## 15-213
## *“The Class That Gives CMU Its Zip!”*

# Bits and Bytes
# Jan. 20, 2000

## Topics

- **Why bits?**
- **Representing information as bits**
  - Binary/Hexadecimal
  - Byte representations
    - » numbers
    - » characters and strings
    - » Instructions
- **Bit-level manipulations**
  - Boolean algebra
  - Expressing in C

# Why Don’t Computers Use Base 10?

## Base 10 Number Representation

- **That’s why fingers are known as “digits”**
- **Natural representation for financial transactions**
  - Floating point number cannot exactly represent $1.20
- **Even carries through in scientific notation**
  - $1.5213 \times 10^4$

## Implementing Electronically

- **Hard to store**
  - ENIAC (First electronic computer) used 10 vacuum tubes / digit
- **Hard to transmit**
  - Need high precision to encode 10 signal levels on single wire
- **Messy to implement digital logic functions**
  - Addition, multiplication, etc.

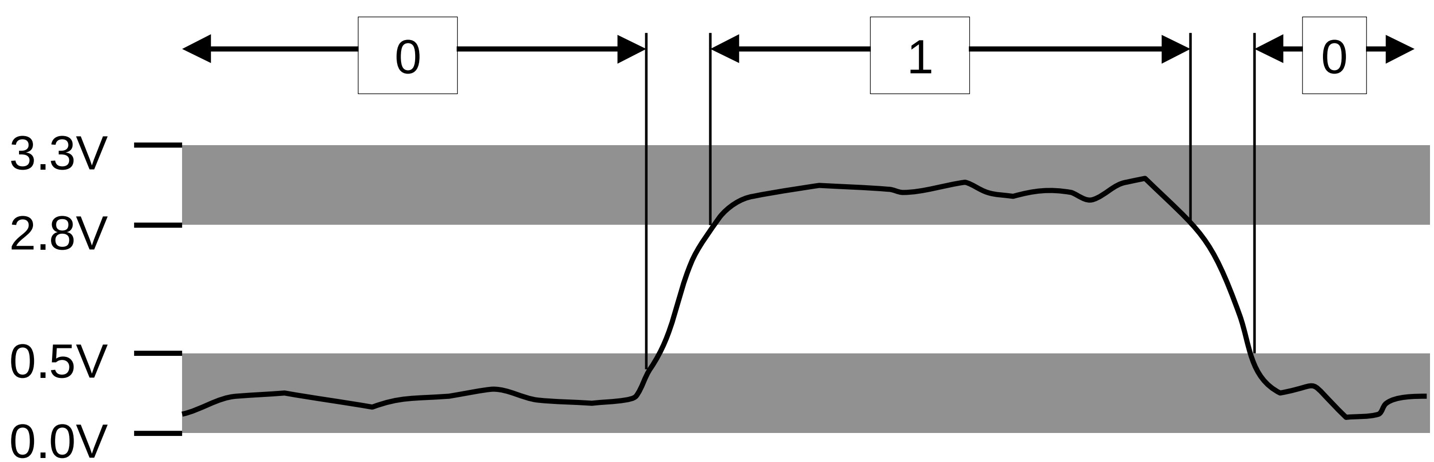
# Binary Representations

## Base 2 Number Representation

- Represent $15213_{10}$ as $11101101101101_2$
- Represent $1.20_{10}$ as $1.0011001100110011[0011]\ldots_2$
- Represent $1.5213 \times 10^4$ as $1.1101101101101_2 \times 2^{13}$

## Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires




- Straightforward implementation of arithmetic functions

# Byte-Oriented Memory Organization

## Programs Refer to Virtual Addresses

- **Conceptually very large array of bytes**
- **Actually implemented with hierarchy of different memory types**
  - SRAM, DRAM, disk
  - Only allocate for regions actually used by program
- **In Unix and Windows NT, address space private to particular “process”**
  - Program being executed
  - Program can clobber its own data, but not that of others

## Compiler + Run-Time System Control Allocation

- **Where different program objects should be stored**
- **Multiple mechanisms: static, stack, and heap**
- **In any case, all allocation within single virtual address space**

