

15-251

Great Theoretical Ideas in Computer Science

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www.cs.cmu.edu/~15251

Course Staff

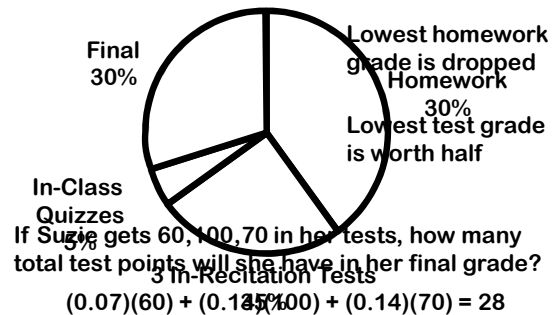
Instructors

Anupam Gupta
Danny Sleator

TAs

Adam Blank
Dmitriy Chernyak
Tim Wilson

Grading



9 Homeworks

Homeworks handed out Tuesdays or Thursdays
due on the stated date, at midnight.

Ten points per day late penalty

No homework will be accepted more than three days late

Homework **MUST** be typeset, and a single PDF file





Collaboration + Cheating

Groups of at most 3 people

You may NOT share written work

You may NOT use Google (or other search engines)

You may NOT use solutions to previous years' homework.

You MUST sign the class honor code

Question 3

(i)

A is a regular language

$\Rightarrow \exists$ a DFA which accepts the language A .

Let this DFA be defined as $M_A = \{Q, q_0, F, \delta, \Sigma_A\}$

Suppose the first input string is uv where $u \in \text{language } A$, and uv is

Problem 5

A .

A is a regular language

$\Rightarrow \exists$ a DFA which accepts the language A .

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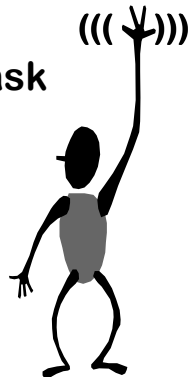
Textbook

There is NO textbook for this class

We have class notes in wiki format

You too can edit the wiki!!! (soon)

Feel free to ask questions

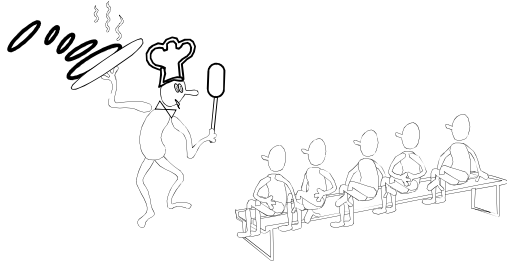


15-251

Cooking for
Computer Scientists

Pancakes With A Problem!

Lecture 1 (August 24, 2010)



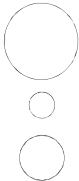
The chefs at our place are sloppy:
when they prepare pancakes, they
come out all different sizes

When the waiter delivers them to a customer,
he rearranges them (so that smallest is on top,
and so on, down to the largest at the bottom)

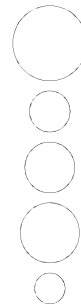
He does this by grabbing several
from the top and flipping them
over, repeating this (varying the
number he flips) as many times as
necessary



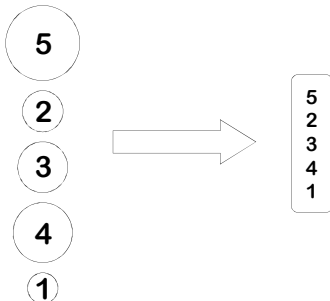
How do we sort this stack?
How many flips do we need?



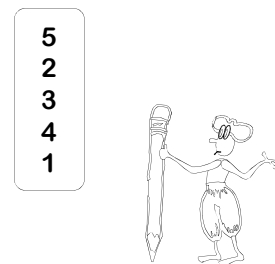
How do we sort this stack?
How many flips do we need?



Developing A Notation:
Turning pancakes into numbers



How do we sort this stack?
How many flips do we need?



5
2
3
4
1

4 Flips Are Sufficient

5
2
3
4
1

1
4
3
2
5

2
3
4
1
5

4
3
2
1
5

1
2
3
4
5

Best Way to Sort

X = Smallest number of
flips required to sort:

5
2
3
4
1

Lower
Bound

$? \leq X \leq 4$

Upper
Bound

X = Smallest number of
flips required to sort:

5
2
3
4
1

Lower
Bound

$? \leq X \leq 4$

Upper
Bound

4 is an upper bound: there exists a way to
sort this stack using at most 4 flips.

