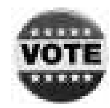
15-251

Great Theoretical Ideas in Computer Science

Social Choice, Voting and Auctions

Lecture 17 (October 21, 2008)



Big picture

Say you like pizza > hotdogs >> burgers.

We ask you for your preferences, so that we can order food for the review session.

I'll choose the option that gets most 1st place votes.

What should you tell me?

What if you know: the rest of the class has 20 votes each for hotdogs and burgers, and 15 for pizza?

Big picture (some more)

People have private information

You want to get that information

If you ask them, will they tell you the truth?

Well, they might lie, if telling a lie helps them. ("strategic behavior")

How do you elicit the truth?

let's start simple

Who's the winner?

Three candidates in town
Three voters in town

1st: a > b > c
2nd: b > c > a
3rd: c > a > b

Who's the winner?
What's the best ordering of the candidates?

Condorcet's Paradox

Marie Jean Antoine Nicolas de Caritat,
Marquis de Condorcet

1st: a > b > c
2nd: b > c > a
3rd: c > a > b

Given any potential winner (say a),
a majority prefer another candidate (c) to this person.

Social Choice/Ranking

A: set of alternatives (say, {a,b,c,d})

L: set of all possible rankings or linear orderings of these alternatives
e.g., a > d > b > c, or b > c > d > a

N people each with their ranking

Want to combine these into one "social ranking"

Two questions

Given the rankings of the N individuals:

Social Choice:
Output the alternative that's the "winner"
i.e., output an element of A

Social Ranking:
Output a ranking that "best captures" the rankings of the individuals.

E.g. 1

A = {a, b}
In this case, L = { (a>b), (b>a)}

Population:
person 1: a>b
person 2: a>b
person 3: b>a
person 4: a>b

Social Choice: maybe use "plurailty" and output a
Social Ranking: (a>b)

E.g. 2

A = {a, b, c}
L = { (a>b>c), (a>c>b),(b>c>a),(b>a>c), (c>a>b), (c>b>a)}

Population:
person 1: a>b>c
person 2: a>b>c
person 3: b>a>c
person 4: c>a>b

Social Choice: maybe use "plurailty" and output a
Social Ranking: maybe (a>b>c)

E.g. 3

A = {a, b, c}
L = { (a>b>c), (a>c>b),(b>c>a),(b>a>c), (c>a>b), (c>b>a)}

Population:
person 1: a>b>c
person 2: b>c>a
person 3: c>a>b

Social Choice: For each output, majority of people prefer some other candidate
Social Ranking: ???

what are some properties we'd like?

Notation

Social ranking function
$F: L^N \to L$
takes $(>_1, >_2, \ldots, >_N) \to >_{output}$

Social choice function
$G: L^N \to A$
takes $(>_1, >_2, \ldots, >_N) \to a$

Some properties

Unanimity:

F is unanimous if when all individuals have a>b for some a,b in A, then the output satisfies a>b.

Some properties

Independent of Irrelevant Alternatives:

F is IIA if the relative ranking of a and b in the outcome depends only on the voters' rankings of a and b.

I.e., whenever all voters rank a and b the same, the output is the same, regardless of the other alternatives.

Some properties

Dictator

Voter j is a dictator in F if
$F(>_1, >_2, \ldots, >_N) = >_j$

F is a dictatorship if there is some j that is a dictator in F.

The case for |A| = 2

Fact:
If there are 2 alternatives, the IIA property is trivially satisfied.

Facts:
Note that plurality satisfies unanimity, and is not a dictatorship.

The case for |A| = 3 (or more)

Here are two ways to output an ordering:

Copeland's method
Borda's method

Copeland's Method

The Borda system

Social Choice functions…

What about social-choice functions?

Remember a social choice function
outputs a single choice

I.e., G: $L^N \to A$,
takes $(>_1, >_2, \ldots, >_N) \to a$

Some properties

Unanimity:

G is unanimous if when all individuals have a at the top of their rankings, then G outputs a.