# Encoding Byte Values

## Byte = 8 bits

- **Binary** **$00000000_2$** **to** **$11111111_2$**
- **Decimal:** **$0_{10}$** **to** **$255_{10}$**
- **Hexadecimal** **$00_{16}$** **to** **$FF_{16}$**
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

| Hex | Decimal | Binary |
|---|---|---|
| **0** | 0 | 0000 |
| **1** | 1 | 0001 |
| **2** | 2 | 0010 |
| **3** | 3 | 0011 |
| **4** | 4 | 0100 |
| **5** | 5 | 0101 |
| **6** | 6 | 0110 |
| **7** | 7 | 0111 |
| **8** | 8 | 1000 |
| **9** | 9 | 1001 |
| **A** | 10 | 1010 |
| **B** | 11 | 1011 |
| **C** | 12 | 1100 |
| **D** | 13 | 1101 |
| **E** | 14 | 1110 |
| **F** | 15 | 1111 |

# Machine Words

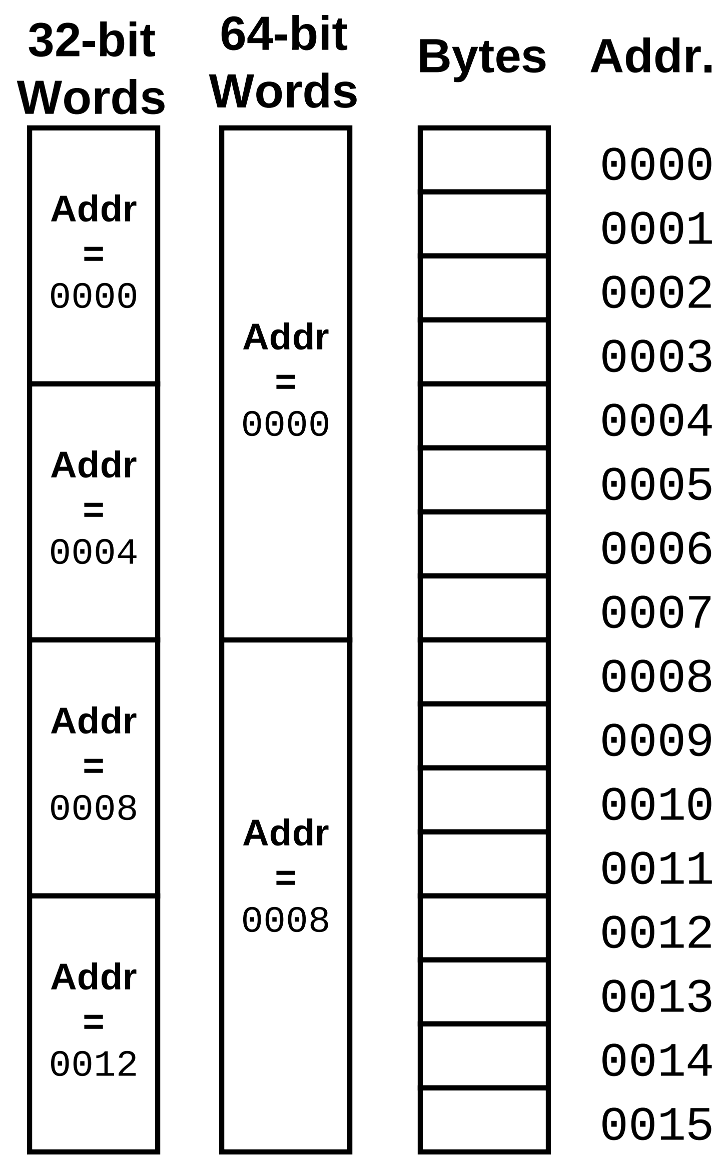
## Machine Has “Word Size”

- **Nominal size of integer-valued data**
  - Including addresses
- **Most current machines are 32 bits (4 bytes)**
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- **High-end systems are 64 bits (8 bytes)**
  - Potentially address $\approx 1.8 \times 10^{19}$ bytes
- **Machines support multiple data formats**
  - Fractions or multiples of word size
  - Always integral number of bytes

# Word-Oriented Memory Organization

## Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



# Data Representations

## Sizes of C Objects (in Bytes)

| C Data Type | Compaq Alpha | Typical 32-bit |
|---|---|---|
| `int` | 4 | 4 |
| `long int` | 8 | 4 |
| `char` | 1 | 1 |
| `short` | 2 | 2 |
| `float` | 4 | 4 |
| `double` | 8 | 8 |
| `char *` | 8 | 4 |