Which of the following statements would show a lower bound of 4?

a. there exists a way to sort
this stack using at least 4 flips?

b. for every way to sort this stack,
you use at least 4 flips.

Can we do better?

5
2
3
4
1

Four Flips Are Necessary

5
2
3
4
1

1
4
3
2
5

4
1
3
2
5

If we could do it in three flips:

Flip 1 has to put 5 on bottom (else we
would take 3 flips just to get 5 to bottom)

Flip 2 must bring 4 to top (if it didn't,
we would take more than three flips)

4

$$4 \leq X \leq 4$$

Lower
Bound

}

Upper
Bound

$X = 4$

where
 $X =$ Smallest number of
 flips required to sort:

5
2
3
4
1

1
2
3
5
4

5th Pancake Number

$P_5 =$ Number of flips required to sort the
 MAX over $s \in$ stacks of 5
 worst case stack of 5 pancakes
 of MIN # of flips to sort s

1
2
3
4
5

X_1

1
2
3
5
4

X_2

1
2
4
3
5

X_3

...

5
2
3
4
1

4

...

5
4
3
1
2

X_{119}

5
4
3
2
1

X_{120}

5th Pancake Number

Lower
Bound

$4 \leq P_5 \leq ?$

Upper
Bound

“There exists a 5-pancake stack which will make me take this much time”

~~“For every 5-pancake stack, I will take this much time”~~

“For all 5-pancake stacks s , we can sort s in this much time”

$P_n =$ MAX over $s \in$ stacks of n pancakes
 of MIN # of flips to sort s

$P_n =$ The number of flips required to sort
 the worst-case stack of n pancakes

What is P_n for small n ?

Can you do
 $n = 0, 1, 2, 3$?

Initial Values of P_n

n	0	1	2	3
P_n	0	0	1	3

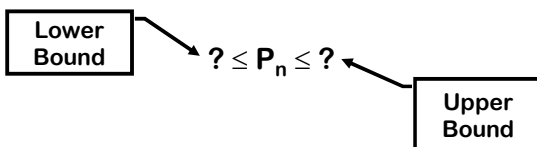
$$P_3 = 3$$

$\begin{matrix} 1 \\ 3 \\ 2 \end{matrix}$ requires 3 Flips, hence $P_3 \geq 3$

ANY stack of 3 can be done by getting the big one to the bottom (≤ 2 flips), and then using ≤ 1 flips to handle the top two

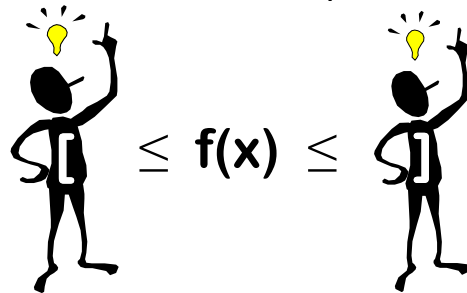
n^{th} Pancake Number

P_n = Number of flips required to sort the worst case stack of n pancakes



Bracketing:

What are the best lower and upper bounds that I can prove?

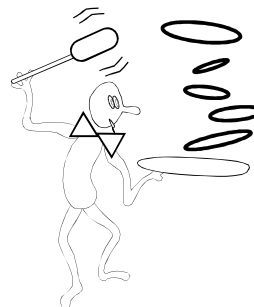


$$? \leq P_n \leq ?$$

Try to find upper and lower bounds on P_n , for $n > 3$



Bring-to-top Method



Bring biggest to top
Place it on bottom

Bring next largest to top
Place second from bottom

And so on...

Upper Bound On P_n :

If $n=1$, no work required — we are done!
Otherwise, flip pancake n to top and
then flip it to position n

Bring To Top Method For n-1 Pancakes

Total Cost: at most $2(n-1) = 2n - 2$ flips

Better Upper Bound On P_n :

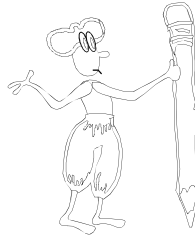
If $n=2$, at most one flip and we are done!
Otherwise, flip pancake n to top and
then flip it to position n

Bring To Top Method For n-1 Pancakes

Total Cost: at most $2(n-2) + 1 = 2n - 3$ flips

$$? \leq P_n \leq 2n-3$$

$$? \leq P_n \leq 2n-3$$



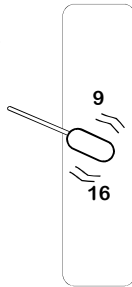
What other bounds can you prove on P_n ?