Some properties

Monotone:

G is monotone if whenever
$G(>_1, >_2, \ldots, >_j, \ldots, >_N) = a$
and
$G(>_1, >_2, \ldots, >_j', \ldots, >_N) = a'$
then it must be the case that voter j moved a' above a in his ranking.

(I.e., G is "incentive-compatible". It does not reward lying.)

Some properties

Dictator

Voter j is a dictator in G if
$G(>_1, >_2, \ldots, >_N)$ = choice at top of $>_j$

G is a dictatorship if there is some j that is a dictator in G.

Again, some simple cases…

Plurality

Output the option at the top of most people's rankings.

Unanimity:

Monotonicity:

Instant-Runoff Voting

Remove alternative with fewest first-place votes, and repeat.

Unanimity:

Monotonicity:

What are some good social ranking and social choice functions for |A| >= 3?

The case for |A| = 3 (or more)

Theorem (Arrow)

Any social ranking function with |A| = 3 or more that satisfies
unanimity and IIA
is a
dictatorship.

The case for |A| = 3 (or more)

Theorem (Gibbard-Satterthwaite)

Any social choice function with |A| = 3 or more that satisfies
unanimity and monotonicity
is a
dictatorship.

Gibbard-Satterthwaite

Note that we wanted to ask people for their (secret) preferences, and use that to pick a social choice.

We don't want them to lie. (Hence we want the social choice function to be monotone.)

But that is impossible. ☹

Arrow's Theorem

Gibbard-Satterthwaite Theorem

Two important results
with
very similar proofs

So what do we do?

How to get around these impossibility results?

Two solutions:

1) Money…

2) Change the representation…

Mechanisms with money

Measure not just that a preferred to b, but also "by how much"…

Each individual j (or player j) has a "valuation" for each alternative a in A. Denoted as $v_j(a)$

Also, each player values money the same.

So, if we choose alternative a, and give \$m to j, then j's "utility" is $v_j(a) + m$

Selfishness

Each player acts to maximize her utility.

Auctions

Suppose there is a single item a to be auctioned.

Each player has value $v_j(a)$ (or just v_j) for it.

If item given to j, and j pays \$p, then
utility(j) = $v_j - p$
and
utility(j') = 0 for all other players j'.

Auctions

However, auctioneer does not know these private valuations.

Auctioneer wants to give the item to the person who values it the most.

(Think of artist giving a painting to the person who wants it the most. Not revenue-maximizing here!)

What should the auctioneer do?

Picture

Auctioneer gets "bids" b_j
which should ideally be the valuations v_j

But may be higher or lower
if it helps players, they'll report something else

Try #1

Ask each person for their valuation ("bids"),
give it to the person j with highest bid b_j.

Try #2

Ask each person for their valuation,
give it to the person j with highest bid b_j,
ask for payment b_j.

Try #3

Ask each person for their valuation,
give it to the person j with highest bid b_j,
ask for payment b_k where k has 2nd-highest bid.

(Called Vickery second-price auction.)

Truth-telling is a good strategy here

Suppose true valuations are $v_1, v_2, \ldots, v_n$
Then j's utility u_j
when he bids $b_j = v_j$
is at least as much as his utility u_j'
when he bids any other b_j'
(regardless of whatever the other players do)

So what do we do?

How to get around these impossibility results?

Two solutions:

1) Money…

2) Change the representation…

Range Voting

How to get around Arrow's paradox:

Each player, instead of giving a ranking of all the alternatives, gives a score in [0…10] to each alternative.

Pick the alternative with maximum average score.

Changing the representation is a powerful idea

I have a number in my left hand
and a different number in my right hand

You don't know what these values are

You choose a hand
I show you the number I have in that hand
You either take that
Or you decline, and I give you the number from other hand

You want to maximize the number you get.
How should you play?

You can get…

If I have X and Y in my two hands,
In expectation, you can get (X+Y)/2.

How can you do better?

It's not the voting that's democracy,
it's the counting…

-Tom Stoppard (Jumpers, 1972)