» Or any other pointer

# Byte Ordering

## Issue

- **How should bytes within multi-byte word be ordered in memory**

## Conventions

- **Alphas, PC's are "Little Endian" machines**
  - Least significant byte has lowest address
- **Sun's, Mac's are "Big Endian" machines**
  - Least significant byte has highest address

## Example

- **Variable x has 4-byte representation `0x1234567`**
- **Address given by &x is `0x100`**

**Big Endian**

| | | 0x100 | 0x101 | 0x102 | 0x103 | | |
|---|---|---|---|---|---|---|---|
| | | 01 | 23 | 45 | 67 | | |

**Little Endian**

| | | 0x100 | 0x101 | 0x102 | 0x103 | | |
|---|---|---|---|---|---|---|---|
| | | 67 | 45 | 23 | 01 | | |

# Examining Data Representations

## Code to Print Byte Representation of Data

- **Cast pointer to unsigned char * creates byte array**

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

**Printf directives:**

%p: Print pointer

%x: Print Hexadecimal

# show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

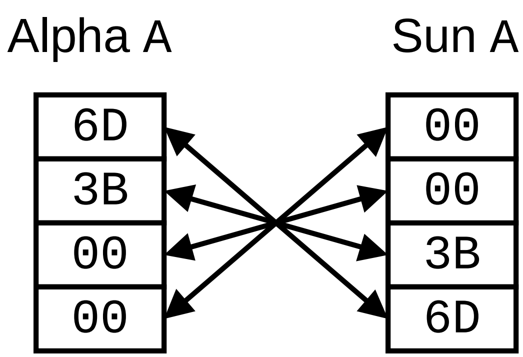
**Result:**

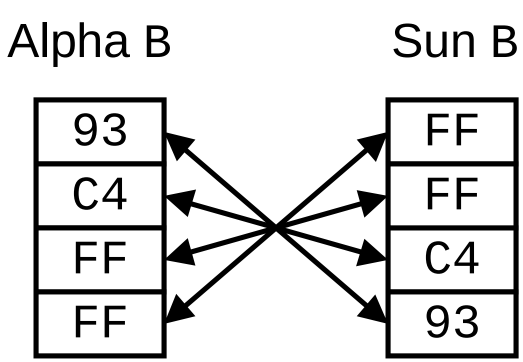
```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```

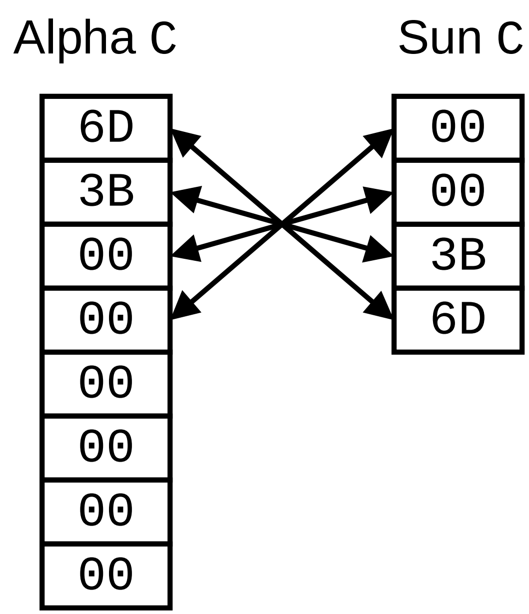
# Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

| | |
|---|---|
| Decimal: | 15213 |
| Binary: | 0011 1011 0110 1101 |
| Hex: | 3 B 6 D |




Two’s complement representation
(Covered next lecture)

# Representing Pointers

