15-213

"The course that gives CMU its Zip!"

Cache Memories
February 26, 2008

Topics

- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance
- The memory mountain

Synchronization

First exam this evening

- If you have not received mail with a Subject line like “15-213 exam: conflict session C2” then we expect you at the main exam session
- Room split by Andrew username (not first/last/middle name!)
  - a-c Wean 7500
  - d-z McConomy Auditorium in University Center
- Bring with you
  - Your TA's name and/or 15-213 section letter
    - If you want your test to be returned in recitation
  - Book and notes, if you wish
    - Suggested: know your powers of 2
  - No calculators

Synchronization - 2

Computer Club movie night

- “Colossus, The Forbin Project”
- Wednesday evening
- Wean 7500
- 19:00 Computer Club Intro, co-op pizza order
- 19:30 Movie

Determinant

Theorem 5. If A and B are square matrices of the same size, then det(AB) = det(A) * det(B).

The elegant simplicity of this result contrasted with the complex nature of both matrix multiplication and the determinant definition is both refreshing and surprising. We shall omit the proof.

Anton, *Elementary Linear Algebra, 4th ed.*, p. 72.

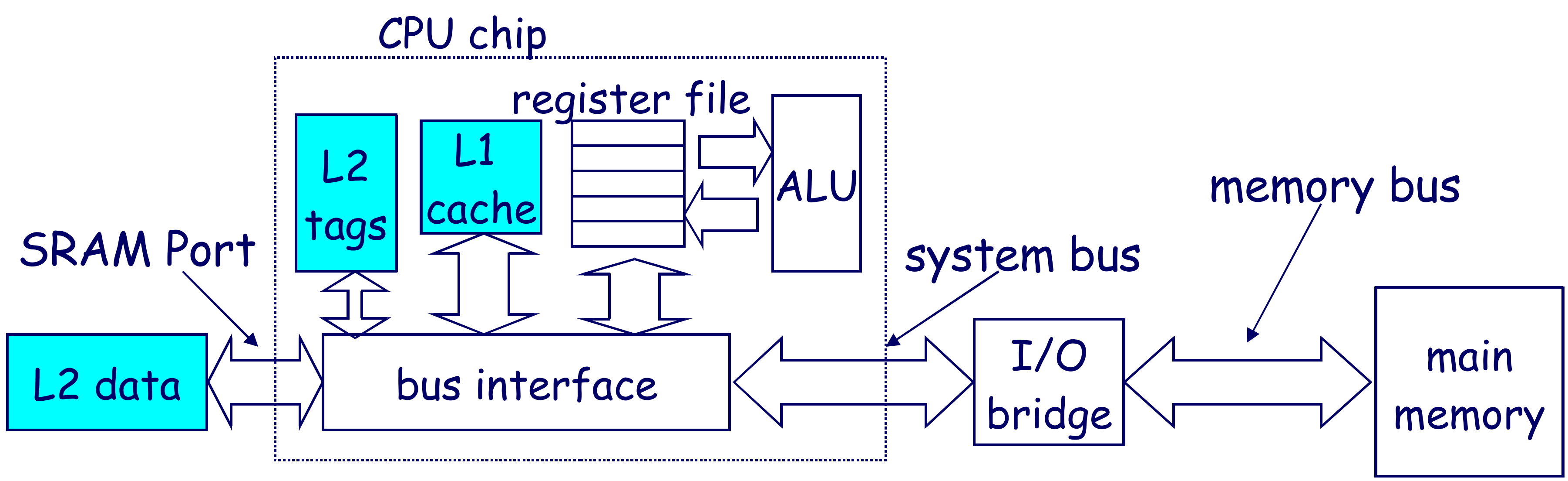
Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware.

- **Hold frequently accessed blocks of main memory**

CPU looks first for data in L1, then in L2, then in main memory.

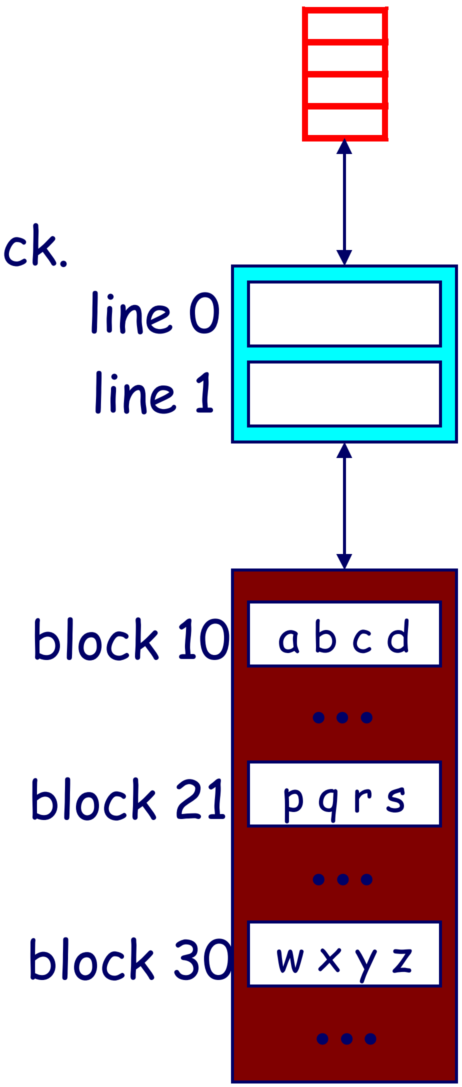
Typical system structure:



Inserting an L1 Cache Between the CPU and Main Memory

The transfer unit between the CPU register file and the cache is a 4-byte block.

The tiny, very fast CPU register file has room for four 4-byte words.



The small fast L1 cache has room for two 4-word blocks.

The transfer unit between the cache and main memory is a 4-word block (16 bytes).

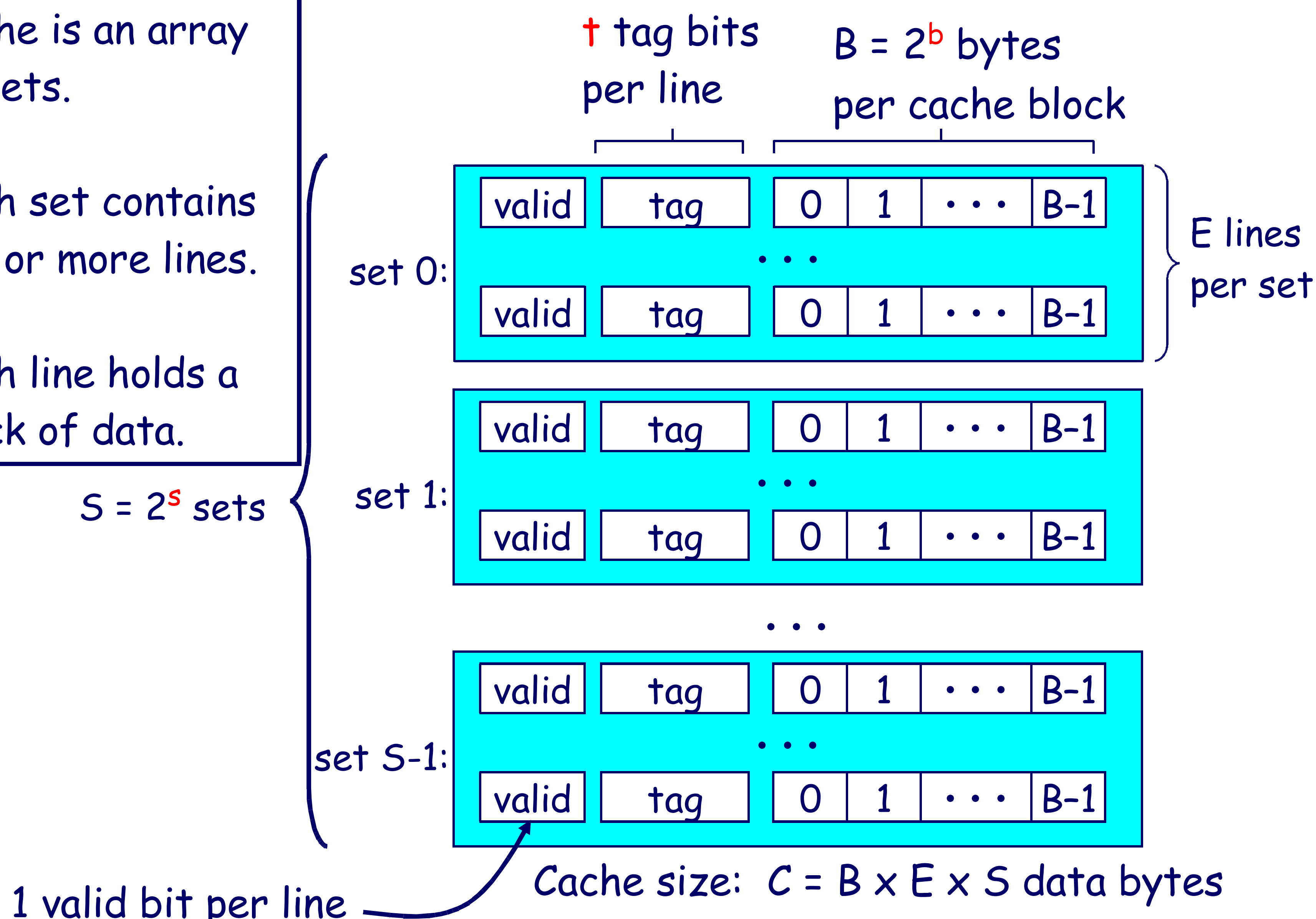
The big slow main memory has room for many 4-word blocks.

General Organization of a Cache

Cache is an array of sets.

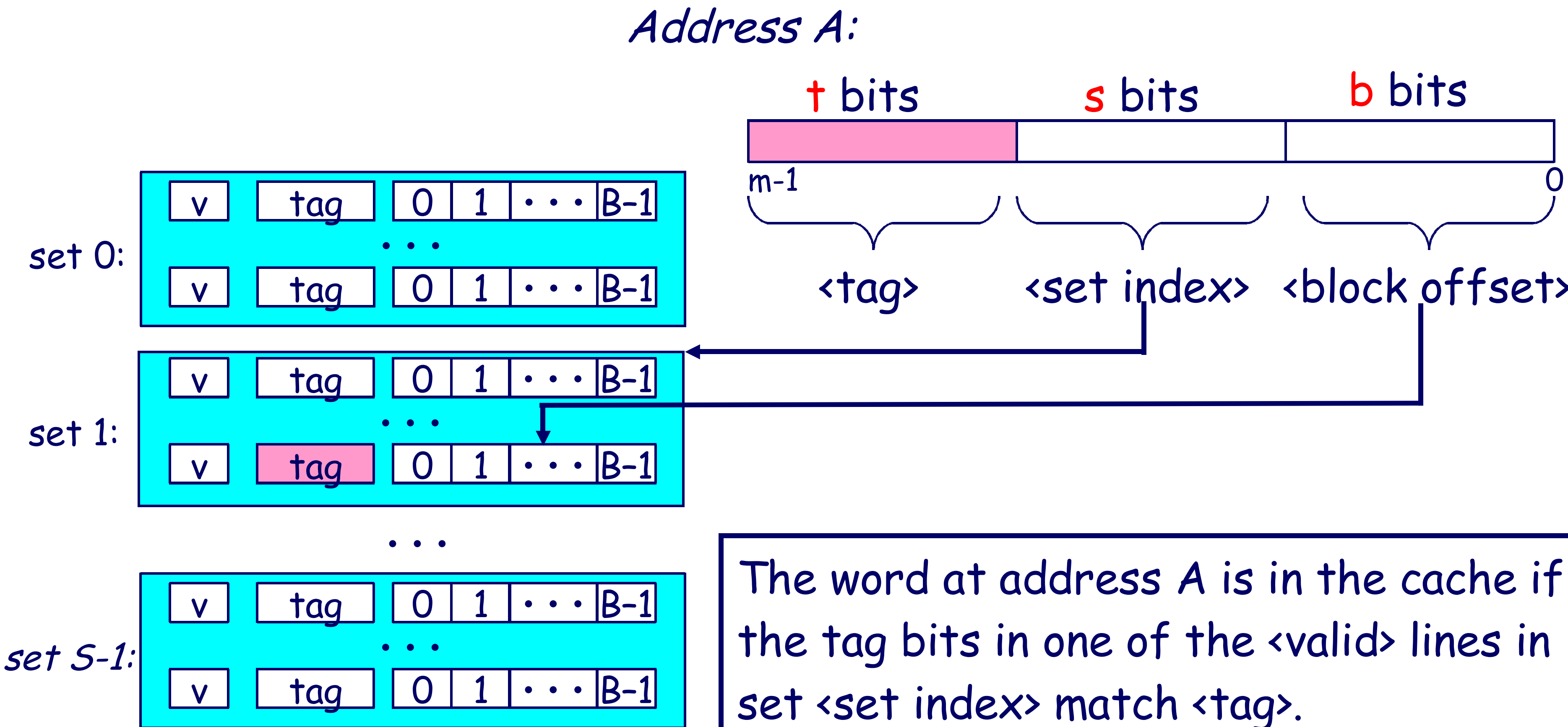
Each set contains one or more lines.

