^*$ is defined to be a <u>language</u>.

L is just a set of strings. It is called a language for historical reasons.

Let $L \subseteq \Sigma^*$ be a language.

L is called a regular language if there is some finite automaton that accepts *L*.

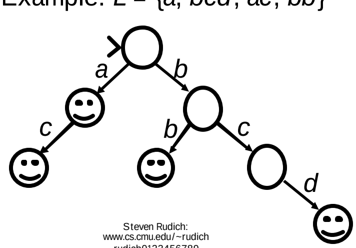
In this lecture we have seen many regular languages.

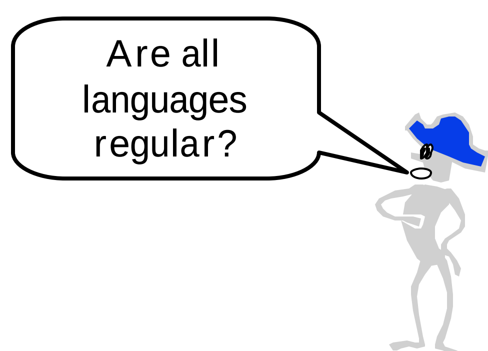
- Σ^*
- even length strings
- strings containing *ababb*

Theorem: Any finite langage is regular.

Proof: Make a machine with a "path" for each string in the language.

Example: *L* = {*a*, *bcd*, *ac*, *bb*}





Consider the language

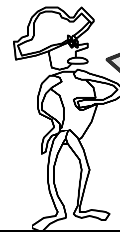
$$a^n b^n = \{e, ab, aabb, aaabbb, \ldots\}$$

i.e., a bunch of *a*'s followed by an equal number of *b*'s

No finite automaton accepts this language.

Can you prove this?

$a^n b^n$ is not regular. No machine has enough states to keep track of the number of *a*'s it might encounter.



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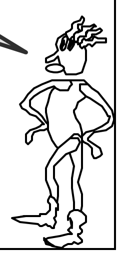
That is a fairly weak argument. Consider the following example…



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L = strings where the # of occurrences of the pattern *ab* is equal to the number of occurrences of the pattern *ba*

Can't be regular. No machine has enough states to keep track of the number of occurrences of *ab*.

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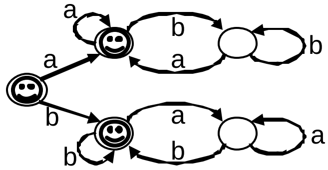
Remember "ABA"?



ABA accepts only the strings with an equal number of *ab*'s and *ba*'s!

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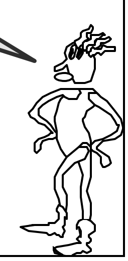
Let me show you a professional strength proof that $a^n b^n$ is not regular….



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Professional Strength Proof

Theorem: $a^n b^n$ is not regular.

Proof: Assume that it is. Then $\exists$ M with k states that accepts it.

For each $0 \le i \le k$, let S_i be the state M is in after reading a^i.

$\exists i,j \le k$ s.t. $S_i = S_j$, but $i \ne j$

M will do the same thing on $a^i b^i$ and $a^j b^i$.

But a valid M must reject $a^j b^i$ and accept $a^i b^i$.

$\Rightarrow\Leftarrow$

MORAL:

Finite automata can't count.

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Advertisement

You can learn much more about these creatures in the FLAC course.

Formal Languages, Automata, and Computation

- **There is a unique smallest automaton for any regular language**
- **It can be found by a fast algorithm.**

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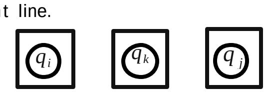
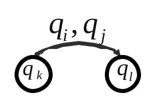
Cellular Automata

- Line up a bunch of ***identical*** finite automata in a straight line.



- Transitions are based on the states of the machine's two neighbors or an indicator that a neighbor is missing. (There is no other input.)

$$Q \times Q \times Q \to Q$$
$$(q_k, q_i, q_j) = q_l$$



- All cells move to their next states at the same time: synchronous transition

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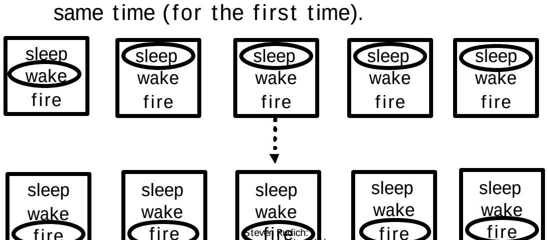
The Firing Squad Problem

- Five "soldiers" all start in the sleep state. You change the one on the left to the wake state.
- All five must get to the fire state at the same time (for the first time).



