

15-213
"The Class That Gives CMU Its Zip!"

Bits and Bytes

Jan. 18, 2001

Topics

- Why bits?
- Representing information as bits
 - Binary/Hexadecimal
 - Byte representations
 - » numbers
 - » characters and strings
 - » Instructions
- Bit-level manipulations
 - Boolean algebra
 - Expressing in C

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Why Don't Computers Use Base 10?

Base 10 Number Representation

- That's why fingers are known as "digits"
- Natural representation for financial transactions
 - Floating point number cannot exactly represent \$1.20
- Even carries through in scientific notation
 - 1.5213×10^4

Implementing Electronically

- Hard to store
 - ENIAC (First electronic computer) used 10 vacuum tubes / digit
- Hard to transmit
 - Need high precision to encode 10 signal levels on single wire
- Messy to implement digital logic functions
 - Addition, multiplication, etc.

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- 2 -

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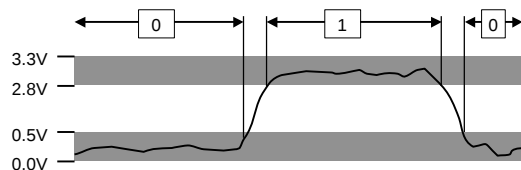
Binary Representations

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



- Straightforward implementation of arithmetic functions

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- 3 -

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Byte-Oriented Memory Organization

Programs Refer to Virtual Addresses

- Conceptually very large array of bytes
- Actually implemented with hierarchy of different memory types
 - SRAM, DRAM, disk
 - Only allocate for regions actually used by program
- In Unix and Windows NT, address space private to particular "process"
 - Program being executed
 - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation

- Where different program objects should be stored
- Multiple mechanisms: static, stack, and heap
- In any case, all allocation within single virtual address space

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Encoding Byte Values

Byte = 8 bits

- Binary 00000000₂ to 11111111₂
- Decimal: 0₁₀ to 255₁₀
- Hexadecimal 00₁₆ to FF₁₆
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write FA1D37B₁₆ in C as 0xFA1D37B
 - » Or 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

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- 5 -

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Machine Words

Machine Has "Word Size"

- Nominal size of integer-valued data
 - Including addresses
- Most current machines are 32 bits (4 bytes)
 - Limits addresses to 4GB
 - Becoming too small for memory-intensive applications
- High-end systems are 64 bits (8 bytes)
 - Potentially address $\approx 1.8 \times 10^{19}$ bytes
- Machines support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

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Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)

32-bit Words	64-bit Words	Bytes	Addr.
Addr = 0000			0000
			0001
			0002
	Addr = 0000		0003
			0004
Addr = 0004			0005
			0006
			0007
			0008
	Addr = 0008		0009
			0010
			0011
			0012
Addr = 0012			0013
			0014
			0015

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- 7 -

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Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Compaq Alpha	Typical 32-bit
int	4	4
long int	8	4
char	1	1
short	2	2
float	4	4
double	8	8
char *	8	4

» Or any other pointer

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Byte Ordering

Issue

- How should bytes within multi-byte word be ordered in memory

Conventions

- Alphas, PC's are "Little Endian" machines
 - Least significant byte has lowest address
- Sun's, Mac's are "Big Endian" machines
 - Least significant byte has highest address

Example

- Variable x has 4-byte representation 0x1234567
- Address given by &x is 0x100

Big Endian

0x100	0x101	0x102	0x103
01	23	45	67

Little Endian

0x100	0x101	0x102	0x103
67	45	23	01

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- 9 -

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Examining Data Representations

Code to Print Byte Representation of Data

- Cast pointer to unsigned char * creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
    int i;
    for (i = 0; i < len; i++)
        printf("0x%p\t0x%.2x\n",
               start+i, start[i]);
    printf("\n");
}
```

Printf directives:

%p: Print pointer
%x: Print Hexadecimal

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- 10 -

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show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result:

```
int a = 15213;
0x11ffffcb8 0x6d
0x11ffffcb9 0x3b
0x11ffffcba 0x00
0x11ffffcbb 0x00
```

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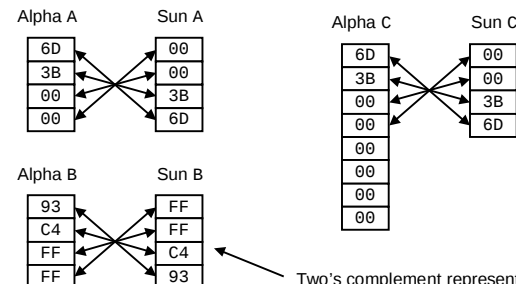
- 11 -

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Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

Decimal: 15213
Binary: 0011 1011 0110 1101
Hex: 3 B 6 D



Two's complement representation
(Covered next lecture)

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- 12 -

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Representing Pointers

