15-213

"The course that gives CMU its Zip!"

Cache Memories
February 26, 2008

Topics

- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance
- The memory mountain

Synchronization

First exam this evening

- If you have not received mail with a Subject line like "15-213 exam: conflict session C2" then we expect you at the main exam session
- Room split by Andrew username (not first/last/middle name!)
  - a-c Wean 7500
  - d-z McConomy Auditorium in University Center
- Bring with you
  - Your TA's name and/or 15-213 section letter
    - If you want your test to be returned in recitation
  - Book and notes, if you wish
    - Suggested: know your powers of 2
  - No calculators

Synchronization - 2

Computer Club movie night

- "Colossus, The Forbin Project"
- Wednesday evening
- Wean 7500
- 19:00 Computer Club Intro, co-op pizza order
- 19:30 Movie

Determinant

Theorem 5. If A and B are square matrices of the same size, then det(AB) = det(A) * det(B).

The elegant simplicity of this result contrasted with the complex nature of both matrix multiplication and the determinant definition is both refreshing and surprising. We shall omit the proof.

Anton, *Elementary Linear Algebra, 4th ed.*, p. 72.

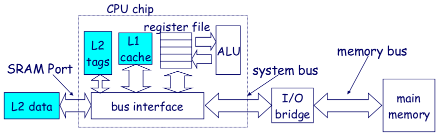
Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware.

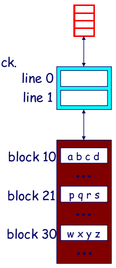
- Hold frequently accessed blocks of main memory

CPU looks first for data in L1, then in L2, then in main memory.

Typical system structure:



Inserting an L1 Cache Between the CPU and Main Memory



The transfer unit between the CPU register file and the cache is a 4-byte block.

The tiny, very fast CPU register file has room for four 4-byte words.

The small fast L1 cache has room for two 4-word blocks.

The transfer unit between the cache and main memory is a 4-word block (16 bytes).

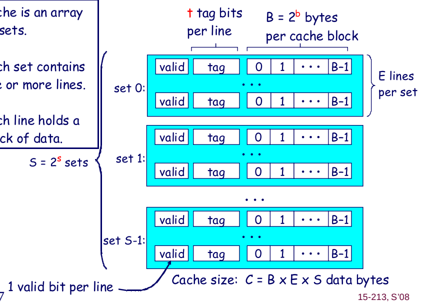
The big slow main memory has room for many 4-word blocks.

General Organization of a Cache

Cache is an array of sets.

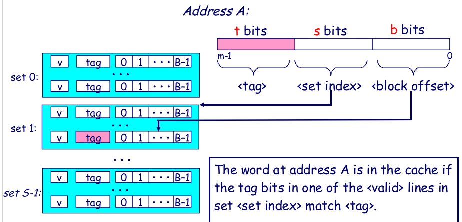
Each set contains one or more lines.

Each line holds a block of data.



Cache size: $C = B \times E \times S$ data bytes

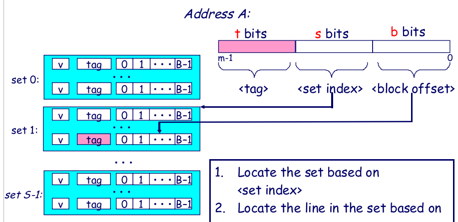
Addressing Caches



The word at address A is in the cache if the tag bits in one of the <valid> lines in set <set index> match <tag>.

The word contents begin at offset <block offset> bytes from the beginning of the block.

Addressing Caches

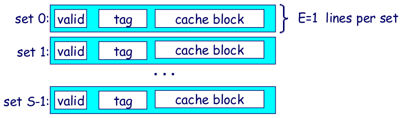


1. Locate the set based on <set index>
2. Locate the line in the set based on <tag>
3. Check that the line is valid
4. Locate the data in the line based on <block offset>

Direct-Mapped Cache

Simplest kind of cache, easy to build
(only 1 tag compare required per access)

Characterized by exactly one line per set.

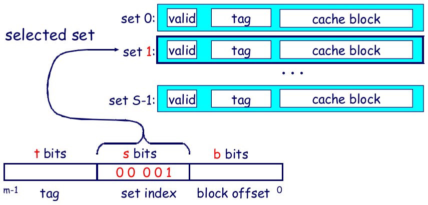


Cache size: $C = B \times S$ data bytes

Accessing Direct-Mapped Caches

Set selection

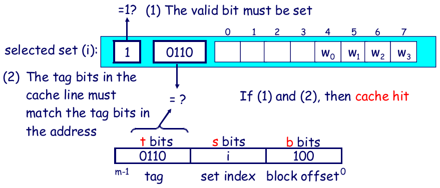
- **Use the set index bits to determine the set of interest.**



Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching: Find a valid line in the selected set with a matching tag**
- **Word selection: Then extract the word**



(1) The valid bit must be set

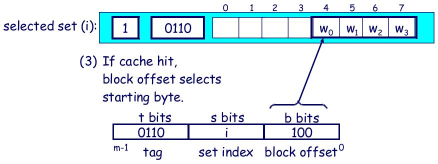
(2) The tag bits in the cache line must match the tag bits in the address

If (1) and (2), then cache hit

Accessing Direct-Mapped Caches

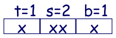
Line matching and word selection

- **Line matching: Find a valid line in the selected set with a matching tag**
- **Word selection: Then extract the word**



(3) If cache hit,
block offset selects
starting byte.

Direct-Mapped Cache Simulation



M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 entry/set

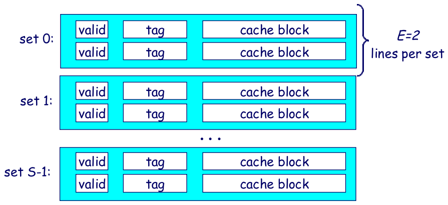
Address trace (reads):

0	$[0000_2]$,	miss
1	$[0001_2]$,	hit
7	$[0111_2]$,	miss
8	$[1000_2]$,	miss
0	$[0000_2]$	miss

v	tag	data
1	0	M[0-1]
1	0	M[6-7]

Set Associative Caches

Characterized by more than one line per set

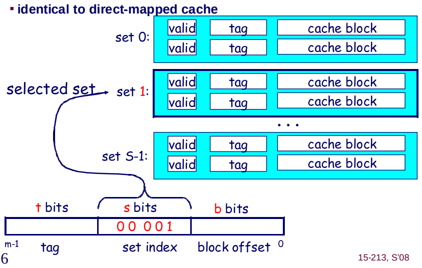


E-way associative cache

Accessing Set Associative Caches

Set selection

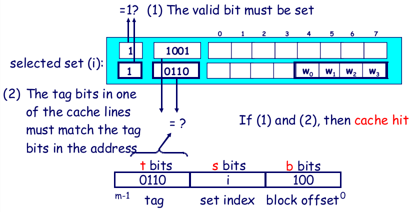
- **identical to direct-mapped cache**



Accessing Set Associative Caches

Line matching and word selection

- **must compare the tag in each valid line in the selected set.**



(1) The valid bit must be set

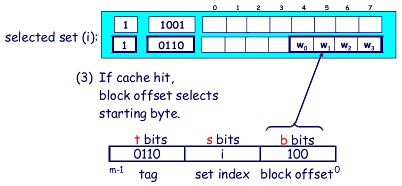
(2) The tag bits in one
of the cache lines
must match the tag
bits in the address

If (1) and (2), then cache hit

Accessing Set Associative Caches

Line matching and word selection

- **Word selection is the same as in a direct mapped cache**



(3) If cache hit,
block offset selects
starting byte.

2-Way Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

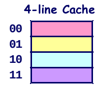
M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 entry/set

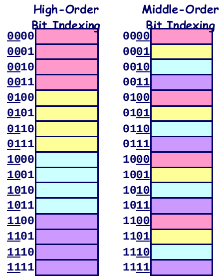
Address trace (reads):

```
0   [0000₂],   miss
1   [0001₂],   hit
7   [0111₂],   miss
8   [1000₂],   miss
0   [0000₂]    hit
```

v	tag	data
1	00	M[0-1]
1	10	M[8-9]
1	01	M[6-7]
0		

Why Use Middle Bits as Index?


4-line Cache
00
01
10
11


High-Order Bit Indexing
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Middle-Order Bit Indexing
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

High-Order Bit Indexing

- Adjacent memory lines would map to same cache entry
- Poor use of spatial locality

Middle-Order Bit Indexing

- Consecutive memory lines map to different cache lines
- Can hold S*B*E-byte region of address space in cache at one time

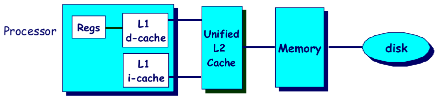
---

Maintaining an Associative Cache

- How to decide which cache line to use in a set?
  - Least Recently Used (LRU), Requires $\lceil \lg_2(E!) \rceil$ extra bits
  - Not recently Used (NRU)
  - Random
- Virtual vs. Physical addresses:
  - The memory system works with physical addresses, but it takes time to translate a virtual to a physical address. So most L1 caches are virtually indexed, but physically tagged.