```
int B = -15213;
int *P = &B;
```

**Alpha Address**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Hex: | 1 | F | F | F | F | F | C | A | 0 |
| Binary: | 0001 | 1111 | 1111 | 1111 | 1111 | 1111 | 1100 | 1010 | 0000 |

Alpha P

| |
|---|
| A0 |
| FC |
| FF |
| FF |
| 01 |
| 00 |
| 00 |
| 00 |

Sun P

| |
|---|
| EF |
| FF |
| FB |
| 2C |

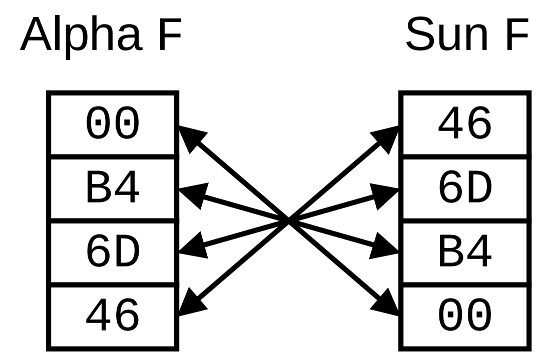
**Sun Address**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Hex: | E | F | F | F | F | B | 2 | C |
| Binary: | 1110 | 1111 | 1111 | 1111 | 1111 | 1011 | 0010 | 1100 |

*Different compilers & machines assign different locations to objects*

# Representing Floats

**`Float F = 15213.0;`**




**IEEE Single Precision Floating Point Representation**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Hex: | 4 | 6 | 6 | D | B | 4 | 0 | 0 |
| Binary: | 0100 | 0110 | 0110 | 1101 | 1011 | 0100 | 0000 | 0000 |
| 15213: | | | 1110 | 1101 | 1011 | 01 | | |

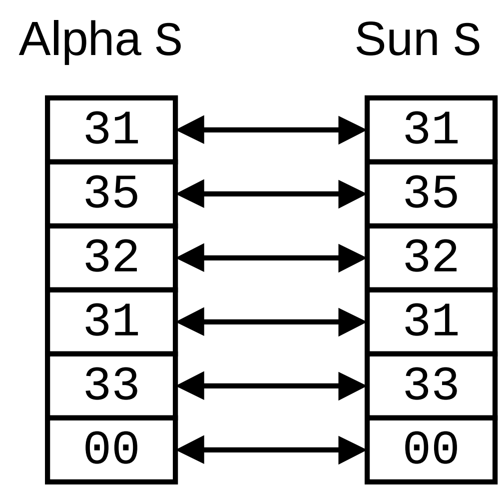
*Not same as integer representation, but consistent across machines*

# Representing Strings

## Strings in C

`char S[6] = "15213";`

- **Represented by array of characters**
- **Each character encoded in ASCII format**
  - Standard 7-bit encoding of character set
  - Other encodings exist, but uncommon
  - Character “0” has code `0x30`
    - » Digit *i* has code `0x30`+*i*
- **String should be null-terminated**
  - Final character = 0




## Compatibility

- **Byte ordering not an issue**
  - Data are single byte quantities
- **Text files generally platform independent**
  - Except for different conventions of line termination character!

# Machine-Level Code Representation

## Encode Program as Sequence of Instructions

- **Each simple operation**
  - Arithmetic operation
  - Read or write memory
  - Conditional branch
- **Instructions encoded as bytes**
  - Alpha’s, Sun’s, Mac’s use 4 byte instructions
    - » Reduced Instruction Set Computer (RISC)
  - PC’s use variable length instructions
    - » Complex Instruction Set Computer (CISC)
- **Different instruction types and encodings for different machines**
  - Most code not binary compatible

## *Programs are Byte Sequences Too!*

# Representing Instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

- **For this example, Alpha & Sun use two 4-byte instructions**
  - Use differing numbers of instructions in other cases
- **PC uses 7 instructions with lengths 1, 2, and 3 bytes**
  - Same for NT and for Linux
  - NT / Linux not binary compatible

| Alpha sum | Sun sum | PC sum |
|---|---|---|
| 00 | 81 | 55 |
| 00 | C3 | 89 |
| 30 | E0 | E5 |
| 42 | 08 | 8B |
| 01 | 90 | 45 |
| 80 | 02 | 0C |
| FA | 00 | 03 |
| 6B | 09 | 45 |
| | | 08 |
| | | 89 |
| | | EC |
| | | 5D |
| | | C3 |

*Different machines use totally different instructions and encodings*

# Boolean Algebra

## Developed by George Boole in 19th Century

- **Algebraic representation of logic**
  - Encode “True” as 1 and “False” as 0

## And

- **A&B = 1 when both A=1 and B=1**

| & | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

