

## Weekly Homework

Homework will go out every Tuesday and is due the Tuesday after
Ten points per day late penalty
No homework will be accepted more than three days late

## Assignment 1: The Great 251 Hunt!

You will work in randomly chosen groups of 4
The actual Puzzle Hunt will start at 8pm tonight
You will need at least one digital camera per group

Can buy a digital camera for $\$ 8$ nowadays!

## Textbook

There is NO textbook for this class
We have class notes in wiki format
You too can edit the wiki!!!

## Collaboration + Cheating

You may NOT share written work
You may NOT use Google, or solutions to previous years' homework

You MUST sign the class honor code

| TextboOk |
| :---: |
| There is NO textbook for this class |
| We have class notes in wiki format |
| You too can edit the wiki!!! |
|  |



## Pancakes With A Problem!

Lecture 1 (August 28, 2007)


Developing A Notation:
Turning pancakes into numbers

(2)


4
(1)

How do we sort this stack? How many flips do we need?



## Four Flips Are Necessary



If we could do it in three flips:
Flip 1 has to put 5 on bottom
Flip 2 must bring 4 to top (if it didn't, we would spend more than 3)


$\mathbf{P}_{\mathrm{n}}=$ MAX over $\mathbf{s} \in$ stacks of n pancakes of MIN \# of flips to sort s
$P_{n}=$ The number of flips required to sort the worst-case stack of $n$ pancakes



## Upper Bound On $\mathrm{P}_{\mathrm{n}}$ : <br> Bring-to-top Method For $n$ Pancakes



Total Cost: at most $2(n-1)=2 n-2$ flips


## Breaking Apart Argument

Suppose a stack $S$ has a pair of adjacent pancakes that will not be adjacent in the sorted stack

Any sequence of flips that sorts stack $S$ must have one flip that inserts the spatula between that pair and breaks them apart

Furthermore, this is true of the "pair" formed by the bottom pancake of $S$ and the plate

-


From ANY stack to sorted stack in $\leq P_{n}$
From sorted stack to ANY stack in $\leq P_{\mathrm{n}}$ ?


ANY Stack $S$ to ANY stack $T$ in $\leq P_{n}$


Rename the pancakes in $S$ to be 1,2,3,..,n
Rewrite T using the new naming scheme that you used for $S$
The sequence of flips that brings the sorted stack to the "new T" will bring $S$ to $T$

The Known Pancake Numbers

| $n$ | $P_{n}$ |
| :---: | :---: |
| 1 | 0 |
| 2 | 1 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 7 |
| 7 | 8 |
| 8 | 9 |
| 9 | 10 |
| 10 | 11 |
| 11 | 13 |
| 12 | 14 |
| 13 | 15 |


| $P_{14}$ is Unknown |
| :---: |
| $1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot 13 \cdot 14=14$ ! orderings of 14 pancakes |
| $14!=87,178,291,200$ |
|  |


$(17 / 16) n \leq P_{n} \leq(5 n+5) / 3$
William Gates and Christos Papadimitriou. Bounds For Sorting By Prefix Reversal.
Discrete Mathematics, vol 27, pp 47-57, 1979.

$(15 / 14) n \leq P_{n} \leq(5 n+5) / 3$
H. Heydari and H. I. Sudborough. On the Diameter of the Pancake Network. Journal of Algorithms, vol 25, pp 67-94, 1997.


## Pancake Network:

 Definition For $n$ ! NodesFor each node, assign it the name of one of the $n$ ! stacks of $n$ pancakes

Put a wire between two nodes if they are one flip apart

Network For $\mathrm{n}=3$



## Pancake Network: Message Routing Delay

What is the maximum distance between two nodes in the pancake network?

$P_{n}$

## Pancake Network: Reliability

If up to $\mathrm{n}-2$ nodes get hit by lightning, the network remains connected, even though each node is connected to only $n$ - 1 others

The Pancake Network is optimally reliable for its number of edges and nodes

## Mutation Distance




## High Level Point

Computer Science is not merely about computers and programming, it is about mathematically modeling our world, and about finding better and better ways to solve problems

Today's lecture is a microcosm of this exercise