Each line holds a block of data.



Cache size: C = B x E x S data bytes

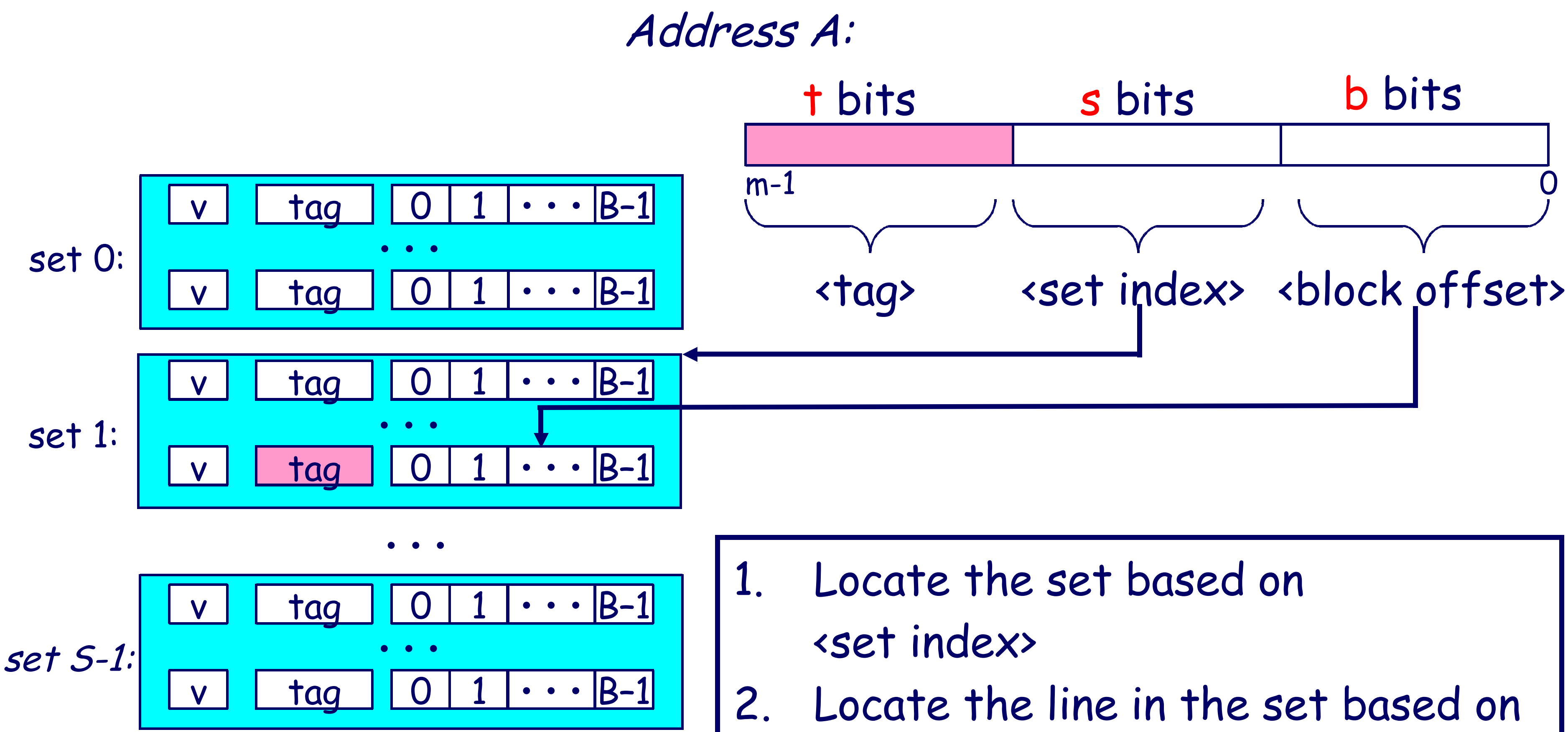
Addressing Caches



The word at address A is in the cache if the tag bits in one of the <valid> lines in set <set index> match <tag>.

The word contents begin at offset <block offset> bytes from the beginning of the block.

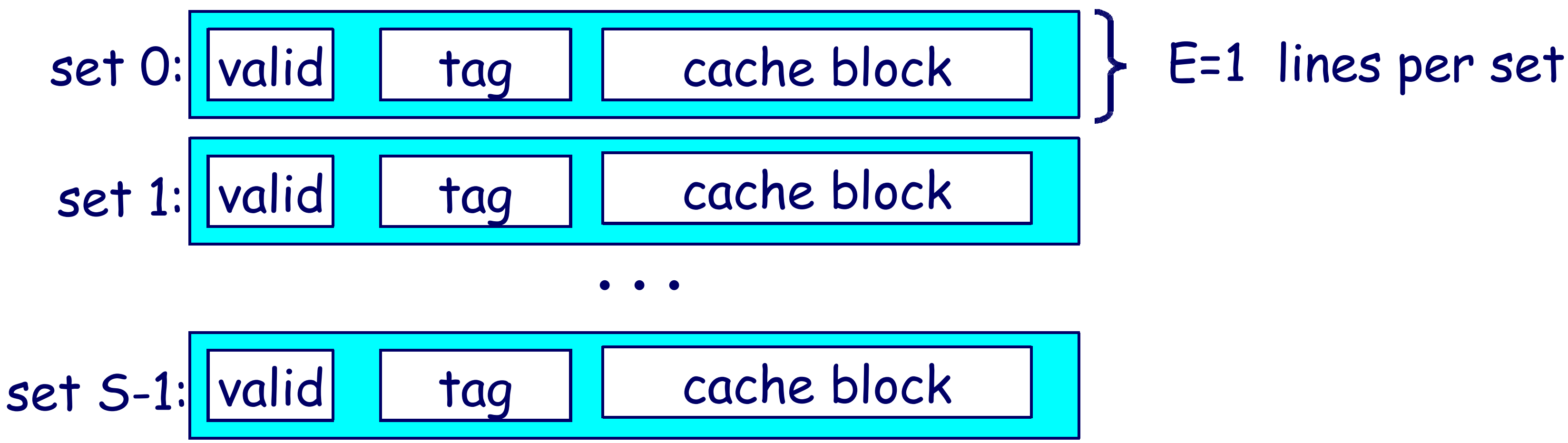
Addressing Caches



1. Locate the set based on <set index>
2. Locate the line in the set based on <tag>
3. Check that the line is valid
4. Locate the data in the line based on <block offset>

Direct-Mapped Cache

- **Simplest kind of cache, easy to build (only 1 tag compare required per access)**
- **Characterized by exactly one line per set.**