## Or

- **A|B = 1 when either A=1 or B=1**

| \| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

## Not

- **~A = 1 when A=0**

| ~ | |
|---|---|
| 0 | 1 |
| 1 | 0 |

## Exclusive-Or (Xor)

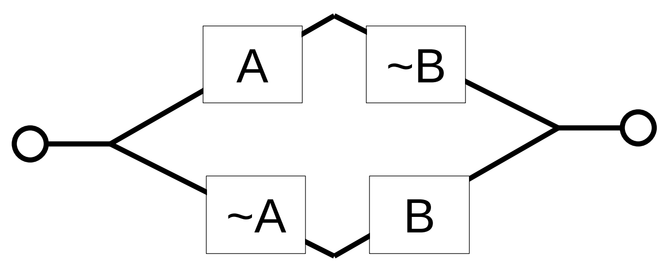
- **A^B = 1 when either A=1 or B=1, but not both**

| ^ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

# Application of Boolean Algebra

## Applied to Digital Systems by Claude Shannon

- **1937 MIT Master's Thesis**
- **Reason about networks of relay switches**
  - Encode closed switch as 1, open switch as 0




Connection when
A&~B | ~A&B
= A^B

# Properties of & and | Operations

## Integer Arithmetic

- $\langle Z, +, *, -, 0, 1\rangle$ **forms a "ring"**
- **Addition is "sum" operation**
- **Multiplication is "product" operation**
- **– is additive inverse**
- **0 is identity for sum**
- **1 is identity for product**

## Boolean Algebra

- $\langle \{0,1\}, |, \&, \sim, 0, 1\rangle$ **forms a "Boolean algebra"**
- **Or is "sum" operation**
- **And is "product" operation**
- **~ is "complement" operation (not additive inverse)**
- **0 is identity for sum**
- **1 is identity for product**

# Properties of Rings & Boolean Algebras

| | Boolean Algebra | Integer Ring |
|---|---|---|
| *Commutativity* | | |
| | A \| B = B \| A | A + B = B + A |
| | A & B = B & A | A * B = B * A |
| *Associativity* | | |
| | (A \| B) \| C = A \| (B \| C) | (A + B) + C = A + (B + C) |
| | (A & B) & C = A & (B & C) | (A * B) * C = A * (B * C) |
| *Product distributes over sum* | | |
| | A & (B \| C) = (A & B) \| (A & C) | A * (B + C) = A * B + B * C |
| *Sum and product identities* | | |
| | A \| 0 = A | A + 0 = A |
| | A & 1 = A | A * 1 = A |
| *Zero is product annihilator* | | |
| | A & 0 = 0 | A * 0 = 0 |
| *Cancellation of negation* | | |
| | ~ (~ A) = A | – (– A) = A |

# Ring ≠ Boolean Algebra

| Boolean Algebra | Integer Ring |
|---|---|
| • Boolean: *Sum distributes over product* | |
| A \| (B & C) = (A \| B) & (A \| C) | A + (B * C) ≠ (A + B) * (B + C) |
| • Boolean: *Idempotency* | |
| A \| A = A | A + A ≠ A |
| – "A is true" or "A is true" = "A is true" | |
| A & A = A | A * A ≠ A |
| • Boolean: *Absorption* | |
| A \| (A & B) = A | A + (A * B) ≠ A |
| – "A is true" or "A is true and B is true" = "A is true" | |
| A & (A \| B) = A | A * (A + B) ≠ A |
| • Boolean: *Laws of Complements* | |
| A \| ~A = 1 | A + –A ≠ 1 |
| – "A is true" or "A is false" | |
| • Ring: *Every element has additive inverse* | |
| A \| ~A ≠ 0 | A + –A = 0 |

# Properties of & and ^

## Boolean Ring

- $\langle \{0,1\}, \text{^}, \&, I, 0, 1 \rangle$
- **Identical to integers mod 2**
- **$I$ is identity operation: $I$(A) = A**

  A ^ A = 0

| Property | Boolean Ring |
|---|---|
| • Commutative sum | A ^ B = B ^ A |
| • Commutative product | A & B = B & A |
| • Associative sum | (A ^ B) ^ C = A ^ (B ^ C) |
| • Associative product | (A & B) & C = A & (B & C) |
| • Prod. over sum | A & (B ^ C) = (A & B) ^ (B & C) |
| • 0 is sum identity | A ^ 0 = A |