Breaking Apart Argument

Suppose a stack S has a pair of adjacent pancakes that will not be adjacent in the sorted stack

Any sequence of flips that sorts stack S must have one flip that inserts the spatula between that pair and breaks them apart

Furthermore, this is true of the “pair” formed by the bottom pancake of S and the plate



S

$$n \leq P_n$$

Suppose n is even

S contains n pairs that will need to be broken apart during any sequence that sorts it

Detail: This construction only works when $n > 2$

2
1

S

1
3
5
7
.
n
2
4
6
.
n-1

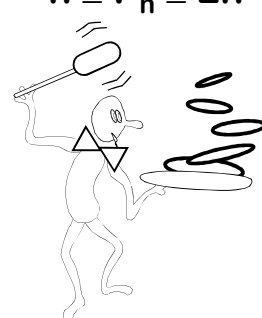
$n \leq P_n$

Suppose n is odd

S contains n pairs that will need to be broken apart during any sequence that sorts it

Detail: This construction only works when $n > 3$

1
3
2

$n \leq P_n \leq 2n - 3 \text{ for } n > 3$


Bring-to-top is within a factor of 2 of the lower bound!

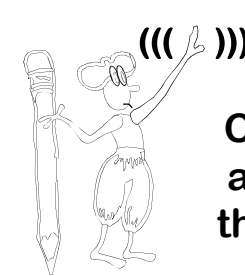
From ANY stack to sorted stack in $\leq P_n$

From sorted stack to ANY stack in $\leq P_n$?



Reverse the sequences we use to sort

Hence, from ANY stack to ANY stack in $\leq 2P_n$



Can you find a faster way than $2P_n$ flips to go from ANY to ANY?

ANY Stack S to ANY stack T in $\leq P_n$

S: 4,3,5,1,2

1,2,3,4,5

T: 5,2,4,3,1

3,5,1,2,4

“new T” →

Rename the pancakes in S to be 1,2,3,...,n

Rewrite T using the new naming scheme that you used for S

The sequence of flips that brings the sorted stack to the “new T” will bring S to T

The Known Pancake Numbers

n	P_n
1	0
2	1
3	3
4	4
5	5
6	7
7	8
8	9
9	10
10	11
11	13
12	14
13	15
14	16
15	17
16	18
17	19

P_{18} is Unknown

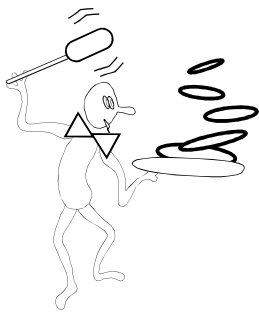
It is either 20 or 21, we don't know which.

$1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot 17 \cdot 18 = 18!$ orderings of 18 pancakes

$$18! = 6.402373 \times 10^{15}$$

To give a lower bound of 21 (say), we would have to find one of these stacks for which no sequences of 20 swaps sort the stack.

Is This Really Computer Science?

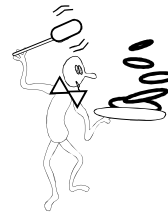


Sorting By Prefix Reversal

Posed in *Amer. Math. Monthly* 82 (1), 1975,
"Harry Dweighter" a.k.a. Jacob Goodman

$$(17/16)n \leq P_n \leq (5n+5)/3$$

William H. Gates and Christos Papadimitriou.
Bounds For Sorting By Prefix Reversal.
Discrete Mathematics, vol 27, pp 47-57, 1979.



$$(15/14)n \leq P_n \leq (5n+5)/3$$

H. Heydari and H. I. Sudborough. On the
Diameter of the Pancake Network. *Journal
of Algorithms*, vol 25, pp 67-94, 1997.



$$(15/14)n \leq P_n \leq (18/11)n$$

B. Chitturi, W. Fahle, Z. Meng, L. Morales,
C. O. Shields, I. H. Sudborough and W. Voit.
An $(18/11)n$ upper bound for sorting by
prefix reversals, to appear in *Theoretical
Computer Science*, 2008.