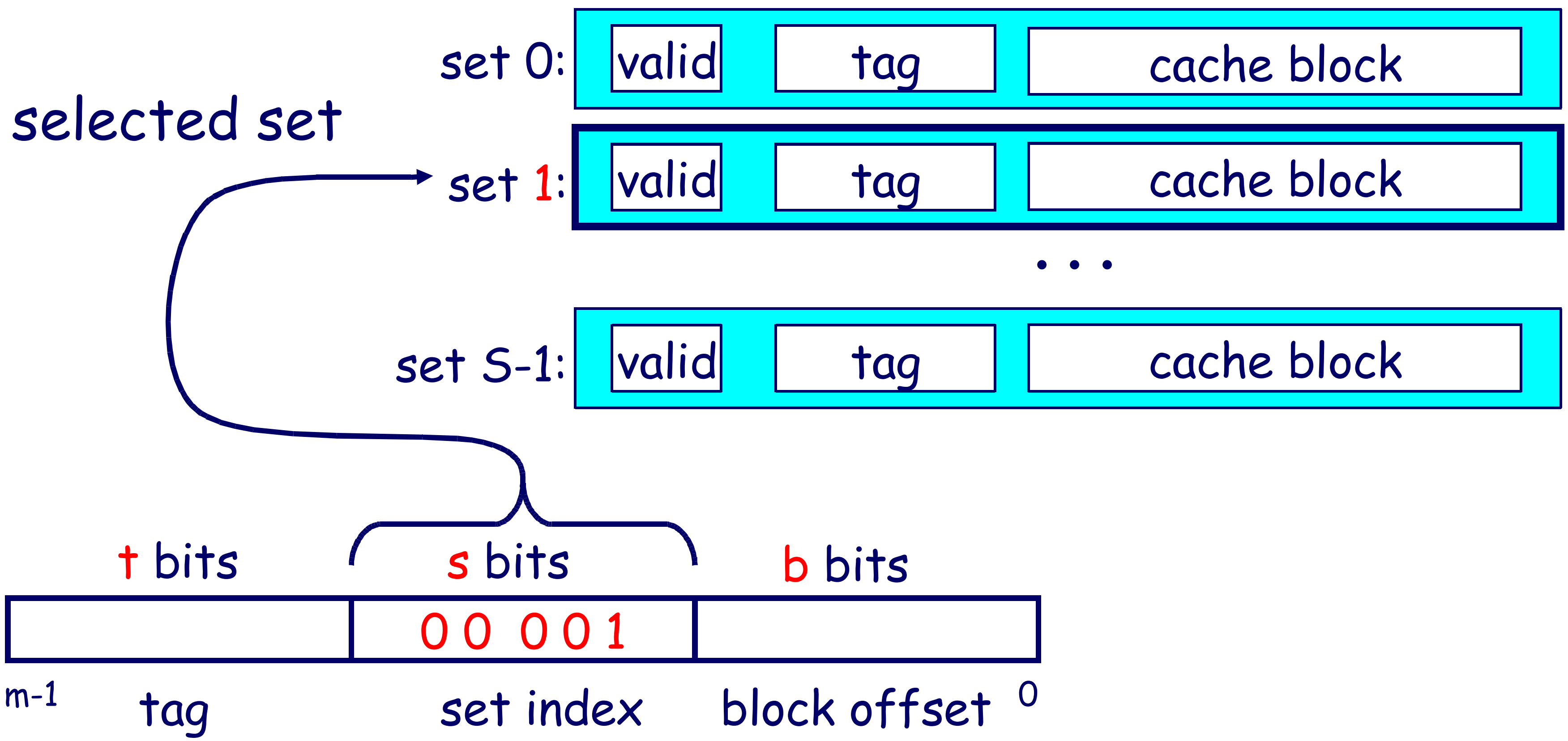


Cache size: $C = B \times S$ data bytes

Accessing Direct-Mapped Caches

Set selection

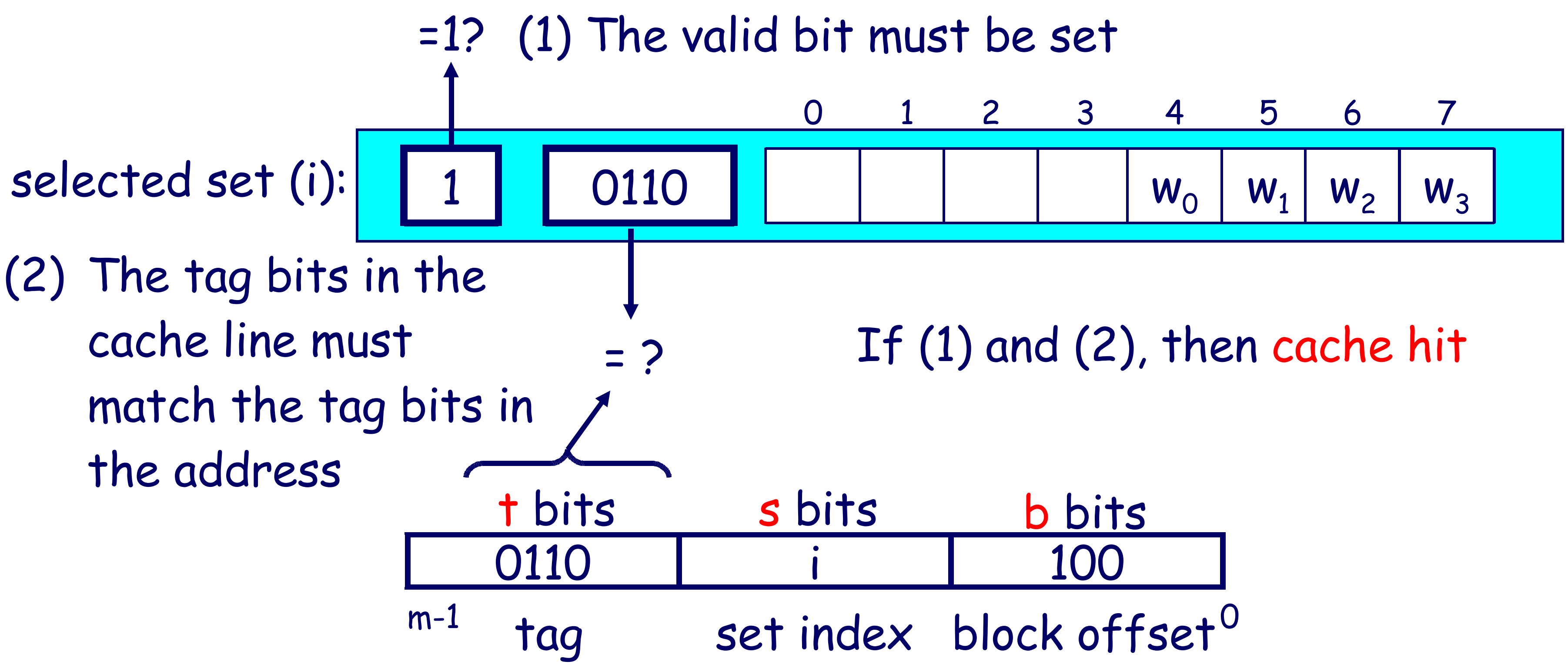
- Use the set index bits to determine the set of interest.



Accessing Direct-Mapped Caches

Line matching and word selection

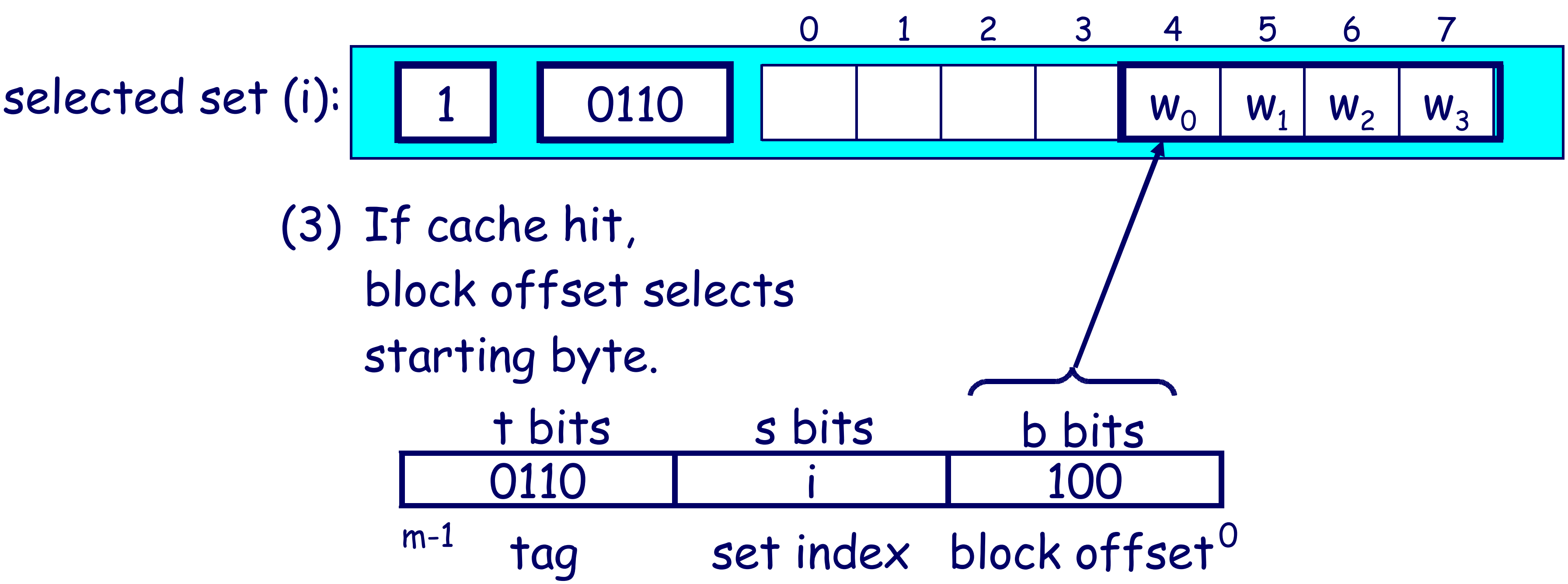
- **Line matching: Find a valid line in the selected set with a matching tag**
- **Word selection: Then extract the word**



Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching**: Find a valid line in the selected set with a matching tag
- **Word selection**: Then extract the word



Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 entry/set

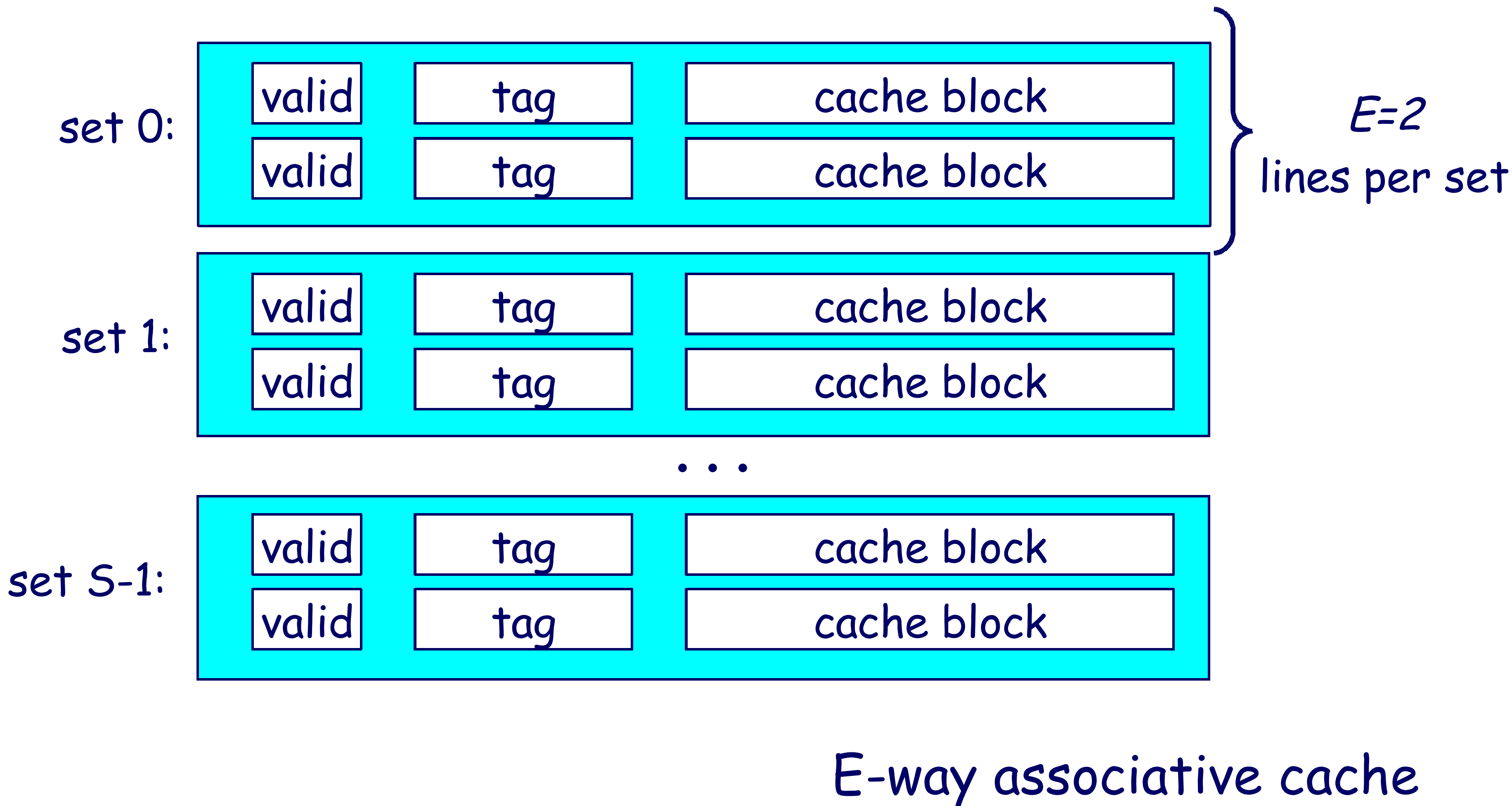
Address trace (reads):

0	**[0000_2],**	miss
1	**[0001_2],**	hit
7	**[0111_2],**	miss
8	**[1000_2],**	miss
0	**[0000_2]**	miss

v	tag	data
1	0	M[0-1]
1	0	M[6-7]

Set Associative Caches

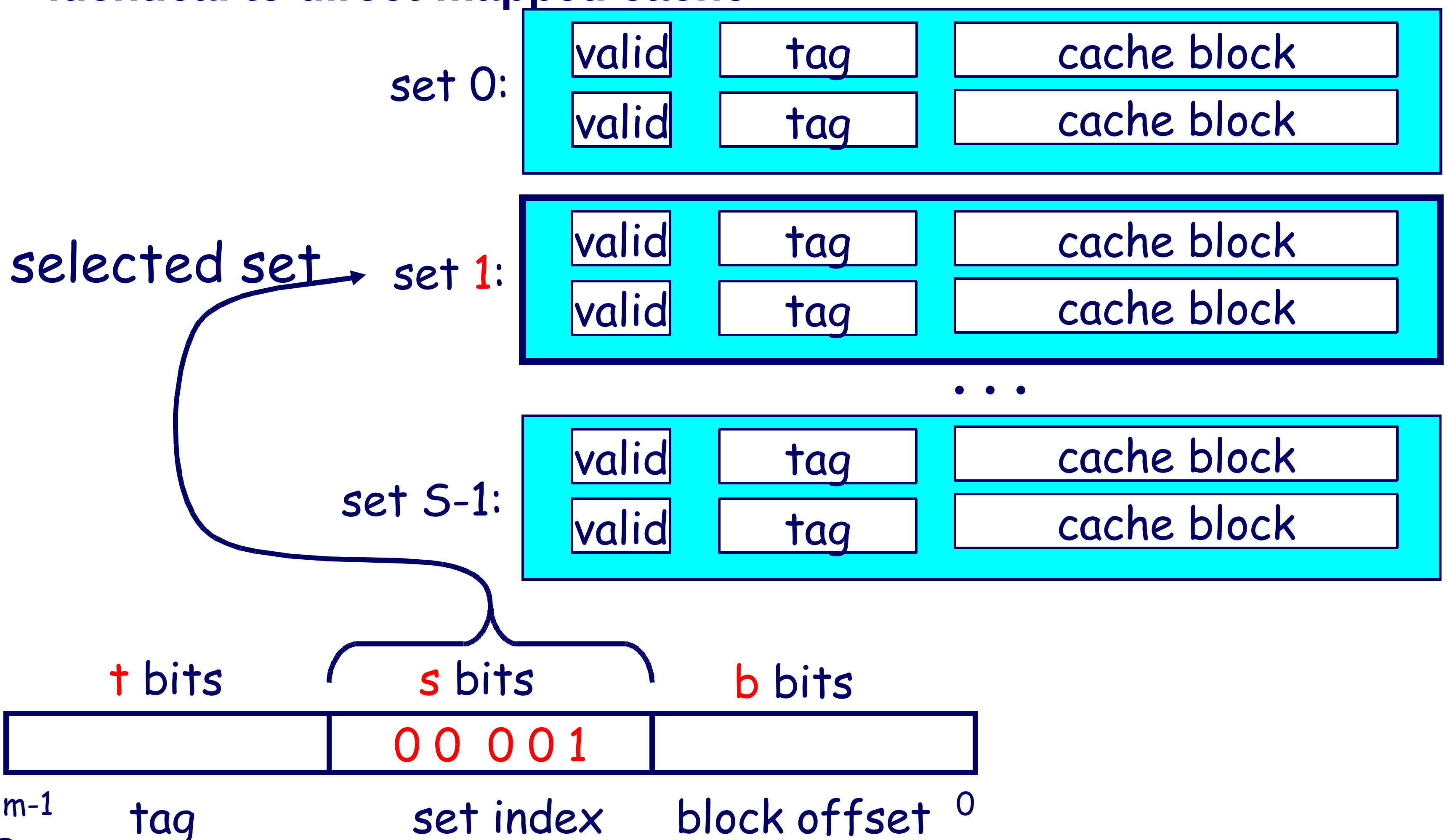
Characterized by more than one line per set