| • 1 is prod. identity | A & 1 = A |
| • 0 is product annihilator | A & 0 = 0 |
| • Additive inverse | A ^ A = 0 |

# Relations Between Operations

## DeMorgan's Laws

- **Express & in terms of |, and vice-versa**

  A & B = ~(~A | ~B)

  » A and B are true if and only if neither A nor B is false

  A | B = ~(~A & ~B)

  » A or B are true if and only if A and B are not both false

## Exclusive-Or using Inclusive Or

A ^ B = (~A & B) | (A & ~B)

» Exactly one of A and B is true

A ^ B = (A | B) & ~(A & B)

» Either A is true, or B is true, but not both

# General Boolean Algebras

## Operate on Bit Vectors

- **Operations applied bitwise**

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

## Representation of Sets

- **Width** $w$ **bit vector represents subsets of** $\{0, \ldots, w-1\}$
- $a_j = 1$ **if** $j \in A$
  - `01101001` { 0, 3, 5, 6 }
  - `01010101` { 0, 2, 4, 6 }
- **& Intersection** **`01000001`** { 0, 6 }
- **| Union** **`01111101`** { 0, 2, 3, 4, 5, 6 }
- **^ Symmetric difference** **`00111100`** { 2, 3, 4, 5 }
- **~ Complement** **`10101010`** { 1, 3, 5, 7 }

# Bit-Level Operations in C

## Operations &, |, ~, ^ Available in C

- **Apply to any "integral" data type**
  - `long`, `int`, `short`, `char`
- **View arguments as bit vectors**
- **Arguments applied bit-wise**

## Examples (Char data type)

- **`~0x41 --> 0xBE`**

  ~$01000001_2$ --> $10111110_2$
- **`~0x00 --> 0xFF`**

  ~$00000000_2$ --> $11111111_2$

- **`0x69 & 0x55 --> 0x41`**

  $01101001_2$ & $01010101_2$ --> $01000001_2$
- **`0x69 | 0x55 --> 0x7D`**

  $01101001_2$ | $01010101_2$ --> $01111101_2$

# Contrast: Logic Operations in C

## Contrast to Logical Operators

- `&&`, `||`, `!`
  - View 0 as “False”
  - Anything nonzero as “True”
  - Always return 0 or 1

## Examples (char data type)

- `!0x41  -->  0x00`
- `!0x00  -->  0x01`
- `!!0x41 -->  0x01`

- `0x69 && 0x55  -->  0x01`
- `0x69 || 0x55  -->  0x01`

# Shift Operations

## Left Shift: x << y

- **Shift bit-vector x left y positions**
  - Throw away extra bits on left
  - Fill with 0's on right

## Right Shift: x >> y

- **Shift bit-vector x right y positions**
  - Throw away extra bits on right
- **Logical shift**
  - Fill with 0's on left
- **Arithmetic shift**
  - Replicate most significant bit on right
  - Useful with two's complement integer representation

| | |
|---|---|
| Argument x | `01100010` |
| << 3 | `00010`*`000`* |
| Log. >> 2 | *`00`*`011000` |
| Arith. >> 2 | *`00`*`011000` |

| | |
|---|---|
| Argument x | `10100010` |
| << 3 | `00010`*`000`* |
| Log. >> 2 | *`00`*`101000` |
| Arith. >> 2 | *`11`*`101000` |

# Cool Stuff with Xor

- **Bitwise Xor is form of addition**
- **With extra property that every value is its own additive inverse**

  A ^ A = 0

```
void funny(int *x, int *y)
{
   *x = *x ^ *y;    /* #1 */
   *y = *x ^ *y;    /* #2 */
   *x = *x ^ *y;    /* #3 */
}
```

| Step | `*x` | `*y` |
|---|---|---|
| Begin | A | B |
| 1 | A^B | B |
| 2 | A^B | (A^B)^B = A^(B^B) = A^0 = A |
| 3 | (A^B)^A = (B^A)^A = B^(A^A) = B^0 = B | A |
| End | B | A |