```
int B = -15213;
int *P = &B;
```

Alpha P

A0
FC
FF
FF
01
00
00
00

Alpha Address

Hex: 1 F F F F F C A 0

Binary: 0001 1111 1111 1111 1111 1111 1100 1010 0000

Sun P

EF
FF
FB
2C

Sun Address

Hex: E F F F F B 2 C

Binary: 1110 1111 1111 1111 1111 1011 0010 1100

Different compilers & machines assign different locations to objects

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- 13 -

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Representing Floats

Float F = 15213.0;

Alpha F

00
B4
6D
46

Sun F

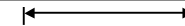
46
6D
B4
00

IEEE Single Precision Floating Point Representation

Hex: 4 6 6 D B 4 0 0

Binary: 0100 0110 0110 1101 1011 0100 0000 0000

15213: 1110 1101 1011 01



Not same as integer representation, but consistent across machines

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- 14 -

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Representing Strings

Strings in C

```
char S[6] = "15213";
```

- Represented by array of characters
- Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Other encodings exist, but uncommon
 - Character "0" has code 0x30
 - » Digit i has code $0x30+i$
- String should be null-terminated
 - Final character = 0

Alpha S

31
35
32
31
33
00

Sun S

31
35
32
31
33
00

Compatibility

- Byte ordering not an issue
 - Data are single byte quantities
- Text files generally platform independent
 - Except for different conventions of line termination character!

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- 15 -

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Machine-Level Code Representation

Encode Program as Sequence of Instructions

- Each simple operation
 - Arithmetic operation
 - Read or write memory
 - Conditional branch
- Instructions encoded as bytes
 - Alpha's, Sun's, Mac's use 4 byte instructions
 - » Reduced Instruction Set Computer (RISC)
 - PC's use variable length instructions
 - » Complex Instruction Set Computer (CISC)
- Different instruction types and encodings for different machines
 - Most code not binary compatible

Programs are Byte Sequences Too!

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- 16 -

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Representing Instructions

```
int sum(int x, int y)
{
    return x+y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
 - Use differing numbers of instructions in other cases
- PC uses 7 instructions with lengths 1, 2, and 3 bytes
 - Same for NT and for Linux
 - NT / Linux not binary compatible

Different machines use totally different instructions and encodings

Alpha sum	Sun sum	PC sum
00	81	55
00	C3	89
30	E0	E5
42	08	8B
01	90	45
80	02	0C
FA	00	03
6B	09	45
		08
		89
		EC
		5D
		C3

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- 17 -

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Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
 - Encode "True" as 1 and "False" as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

&	0	1
0	0	0
1	0	1

Or

- $A|B = 1$ when either $A=1$ or $B=1$

	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

~	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

^	0	1
0	0	1
1	1	0

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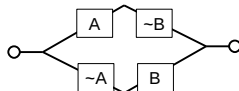
- 18 -

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Application of Boolean Algebra

Applied to Digital Systems by Claude Shannon

- 1937 MIT Master's Thesis
- Reason about networks of relay switches
 - Encode closed switch as 1, open switch as 0



Connection when
 $A \& \sim B | \sim A \& B$
 $= A \wedge B$

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- 19 -

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Properties of & and | Operations

Integer Arithmetic

- $\langle \mathbb{Z}, +, *, -, 0, 1 \rangle$ forms a "ring"
- Addition is "sum" operation
- Multiplication is "product" operation
- is additive inverse
- 0 is identity for sum
- 1 is identity for product

Boolean Algebra

- $\langle \{0,1\}, |, \&, \sim, 0, 1 \rangle$ forms a "Boolean algebra"
- Or is "sum" operation
- And is "product" operation
- $\sim$ is "complement" operation (not additive inverse)
- 0 is identity for sum
- 1 is identity for product

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- 20 -

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Properties of Rings & Boolean Algebras

Boolean Algebra	Integer Ring
- Commutativity $A \mid B = B \mid A$ $A \& B = B \& A$ - Associativity $(A \mid B) \mid C = A \mid (B \mid C)$ $(A \& B) \& C = A \& (B \& C)$ - Product distributes over sum $A \& (B \mid C) = (A \& B) \mid (A \& C)$ - Sum and product identities $A \mid 0 = A$ $A \& 1 = A$ - Zero is product annihilator $A \& 0 = 0$ - Cancellation of negation $\sim(\sim A) = A$	$A + B = B + A$ $A * B = B * A$ $(A + B) + C = A + (B + C)$ $(A * B) * C = A * (B * C)$ $A * (B + C) = A * B + B * C$ $A + 0 = A$ $A * 1 = A$ $A * 0 = 0$ $\sim(\sim A) = A$