Accessing Set Associative Caches

Set selection

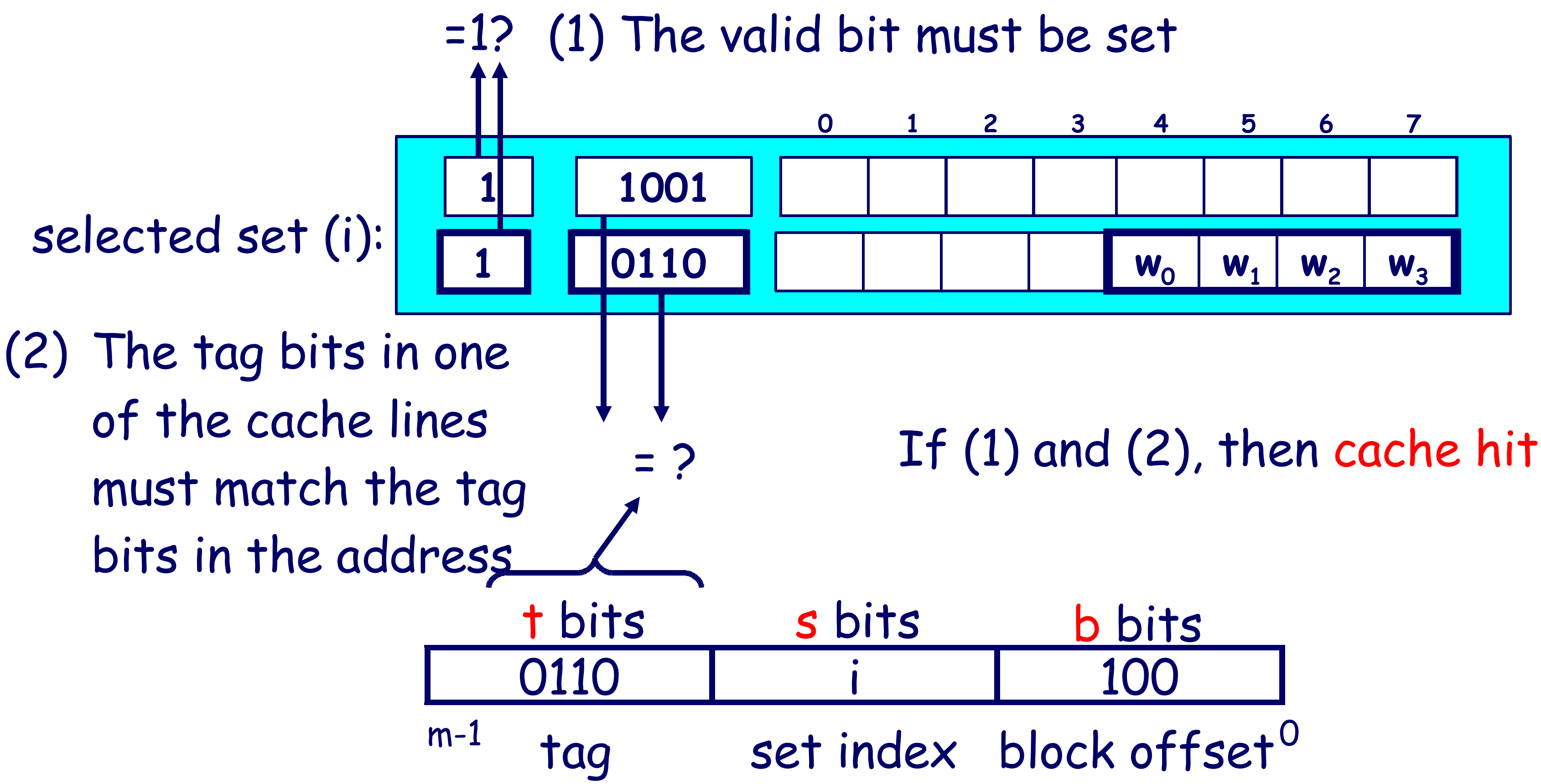
- identical to direct-mapped cache



Accessing Set Associative Caches

Line matching and word selection

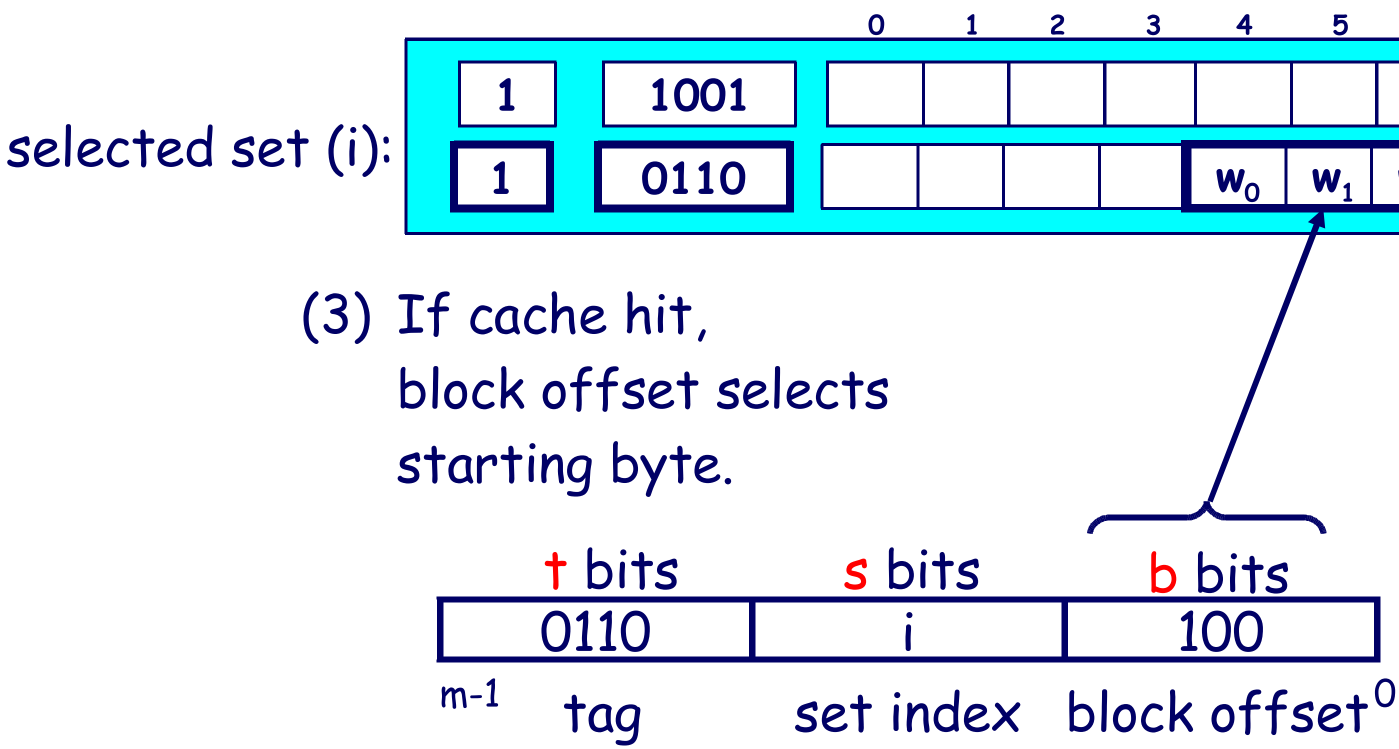
- must compare the tag in each valid line in the selected set.



Accessing Set Associative Caches

Line matching and word selection

- **Word selection is the same as in a direct mapped cache**



2-Way Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

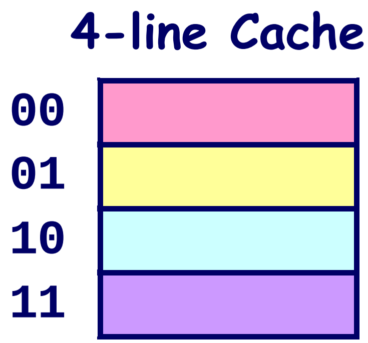
M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 entry/set

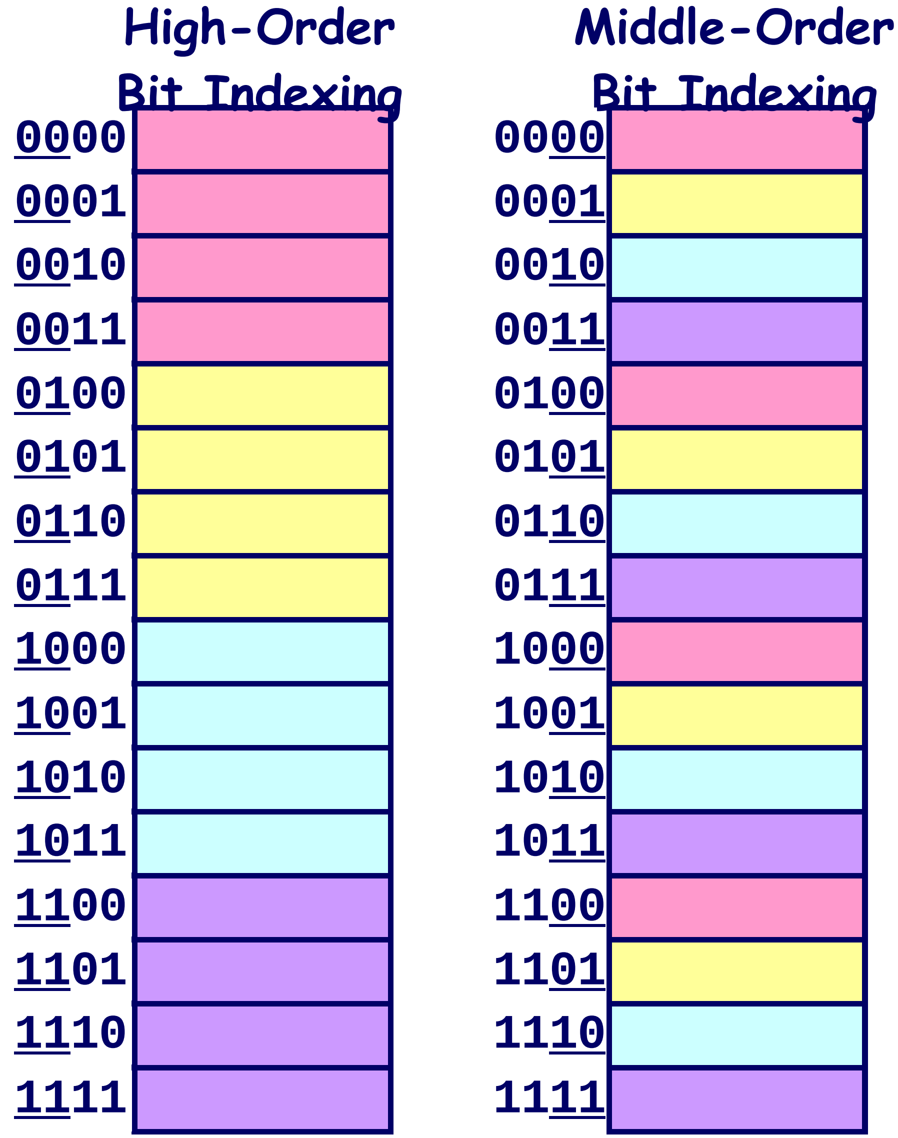
Address trace (reads):

0	$[0000_2]$,	miss
1	$[0001_2]$,	hit
7	$[0111_2]$,	miss
8	$[1000_2]$,	miss
0	$[0000_2]$	hit

v	tag	data
1	00	M[0-1]
1	10	M[8-9]
1	01	M[6-7]
0		

Why Use Middle Bits as Index?