Multi-Level Caches

Options: separate data and instruction caches, or a unified cache


Processor
Regs
L1 d-cache
L1 i-cache
Unified L2 Cache
Memory
disk

size:	200 B	8-64 KB	1-4MB SRAM	128 MB DRAM	30 GB
speed:	3 ns	3 ns	6 ns	60 ns	8 ms
$/Mbyte:			$100/MB	$1.50/MB	$0.05/MB
line size:	8 B	32 B	32 B	8 KB	

larger, slower, cheaper →

What about writes?

Multiple copies of data exist:

- L1
- L2
- Main Memory
- Disk

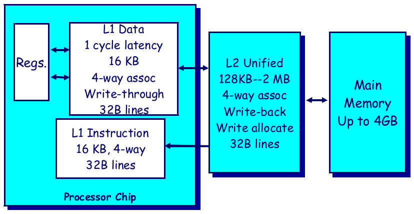
What to do when we write?

- Write-through
- Write-back
  - need a dirty bit
  - What to do on a write-miss?

What to do on a replacement?

- Depends on whether it is write through or write back

Intel Pentium III Cache Hierarchy


Regs.

L1 Data
1 cycle latency
16 KB
4-way assoc
Write-through
32B lines

L1 Instruction
16 KB, 4-way
32B lines

Processor Chip

L2 Unified
128KB--2 MB
4-way assoc
Write-back
Write allocate
32B lines

Main
Memory
Up to 4GB

Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / references)
- Typical numbers:
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
  - 1-2 clock cycle for L1
  - 5-20 clock cycles for L2

Miss Penalty

- Additional time required because of a miss
  - Typically 50-200 cycles for main memory (Trend: increasing!)

Aside for architects:
-Increasing cache size?
-Increasing block size?
-Increasing associativity?

Writing Cache Friendly Code

- Repeated references to variables are good (temporal locality)
- Stride-1 reference patterns are good (spatial locality)
- Examples:
  - cold cache, 4-byte words, 4-word cache blocks

```
int sum_array_rows(int a[M][N])
{
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
     for (j = 0; j < N; j++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 1/4 = 25%

```
int sum_array_cols(int a[M][N])
{
  int i, j, sum = 0;

  for (j = 0; j < N; j++)
     for (i = 0; i < M; i++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 100%

The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)

Memory mountain

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.

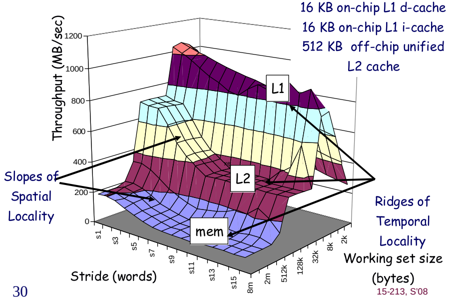
Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride);                     /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0);  /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

Memory Mountain Main Routine

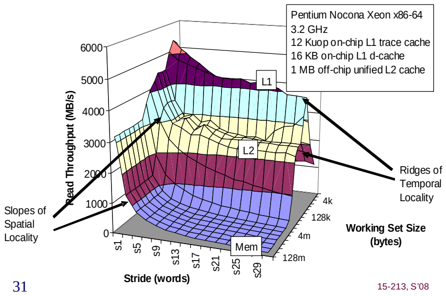
```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10)  /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23)  /* ... up to 8 MB */
#define MAXSTRIDE 16        /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS];         /* The array we'll be traversing */

int main()
{
    int size;        /* Working set size (in bytes) */
    int stride;      /* Stride (in array elements) */
    double Mhz;      /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0);              /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

The Memory Mountain



Pentium III
550 MHz
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified
L2 cache

X86-64 Memory Mountain

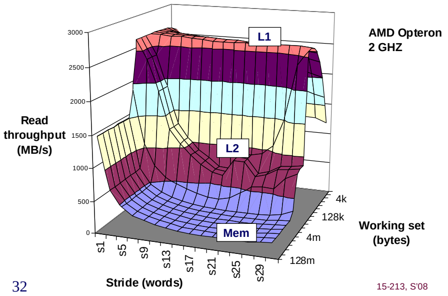


Pentium Nocona Xeon x86-64
3.2 GHz
12 Kuop on-chip L1 trace cache
16 KB on-chip L1 d-cache
1 MB off-chip unified L2 cache

Slopes of Spatial Locality

Ridges of Temporal Locality

Opteron Memory Mountain

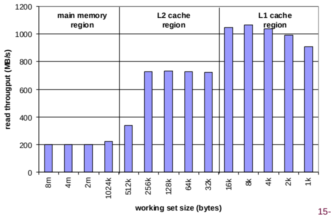


AMD Opteron
2 GHZ

Ridges of Temporal Locality

Slice through the memory mountain with stride=1

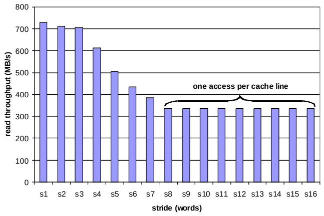
- illuminates read throughputs of different caches and memory



A Slope of Spatial Locality

Slice through memory mountain with size=256KB

- shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider

- Total cache size
  - Exploit temporal locality and keep the working set small (e.g., use blocking)
- Block size
  - Exploit spatial locality

Description:

- Multiply N x N matrices
- O(N^3) total operations
- Accesses
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable **sum** *held in register*

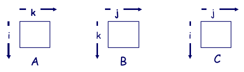
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

C arrays allocated in row-major order

- each row in contiguous memory locations

Stepping through columns in one row:

```
for (i = 0; i < N; i++)
    sum += a[0][i];
```

- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 4 bytes / B

Stepping through rows in one column:

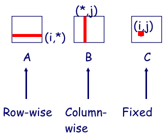
```
for (i = 0; i < n; i++)
    sum += a[i][0];
```

- accesses distant elements
- no spatial locality!
  - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



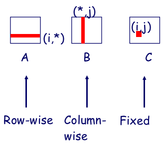
Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:



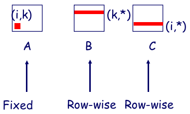
Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



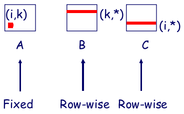
Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



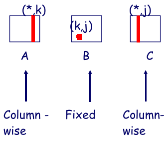
Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:

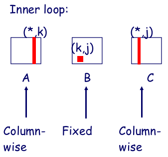


Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
 for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

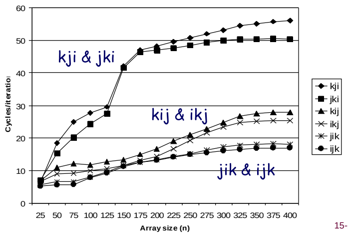
kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

Pentium Matrix Multiply Performance

Miss rates are helpful but not perfect predictors.
- Code scheduling matters, too.



Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication
- "block" (in this context) does not mean "cache block".
- Instead, it mean a sub-block within the matrix.
- Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars.

$C_{11} = A_{11}B_{11} + A_{12}B_{21}$ $C_{12} = A_{11}B_{12} + A_{12}B_{22}$

$C_{21} = A_{21}B_{11} + A_{22}B_{21}$ $C_{22} = A_{21}B_{12} + A_{22}B_{22}$

Equation Rewriting

The math

$C_{11} = A_{11}B_{11} + A_{12}B_{21}$ $C_{12} = A_{11}B_{12} + A_{12}B_{22}$

$C_{21} = A_{21}B_{11} + A_{22}B_{21}$ $C_{22} = A_{21}B_{12} + A_{22}B_{22}$

Straightforward conversion to imperative code

$C = 0$

C_{11} += $A_{11}B_{11}$ C_{11} += $A_{12}B_{21}$ C_{12} += $A_{11}B_{12}$ C_{12} += $A_{12}B_{22}$

C_{21} += $A_{21}B_{11}$ C_{21} += $A_{22}B_{21}$ C_{22} += $A_{21}B_{12}$ C_{22} += $A_{22}B_{22}$

Re-order the code to get more cache hits

$C = 0$

C_{11} += $A_{11}B_{11}$ C_{21} += $A_{21}B_{11}$

C_{11} += $A_{12}B_{21}$ C_{21} += $A_{22}B_{21}$

C_{12} += ...

We use B_{11} twice (with A_{11}, A_{21}), then B_{21} twice...

We can fit 1 B_{xx} in the cache no matter how big the matrices get

---

Blocked Matrix Multiply (bijk)

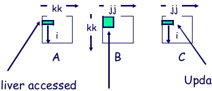
```
for (jj=0; jj<n; jj+=bsize) {

  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;

  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

Blocked Matrix Multiply Analysis

- Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C*
- Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B*

```
for (i=0; i<n; i++) {
    for (j=jj; j < min(jj+bsize,n); j++) {
      sum = 0.0
      for (k=kk; k < min(kk+bsize,n); k++) {
        sum += a[i][k] * b[k][j];
      }
      c[i][j] += sum;
    }
```

Innermost Loop Pair



row sliver accessed *bsize* times

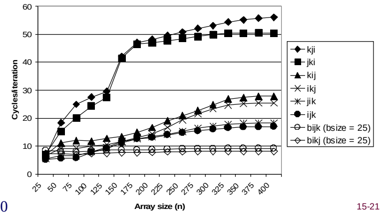
block reused ***n*** times in succession

Update successive elements of sliver

Pentium Blocked Matrix Multiply Performance

Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- relatively insensitive to array size.



Concluding Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
  - Nested loop structure
  - Blocking is a general technique

All systems favor "cache friendly code"

- Getting absolute optimum performance is very platform specific
  - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)