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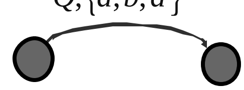
Shorthand

$$Q, \{a, b, d\}$$



Means use this transition when your left neighbor is in any state at all and your right neighbor is in state a,b, or d.

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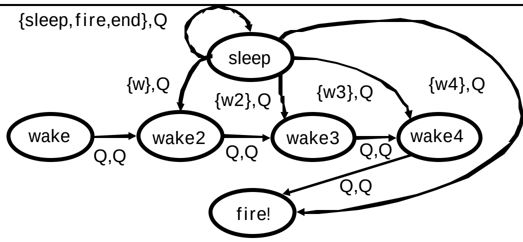
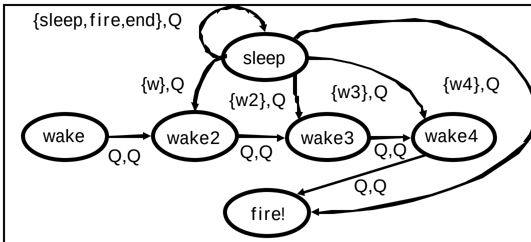
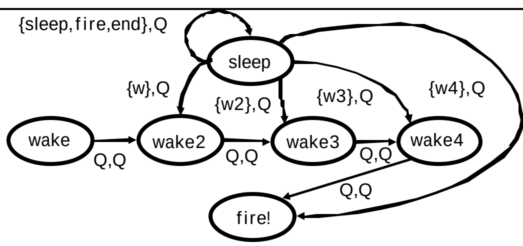


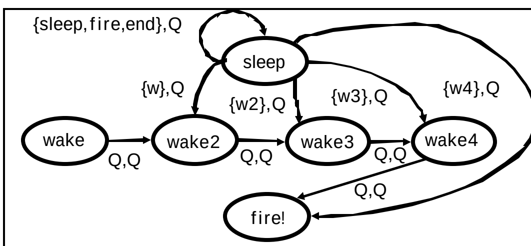
	1	2	3	4	5
	sleep	sleep	sleep	sleep	sleep
	sleep	sleep	sleep	sleep	sleep
	sleep	sleep	sleep	sleep	sleep
	sleep	sleep	sleep	sleep	sleep
	sleep	sleep	sleep	sleep	sleep



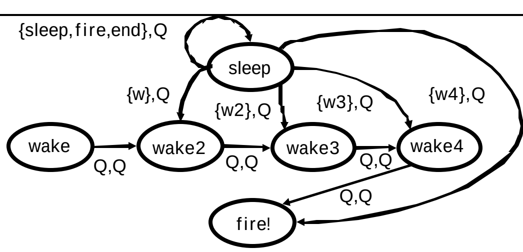
	1	2	3	4	5
	wake	sleep	sleep	sleep	sleep



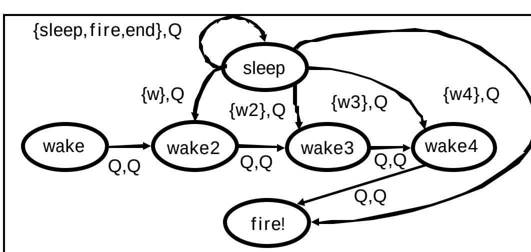
	1	2	3	4	5
	wake	sleep	sleep	sleep	sleep
	wake2	wake2	sleep	sleep	sleep



	1	2	3	4	5
	wake	sleep	sleep	sleep	sleep
	wake2	wake2	sleep	sleep	sleep
	wake3	wake3	wake3	sleep	sleep



	1	2	3	4	5
	wake	sleep	sleep	sleep	sleep
	wake2	wake2	sleep	sleep	sleep
	wake3	wake3	wake3	sleep	sleep
	wake4	wake4	wake4	wake4	sleep



	1	2	3	4	5
	wake	sleep	sleep	sleep	sleep
	wake2	wake2	sleep	sleep	sleep
	wake3	wake3	wake3	sleep	sleep
	wake4	wake4	wake4	wake4	sleep
	fire	fire	fire	fire	fire

Question

Can you build the soldier's finite automaton brain before you know how many soldiers will be in the line?