High-Order Bit Indexing

- Adjacent memory lines would map to same cache entry
- Poor use of spatial locality

Middle-Order Bit Indexing

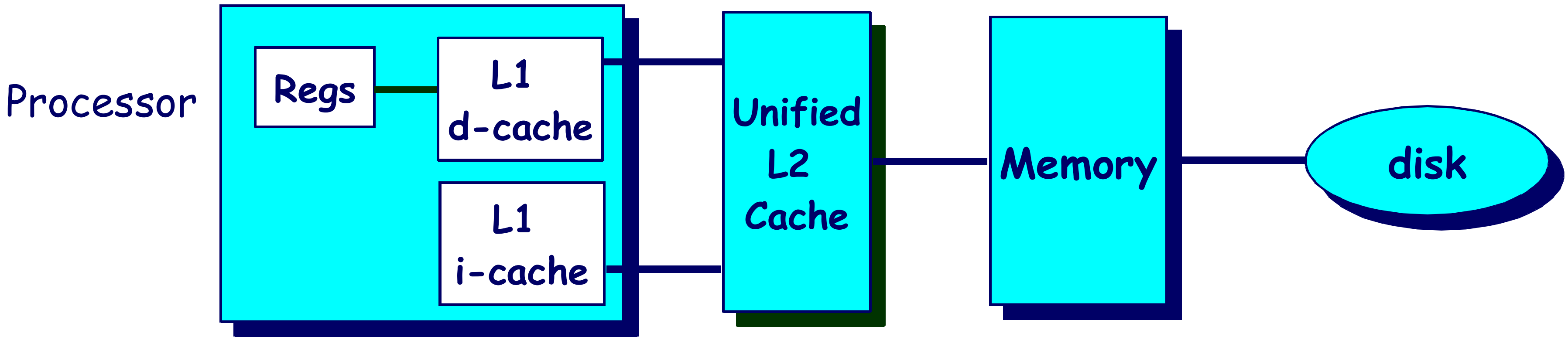
- Consecutive memory lines map to different cache lines
- Can hold S*B*E-byte region of address space in cache at one time

Maintaining an Associative Cache

- **How to decide which cache line to use in a set?**
  - Least Recently Used (LRU), Requires $\lceil lg_2(E!) \rceil$ extra bits
  - Not recently Used (NRU)
  - Random
- **Virtual vs. Physical addresses:**
  - The memory system works with physical addresses, but it takes time to translate a virtual to a physical address. So most L1 caches are virtually indexed, but physically tagged.

Multi-Level Caches

Options: separate data and instruction caches, or a unified cache



size:	200 B	8-64 KB	1-4MB SRAM	128 MB DRAM	30 GB
speed:	3 ns	3 ns	6 ns	60 ns	8 ms
$/Mbyte:			$100/MB	$1.50/MB	$0.05/MB
line size:	8 B	32 B	32 B	8 KB	

larger, slower, cheaper →

What about writes?

Multiple copies of data exist:

- L1
- L2
- Main Memory
- Disk

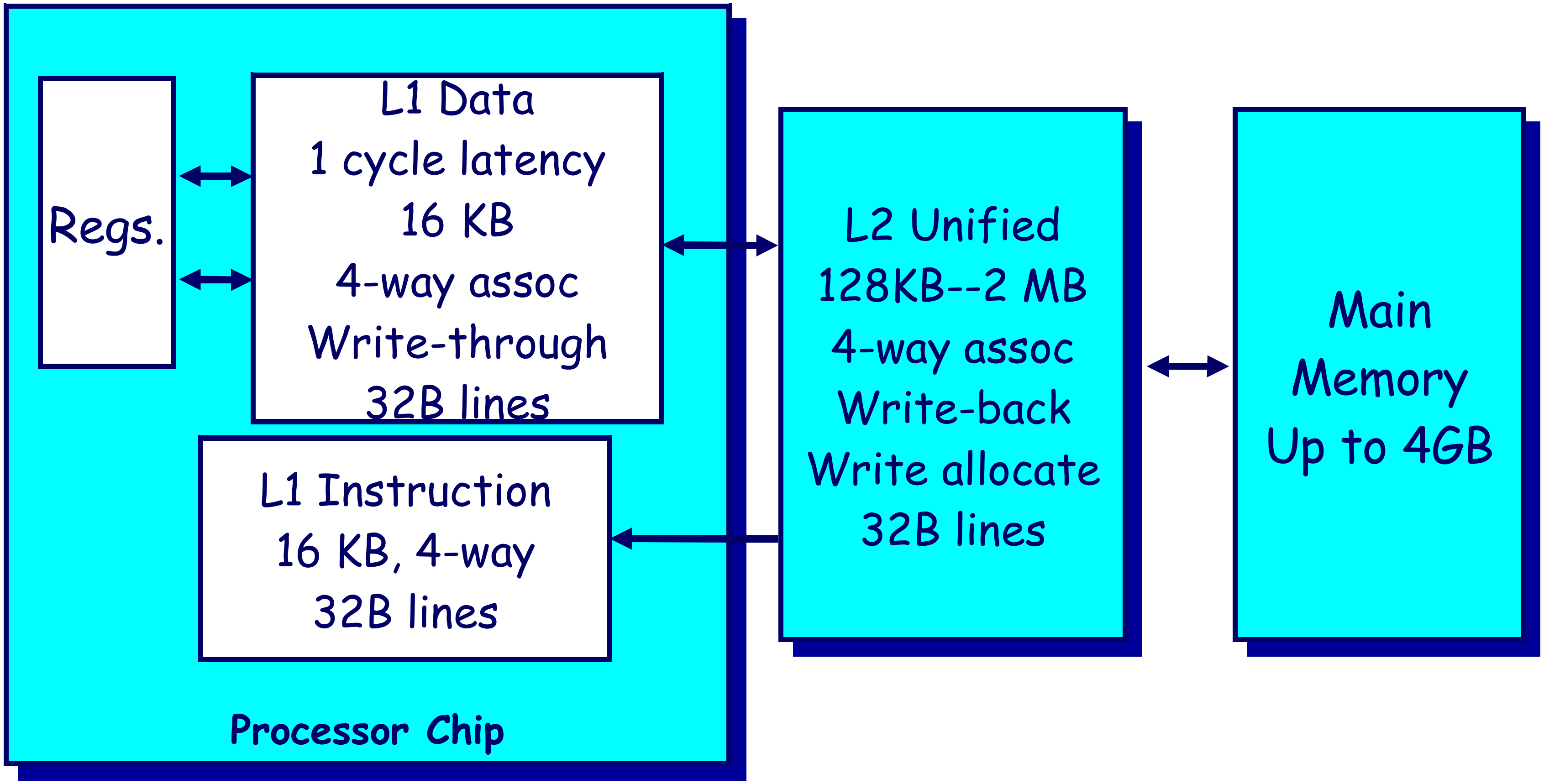
What to do when we write?

- Write-through
- Write-back
  - need a dirty bit
  - What to do on a write-miss?

What to do on a replacement?

- Depends on whether it is write through or write back

Intel Pentium III Cache Hierarchy



Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / references)
- Typical numbers:
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
  - 1-2 clock cycle for L1
  - 5-20 clock cycles for L2

Aside for architects:

-Increasing cache size?

-Increasing block size?

-Increasing associativity?

Miss Penalty

- Additional time required because of a miss
  - Typically 50-200 cycles for main memory (Trend: increasing!)

Writing Cache Friendly Code

- **Repeated references to variables are good (temporal locality)**
- **Stride-1 reference patterns are good (spatial locality)**
- **Examples:**
  - **cold cache, 4-byte words, 4-word cache blocks**

```
int sum_array_rows(int a[M][N])
{
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
     for (j = 0; j < N; j++)
        sum += a[i][j];
  return sum;
}
```

```
int sum_array_cols(int a[M][N])
{
  int i, j, sum = 0;

  for (j = 0; j < N; j++)
     for (i = 0; i < M; i++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 1/4 = 25%

Miss rate = 100%

The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)

Memory mountain

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.

Memory Mountain Test Function

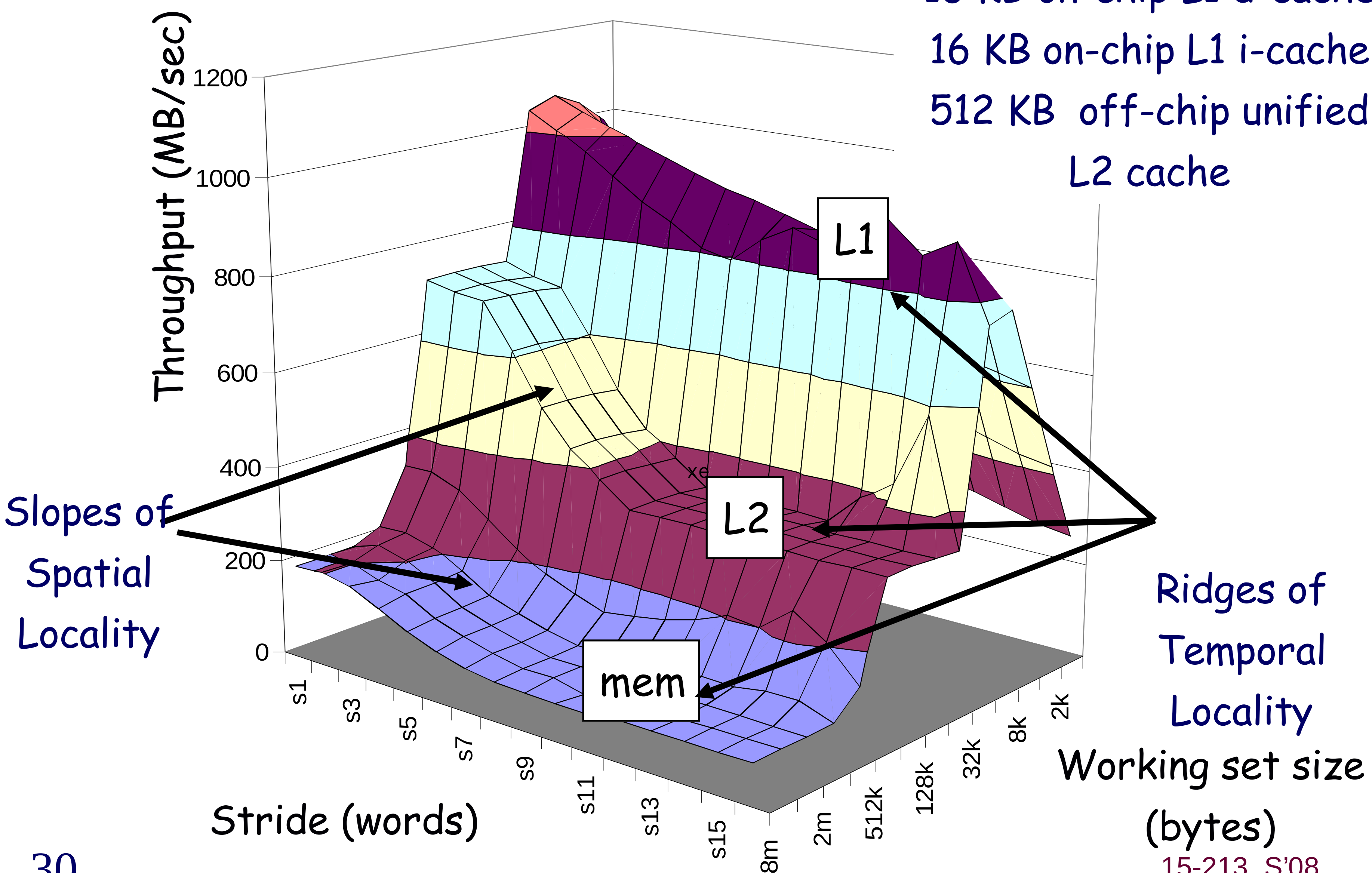
```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride);                     /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0);  /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