Burnt Pancakes

$$(3/2)n \leq BP_n \leq 2n-2$$

David S. Cohen and Manuel Blum.
On the problem of sorting burnt pancakes.
Discrete Applied Mathematics, 1995.



How many different
stacks of n pancakes
are there?

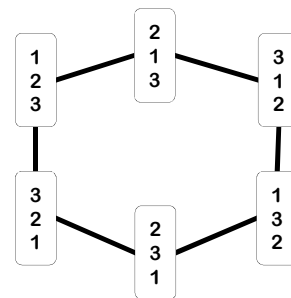
$$n! = 1 \times 2 \times 3 \times \dots \times n$$

Pancake Network: Definition For $n!$ Nodes

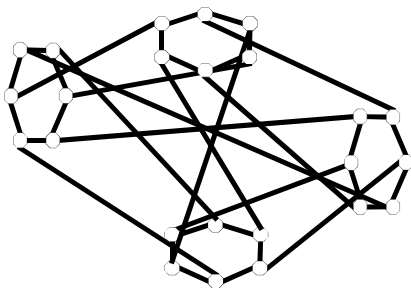
For each node, assign it the name of one
of the $n!$ stacks of n pancakes

Put a wire between two nodes if they
are one flip apart

Network For $n = 3$

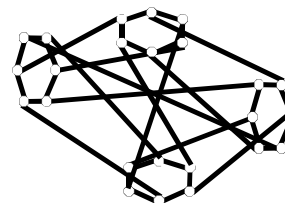


Network For $n=4$



Pancake Network: Message Routing Delay

What is the maximum distance between
two nodes in the pancake network?



P_n

Pancake Network: Reliability

If up to $n-2$ nodes get hit by lightning, the network remains connected, even though each node is connected to only $n-1$ others

The Pancake Network is optimally reliable for its number of edges and nodes

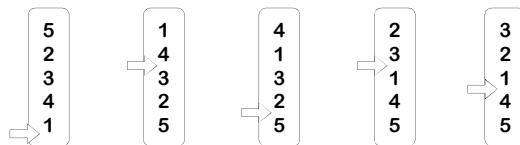
An Algorithms Problem

Suppose I give you a n -stack of pancakes

You want to use the minimum number of flips possible to sort this stack.

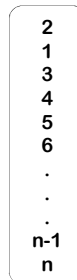
How quickly can you compute this sequence of flips?

For a particular stack
bring-to-top not always optimal

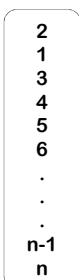


Bring-to-top takes 5 flips,
but we can do in 4 flips

In fact, naïve bring-to-top is not
even close to optimal in general!



In fact, naïve bring-to-top is not
even close to optimal in general!



Can you alter bring-to-top
to take at most
3 * optimal number of flips
in general?

how about
2 * optimal number of flips?

Mutation Distance

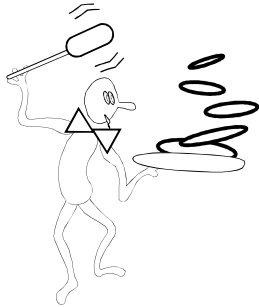
Head Cabbage
(*Brassica oleracea capitata*)



Turnip
(*Brassica rapa*)



One “Simple” Problem

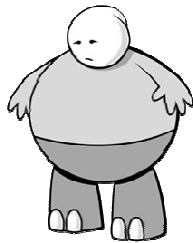
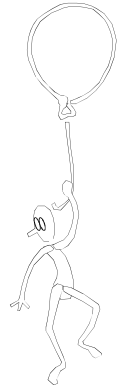


A host of
problems and
applications at
the frontiers of
science

High Level Point

Computer Science is not merely
about computers and programming,
it is about mathematically modeling
our world, and about finding better
and better ways to solve problems

Today’s lecture is a microcosm of
this exercise



Here’s What
You Need to
Know...

Definitions of:
 n^{th} pancake number
lower bound
upper bound

Proof of:
ANY to ANY in $\leq P_n$

Important Technique:
Bracketing