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- 21 -

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Ring $\neq$ Boolean Algebra

Boolean Algebra	Integer Ring
- Boolean: Sum distributes over product $A \mid (B \& C) = (A \mid B) \& (A \mid C)$ - Boolean: Idempotency $A \mid A = A$ – “A is true” or “A is true” = “A is true” $A \& A = A$ - Boolean: Absorption $A \mid (A \& B) = A$ – “A is true” or “A is true and B is true” = “A is true” $A \& (A \mid B) = A$ - Boolean: Laws of Complements $A \mid \sim A = 1$ – “A is true” or “A is false” - Ring: Every element has additive inverse $A \mid \sim A \neq 0$	$A + (B * C) \neq (A + B) * (B + C)$ $A + A \neq A$ $A * A \neq A$ $A + (A * B) \neq A$ $A * (A + B) \neq A$ $A + \sim A \neq 1$ $A + \sim A = 0$

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- 22 -

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Properties of & and ^

Boolean Ring

- $\langle \{0,1\}, \wedge, \&, \vee, 0, 1 \rangle$
- Identical to integers mod 2
- $\vee$ is identity operation: $\vee(A) = A$
- $A \wedge A = 0$

Property

- Commutative sum
- Commutative product
- Associative sum
- Associative product
- Prod. over sum
- 0 is sum identity
- 1 is prod. identity
- 0 is product annihilator
- Additive inverse

Boolean Ring

$$A \wedge B = B \wedge A$$

$$A \& B = B \& A$$

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

$$(A \& B) \& C = A \& (B \& C)$$

$$A \& (B \wedge C) = (A \& B) \wedge (A \& C)$$

$$A \wedge 0 = A$$

$$A \& 1 = A$$

$$A \& 0 = 0$$

$$A \wedge A = 0$$

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- 23 -

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Relations Between Operations

DeMorgan's Laws

- Express & in terms of $\vee$, and vice-versa
$$A \& B = \sim(\sim A \mid \sim B)$$

» A and B are true if and only if neither A nor B is false

$$A \mid B = \sim(\sim A \& \sim B)$$

» A or B are true if and only if A and B are not both false

Exclusive-Or using Inclusive Or

$$A \wedge B = (\sim A \& B) \mid (A \& \sim B)$$

» Exactly one of A and B is true

$$A \wedge B = (A \mid B) \& \sim(A \& B)$$

» Either A is true, or B is true, but not both

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- 24 -

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General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```

01101001    01101001    01101001    ~ 01010101
& 01010101 | 01010101 ^ 01010101 ~ 01010101
01000001    01111101    00111100    10101010

```

Representation of Sets

- Width w bit vector represents subsets of $\{0, \dots, w-1\}$
- $a_j = 1$ if $j \in A$
 - 01101001 {0, 3, 5, 6}
 - 01010101 {0, 2, 4, 6}
- & Intersection 01000001 {0, 6}
- | Union 01111101 {0, 2, 3, 4, 5, 6}
- ^ Symmetric difference 00111100 {2, 3, 4, 5}
- ~ Complement 10101010 {1, 3, 5, 7}

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- 25 -

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Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
 - long, int, short, char
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- ~0x41 --> 0xBE
 - ~01000001₂ --> 10111110₂
- ~0x00 --> 0xFF
 - ~00000000₂ --> 11111111₂
- 0x69 & 0x55 --> 0x41
 - 01101001₂ & 01010101₂ --> 01000001₂
- 0x69 | 0x55 --> 0x7D
 - 01101001₂ | 01010101₂ --> 01111101₂

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- 26 -

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Contrast: Logic Operations in C

Contrast to Logical Operators

- &&, ||, !
 - View 0 as "False"
 - Anything nonzero as "True"
 - Always return 0 or 1

Examples (char data type)

- !0x41 --> 0x00
- !0x00 --> 0x01
- !!0x41 --> 0x01
- 0x69 && 0x55 --> 0x01
- 0x69 || 0x55 --> 0x01

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- 27 -

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Shift Operations

Left Shift: $x \ll y$

- Shift bit-vector x left y positions
 - Throw away extra bits on left
 - Fill with 0's on right

Argument x	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Right Shift: $x \gg y$

- Shift bit-vector x right y positions
 - Throw away extra bits on right
- Logical shift
 - Fill with 0's on left
- Arithmetic shift
 - Replicate most significant bit on right
 - Useful with two's complement integer representation

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

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- 28 -

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Cool Stuff with Xor

- Bitwise Xor is form of addition
- With extra property that every value is its own additive inverse

$$A \oplus A = 0$$

```
void funny(int *x, int *y)
{
    *x = *x ^ *y; /* #1 */
    *y = *x ^ *y; /* #2 */
    *x = *x ^ *y; /* #3 */
}
```

Step	*x	*y
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A^(B^B) = A^0 = A
3	(A^B)^A = (B^A)^A = B^(A^A) = B^0 = B	A
End	B	A

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- 29 -

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