Memory Mountain Main Routine

```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10)  /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23)  /* ... up to 8 MB */
#define MAXSTRIDE 16        /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS];         /* The array we'll be traversing */

int main()
{
    int size;        /* Working set size (in bytes) */
    int stride;      /* Stride (in array elements) */
    double Mhz;      /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0);              /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

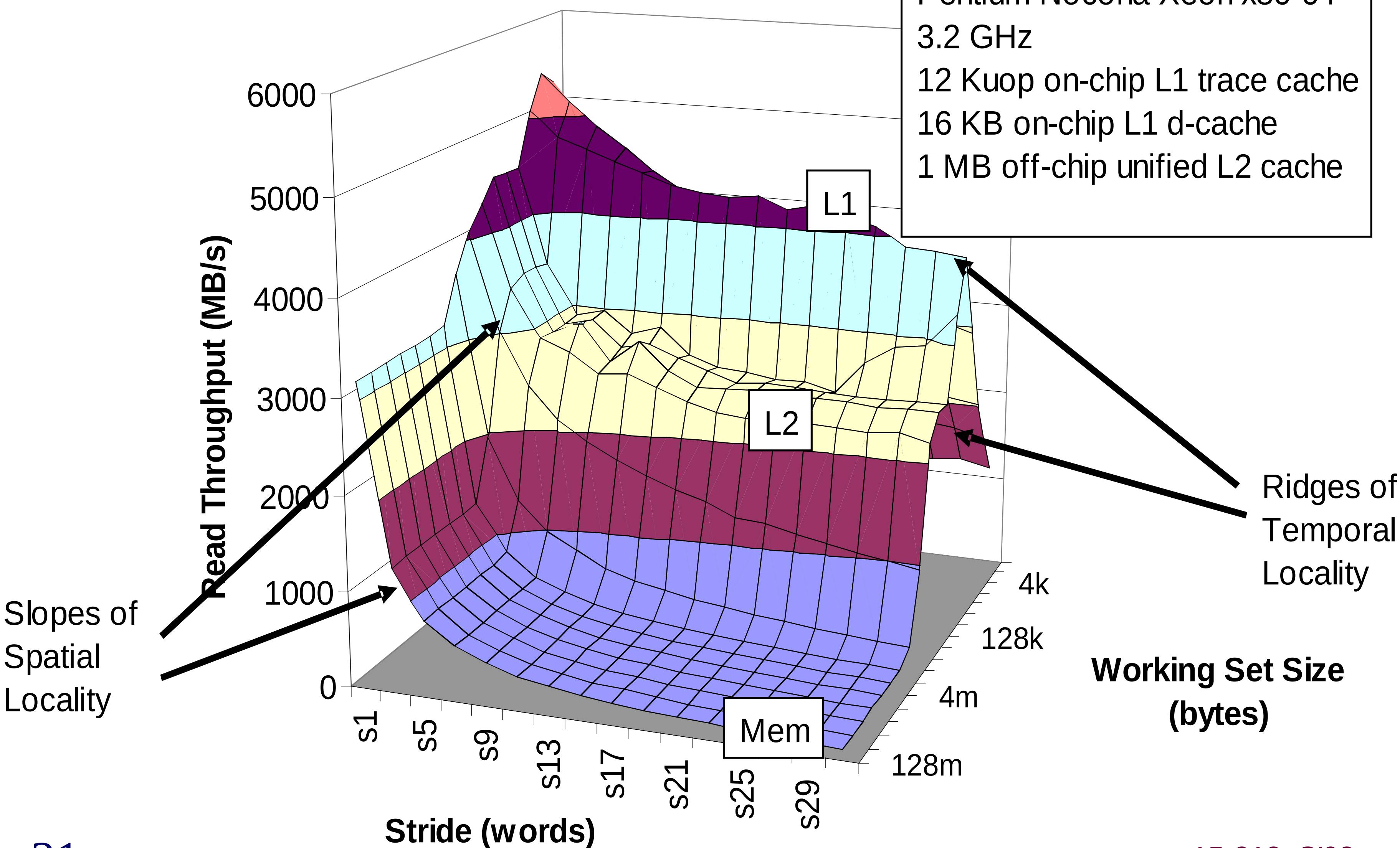
The Memory Mountain

Pentium III
550 MHz
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified
L2 cache

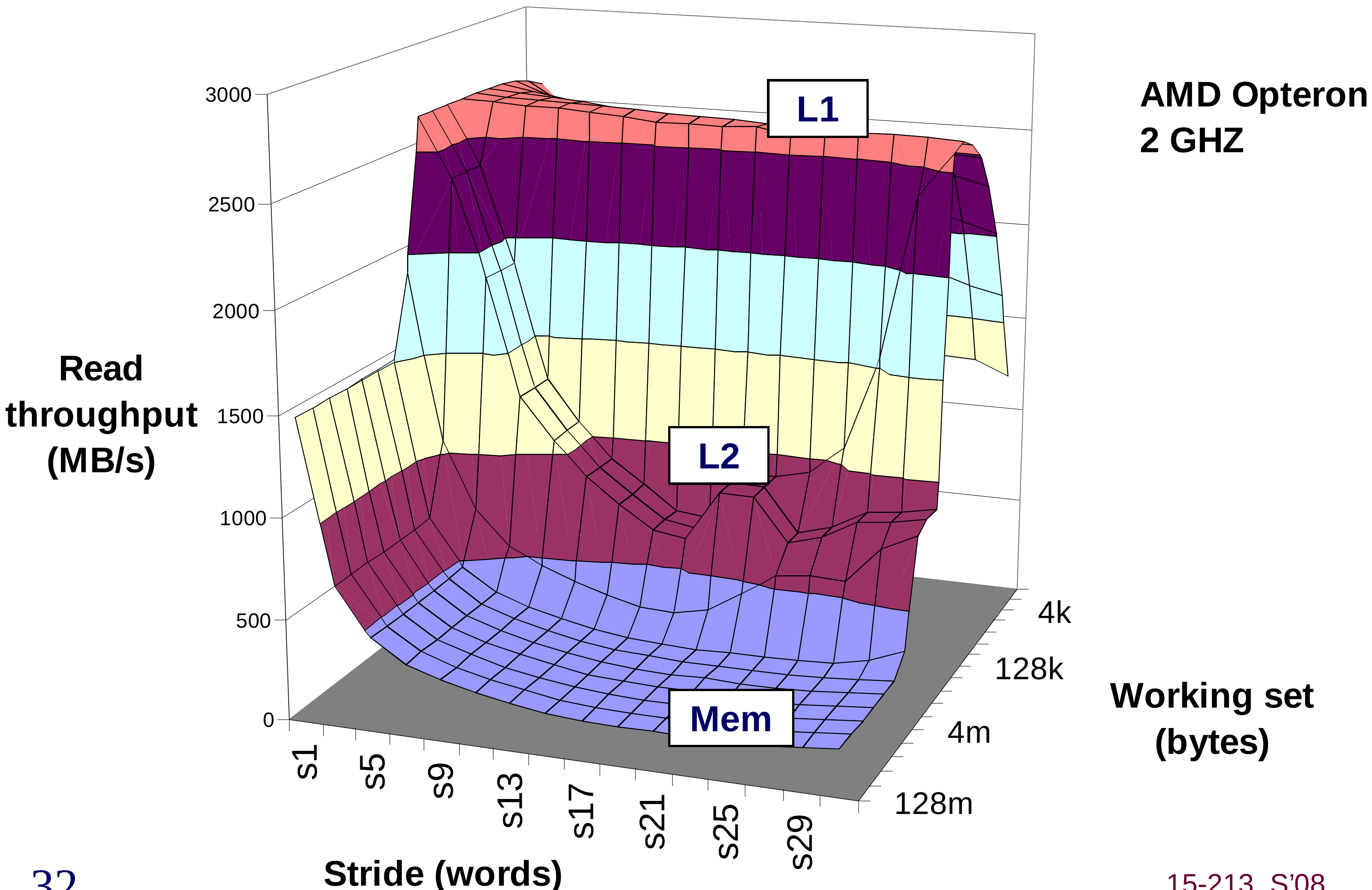


X86-64 Memory Mountain

Pentium Nocona Xeon x86-64
3.2 GHz
12 Kuop on-chip L1 trace cache
16 KB on-chip L1 d-cache
1 MB off-chip unified L2 cache



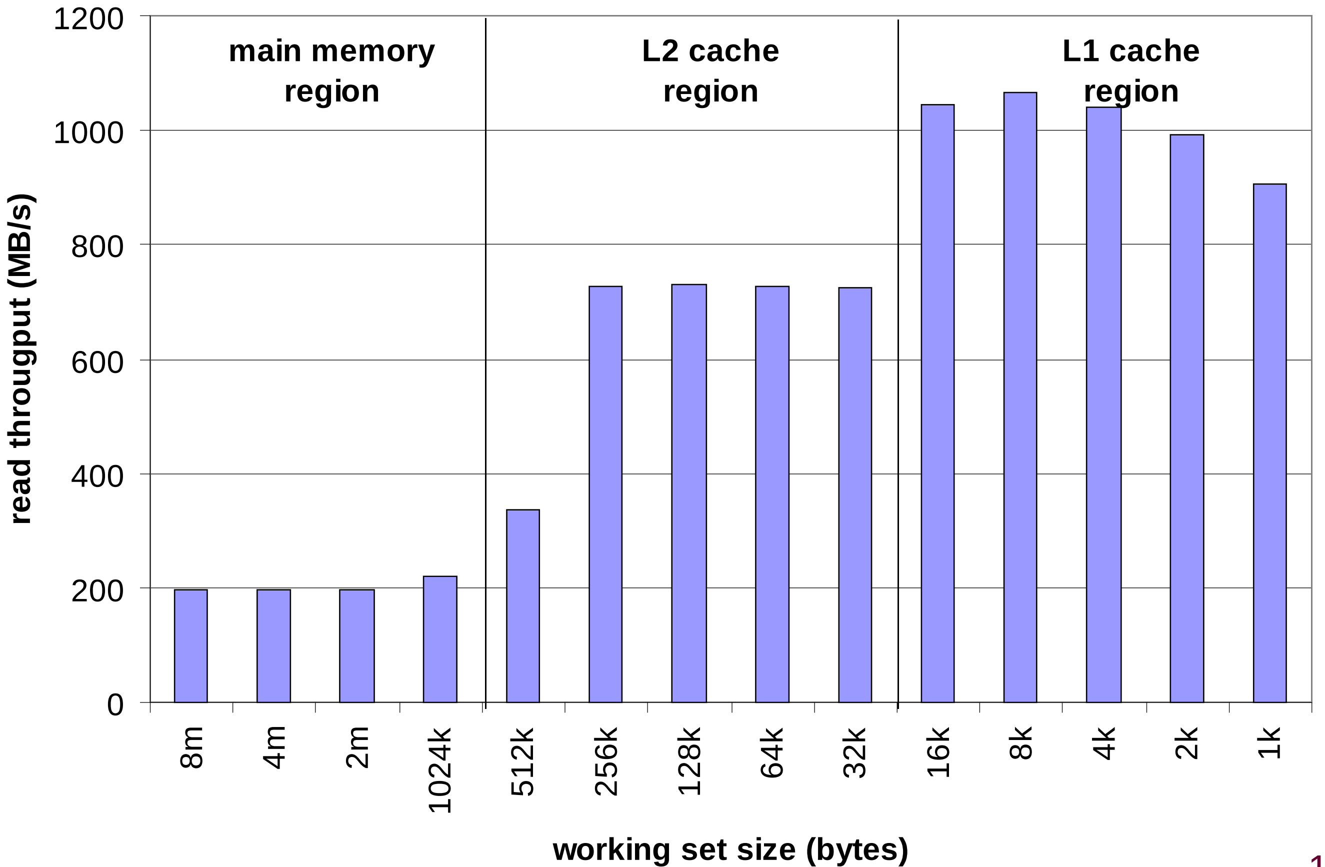
Opteron Memory Mountain



Ridges of Temporal Locality

Slice through the memory mountain with stride=1

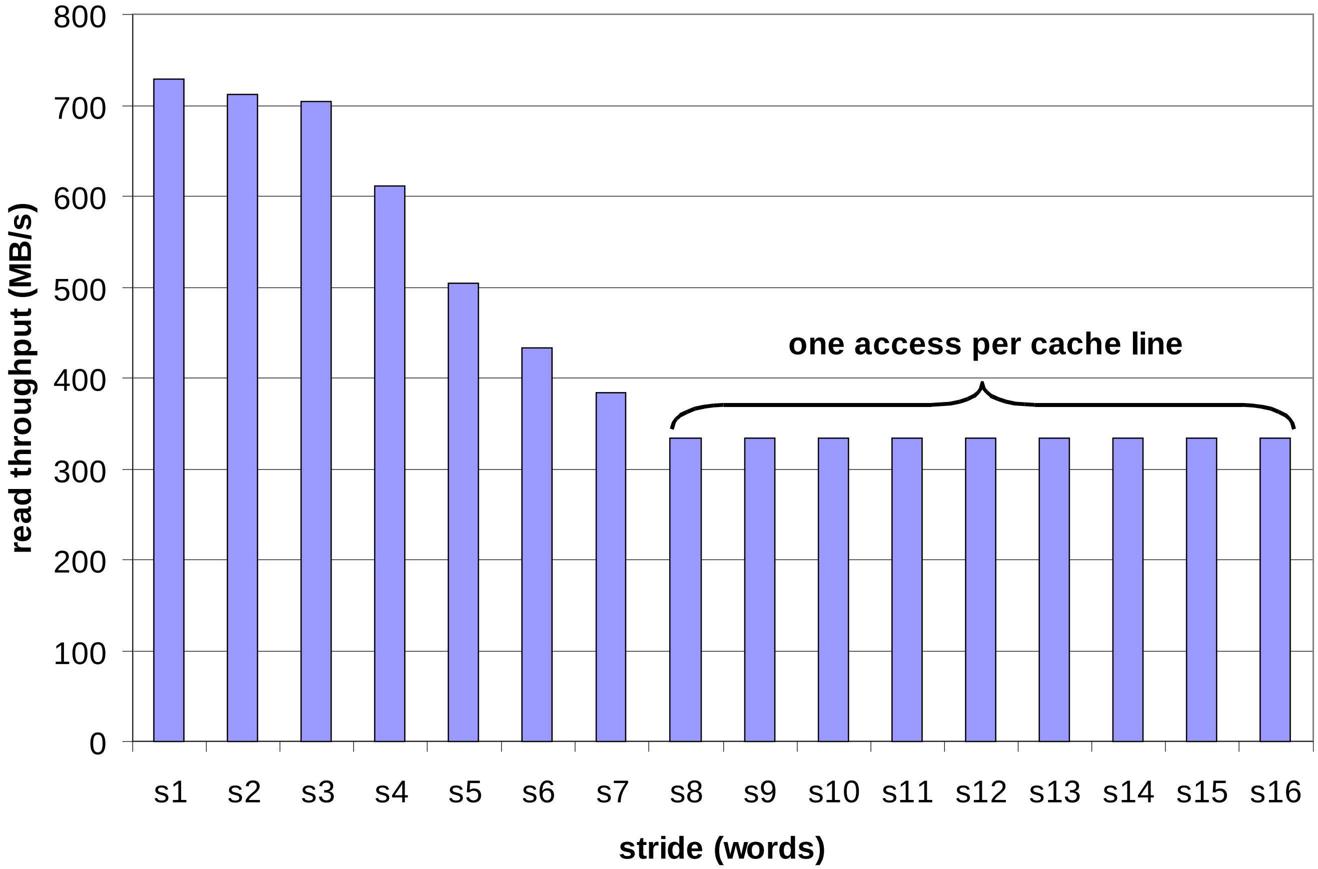
- illuminates read throughputs of different caches and memory



A Slope of Spatial Locality

Slice through memory mountain with size=256KB

- shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider

- Total cache size
  - Exploit temporal locality and keep the working set small (e.g., use blocking)
- Block size
  - Exploit spatial locality

Description:

- Multiply N x N matrices
- O(N^3) total operations
- Accesses
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable **sum** *held in register*

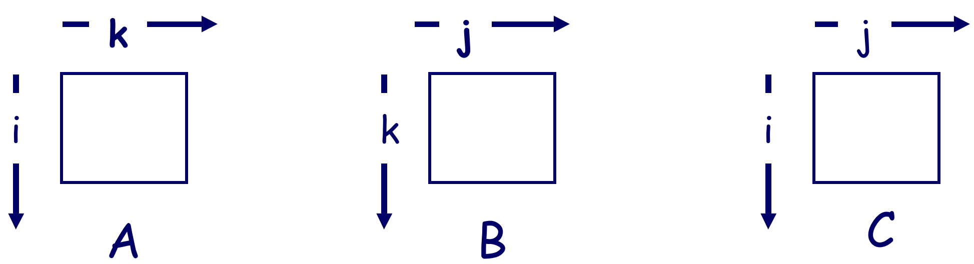
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

C arrays allocated in row-major order

- each row in contiguous memory locations

Stepping through columns in one row:

```
for (i = 0; i < N; i++)
    sum += a[0][i];
```

- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 4 bytes / B

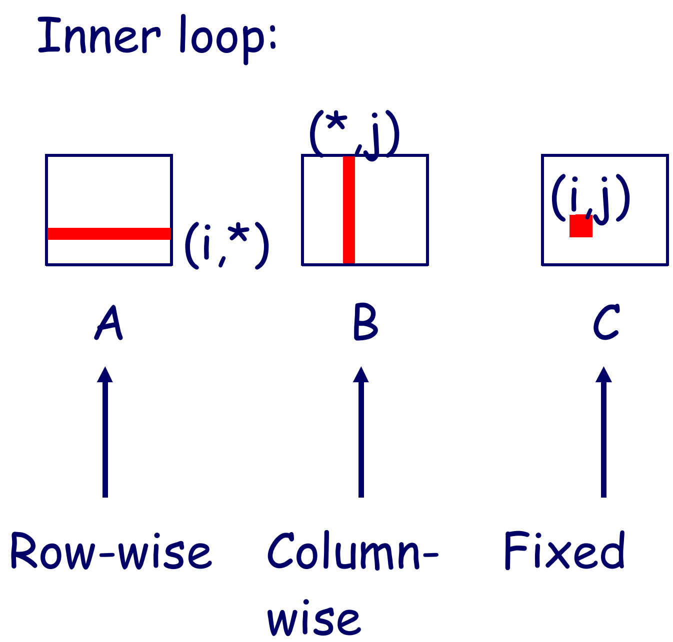
Stepping through rows in one column:

```
for (i = 0; i < n; i++)
    sum += a[i][0];
```

- accesses distant elements
- no spatial locality!
  - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

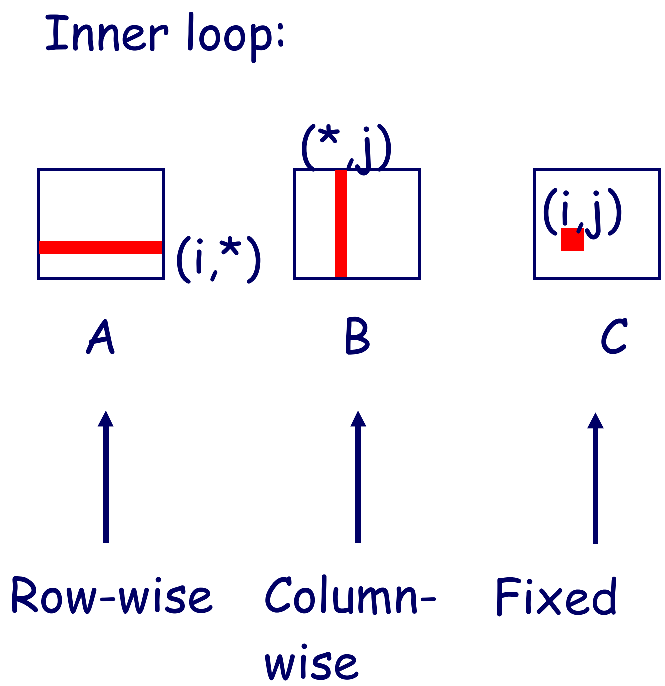


<u>Misses per Inner Loop Iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

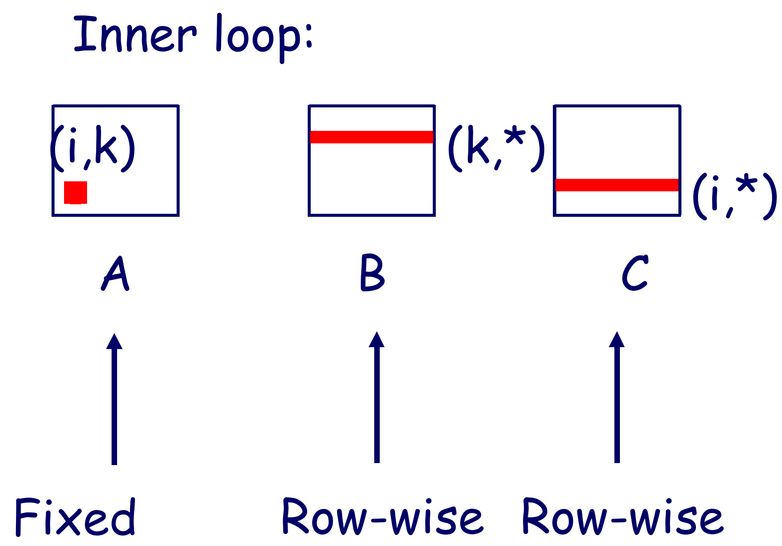


Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

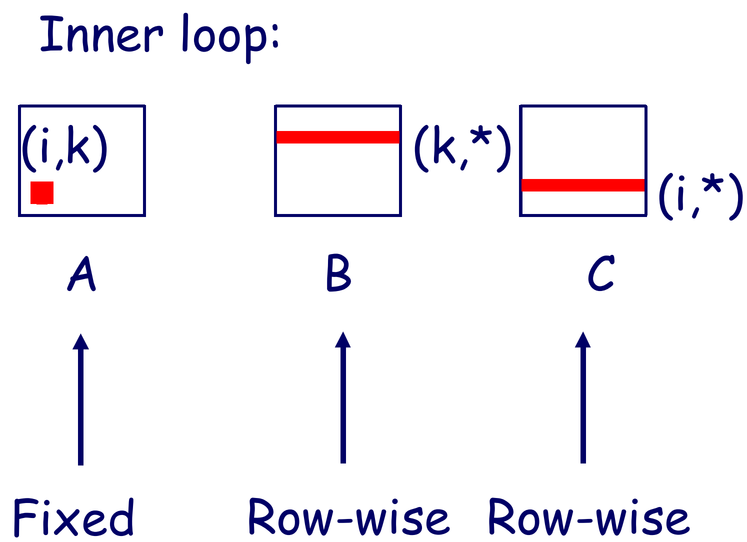


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

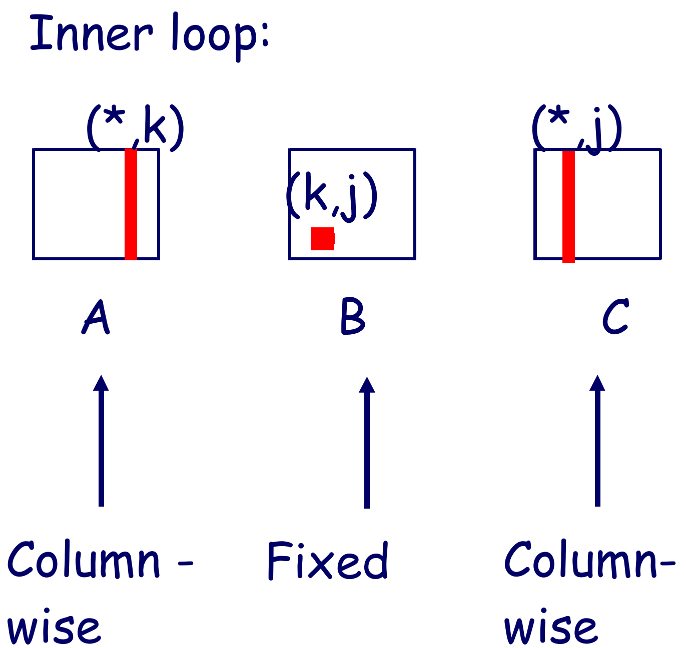


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

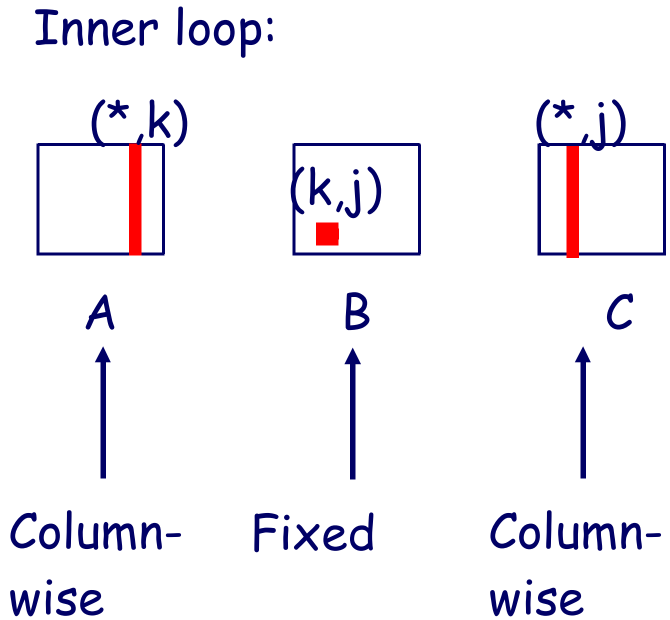


Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
 for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

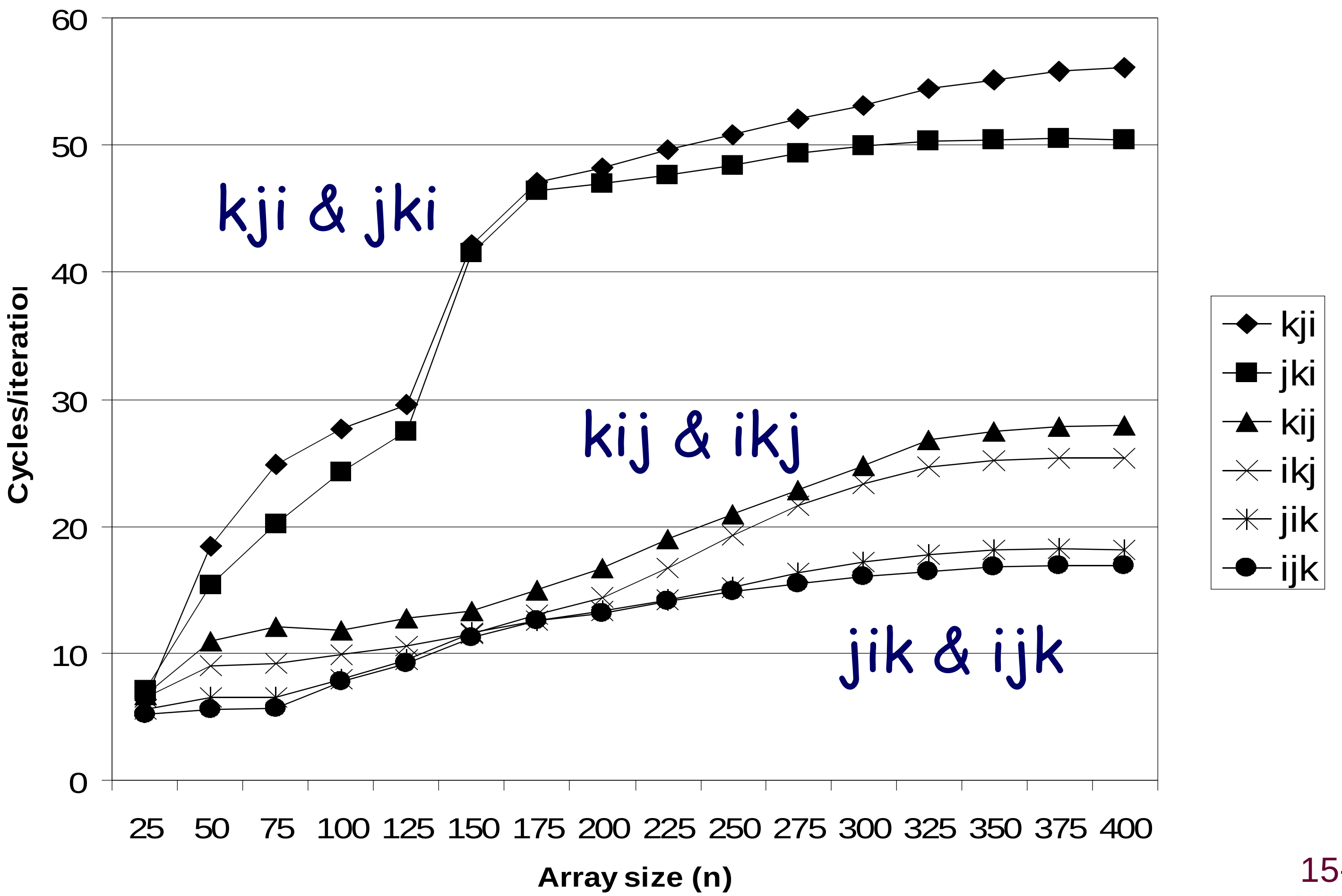
jki (& kji):

- 2 loads, 1 store
- misses/iter = **2.0**

Pentium Matrix Multiply Performance

Miss rates are helpful but not perfect predictors.

- **Code scheduling matters, too.**



Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication

- "block" (in this context) does not mean "cache block".
- Instead, it mean a sub-block within the matrix.
- Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

<u>Key idea:</u> Sub-blocks (i.e., $\mathbf{A}_{xy}$) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Equation Rewriting

The math

$C_{11} = A_{11}B_{11} + A_{12}B_{21}$ $C_{12} = A_{11}B_{12} + A_{12}B_{22}$

$C_{21} = A_{21}B_{11} + A_{22}B_{21}$ $C_{22} = A_{21}B_{12} + A_{22}B_{22}$

Straightforward conversion to imperative code

$C = 0$

$C_{11} \mathrel{+}= A_{11}B_{11}$ $C_{11} \mathrel{+}= A_{12}B_{21}$ $C_{12} \mathrel{+}= A_{11}B_{12}$ $C_{12} \mathrel{+}= A_{12}B_{22}$

$C_{21} \mathrel{+}= A_{21}B_{11}$ $C_{21} \mathrel{+}= A_{22}B_{21}$ $C_{22} \mathrel{+}= A_{21}B_{12}$ $C_{22} \mathrel{+}= A_{22}B_{22}$

Re-order the code to get more cache hits

$C = 0$

$C_{11} \mathrel{+}= A_{11}B_{11}$ $C_{21} \mathrel{+}= A_{21}B_{11}$

$C_{11} \mathrel{+}= A_{12}B_{21}$ $C_{21} \mathrel{+}= A_{22}B_{21}$

$C_{12} \mathrel{+}= \ldots$

We use B_{11} twice (with A_{11}, A_{21}), then B_{21} twice...

We can fit 1 B_{xx} in the cache no matter how big the matrices get

Blocked Matrix Multiply (bijk)

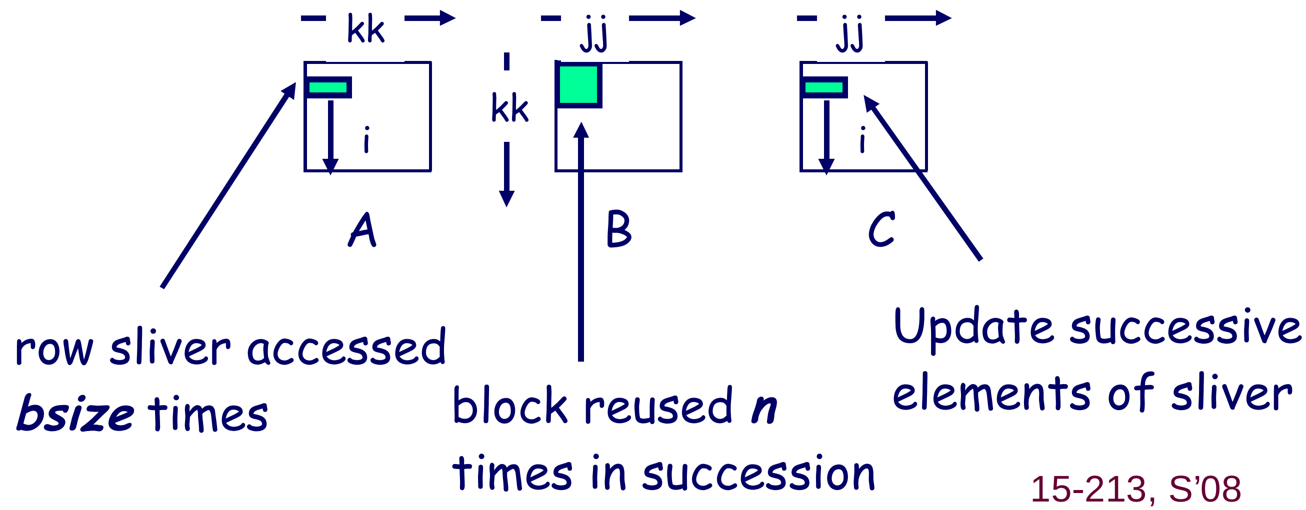
```
for (jj=0; jj<n; jj+=bsize) {

  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;

  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

Blocked Matrix Multiply Analysis

- **Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C***
- **Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B***

```
for (i=0; i<n; i++) {
    for (j=jj; j < min(jj+bsize,n); j++) {
      sum = 0.0
      for (k=kk; k < min(kk+bsize,n); k++) {
        sum += a[i][k] * b[k][j];
      }
      c[i][j] += sum;
    }
```

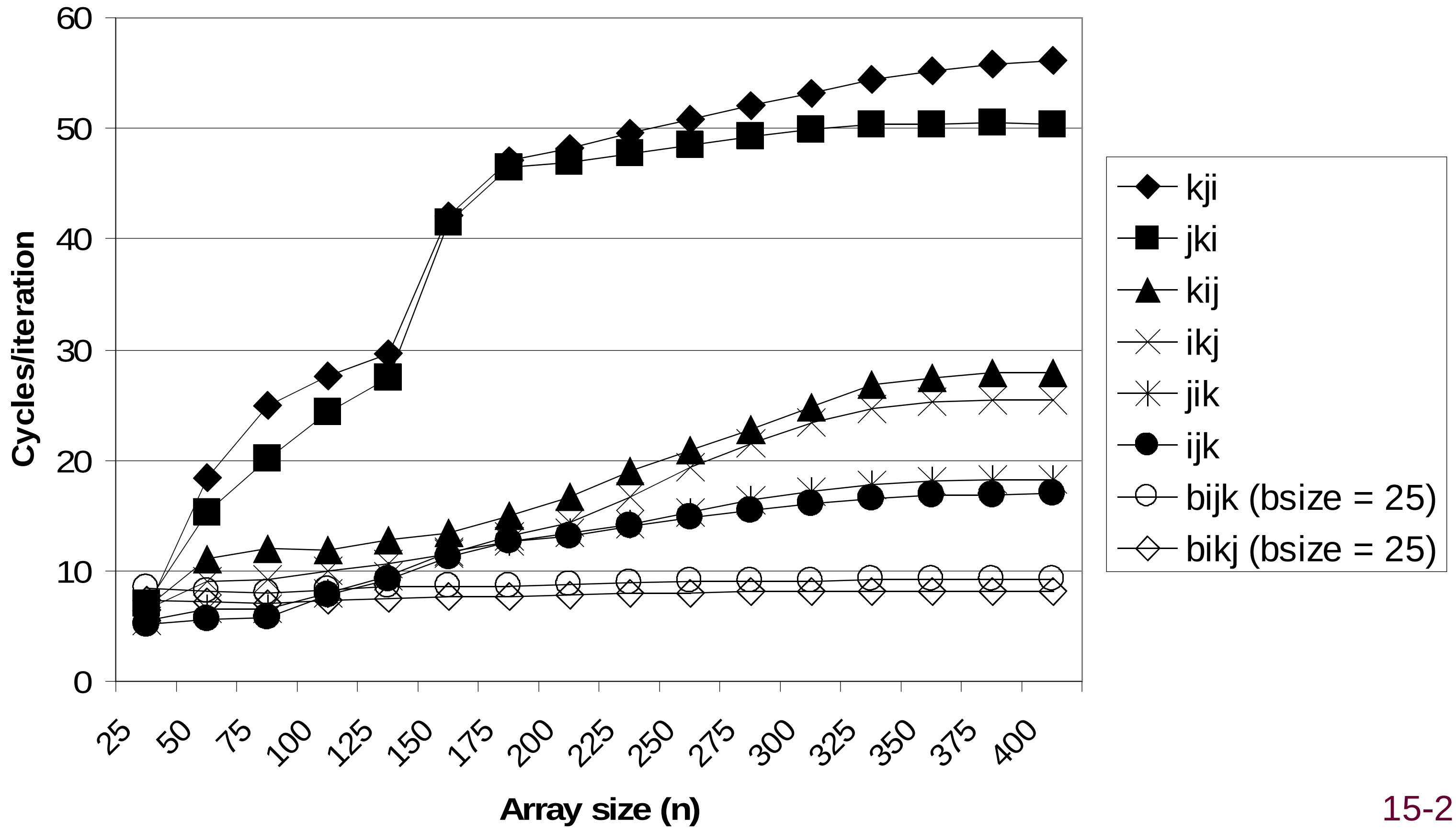
Innermost Loop Pair



Pentium Blocked Matrix Multiply Performance

Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- **relatively insensitive to array size.**



Concluding Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
  - Nested loop structure
  - Blocking is a general technique

All systems favor "cache friendly code"

- Getting absolute optimum performance is very platform specific
  - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)