15-213

Riding the Technology Curve
Dec. 1, 1998

Topics

- Moore's Law
- Are exponential problems intractable?
- Impact on real-world problems
- The verification challenge

Impact of Technology

It's the Technology, Stupid!

- Computer science has ridden the wave

Things Aren't Over Yet

- Technology will continue to progress along current growth curves
- For at least 10 more years
- Difficult technical challenges in doing so

Even Technologists Can't Beat Laws of Physics

Moore's Law

Gordon Moore

- **Co-founded Intel in early 70's**
- **Observed in 1972 that number of transistors / chip doubled ~ every 1.5 years**
- **Really a "trend" rather than a "law"**

Exponential Growth Trends

- **DRAM technology**
  - Capacity 4X every 3 years
  - Speed 3X in 10 years
- **Magnetic disk technology**
  - Capacity 4X every 3 years
- **Microprocessor Performance**
  - SPEC performance 2X every 1.5 years
- **Software complexity**
  - Typical program sizes growing 1.5--2X per year

Semiconductor Industry Forecast

- Semiconductor Industry Association, 1992 Technology Workshop

Year	1992	1995	1998	2001	2004	2007
Feature size	0.5	0.35	0.25	0.18	0.12	0.10
DRAM cap	16M	64M	256M	1G	4G	16G
Gates/chip	300K	800K	2M	5M	10M	20M
Chip cm^2	2.5	4.0	6.0	8.0	10.0	12.5
I/Os	500	750	1500	2000	3500	5000
off chip MHz	60	100	175	250	350	500
on chip MHz	120	200	350	500	700	1000

Impact of Moore's Law

Moore's Law

- Performance factors of systems built with integrated circuit technology follow exponential curve
- E.g., computer speed / memory capacities double every 1.5 years

Implications

- Computers 10 years from now will run 100 X faster
- Problems that appear intractable today will be straightforward
- Must not limit future planning with today's technology

Example Application Domains

- Speech recognition
  - Will be routinely done with handheld devices
- Breaking secret codes
  - Need to use large enough encryption keys

Solving Hard Problems

Conventional Wisdom

- Exponential problems are intractable

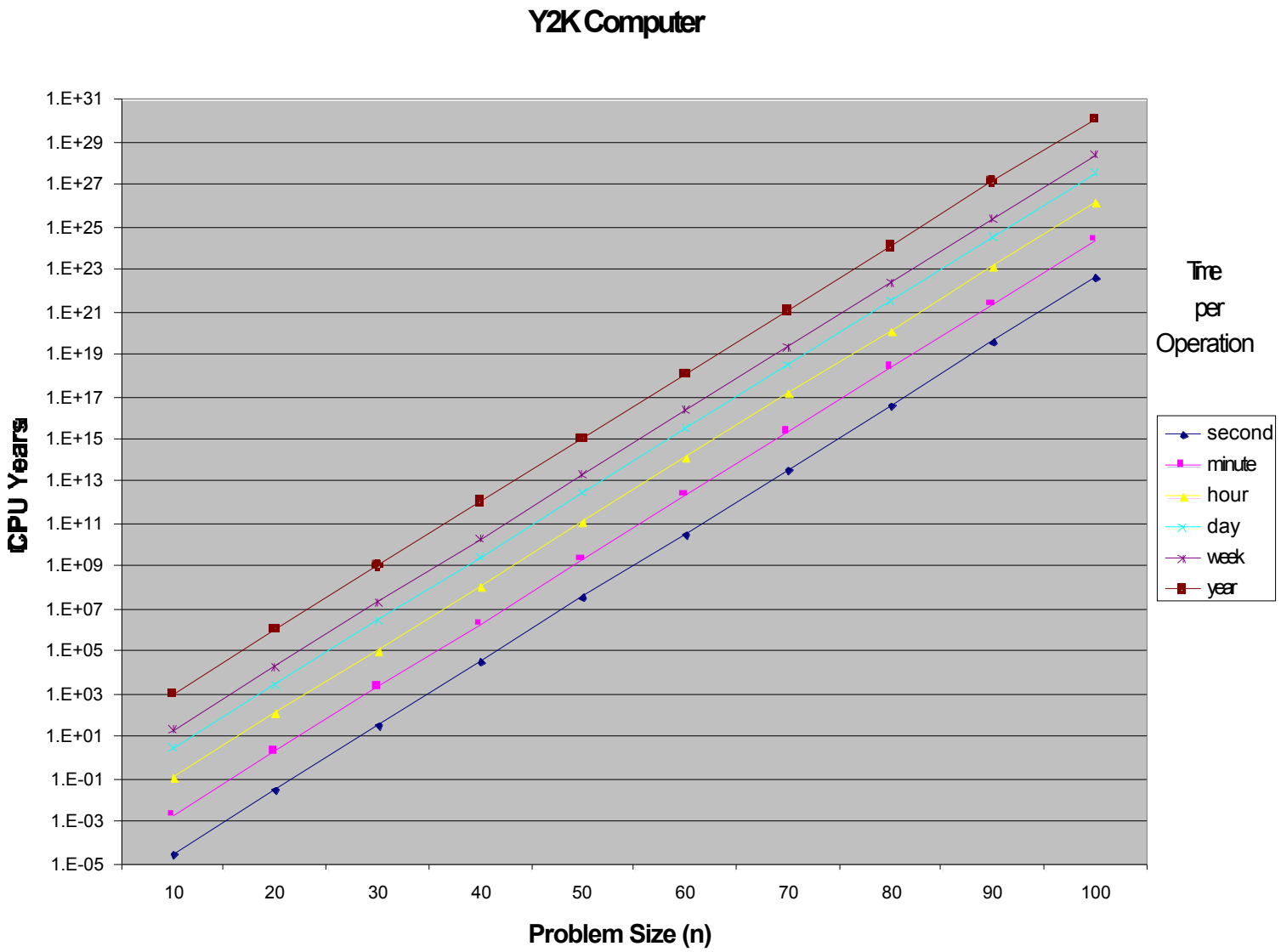
Operation

- Assume problem of size n requires 2^n steps
- Each step takes k years on a Y2K computer

Y2K Computer Performance

- Start computation Jan. 1, 2000
- Keep running same machine until problem solved
- Would take $k\ 2^n$ years

Solving with a Y2K Computer



Moore's Law Computer

Operation

- Start computing on Jan. 1, 2000
- Keep upgrading machine being used
- In year *y*, would have performance 1.587^y relative to Y2K machine

Performance

- After *y* years of operation, would have performed as much computation as Y2K machine would do in time:

$$\int_0^y 1.587^x \, dx = 2.16(1.587^y - 1)$$

- Examples

  y = 1 1.27

  y = 2 3.29

  y = 5 20.

  y = 10 218.

  y = 100 2.53×10^{20}

Solving Hard Problems

Solution Time

- Problem of size *n*
- Running *y* years on Moore's Law computer

$$y = 2.16 \ln(1 + 0.462\, k\, 2^n)$$

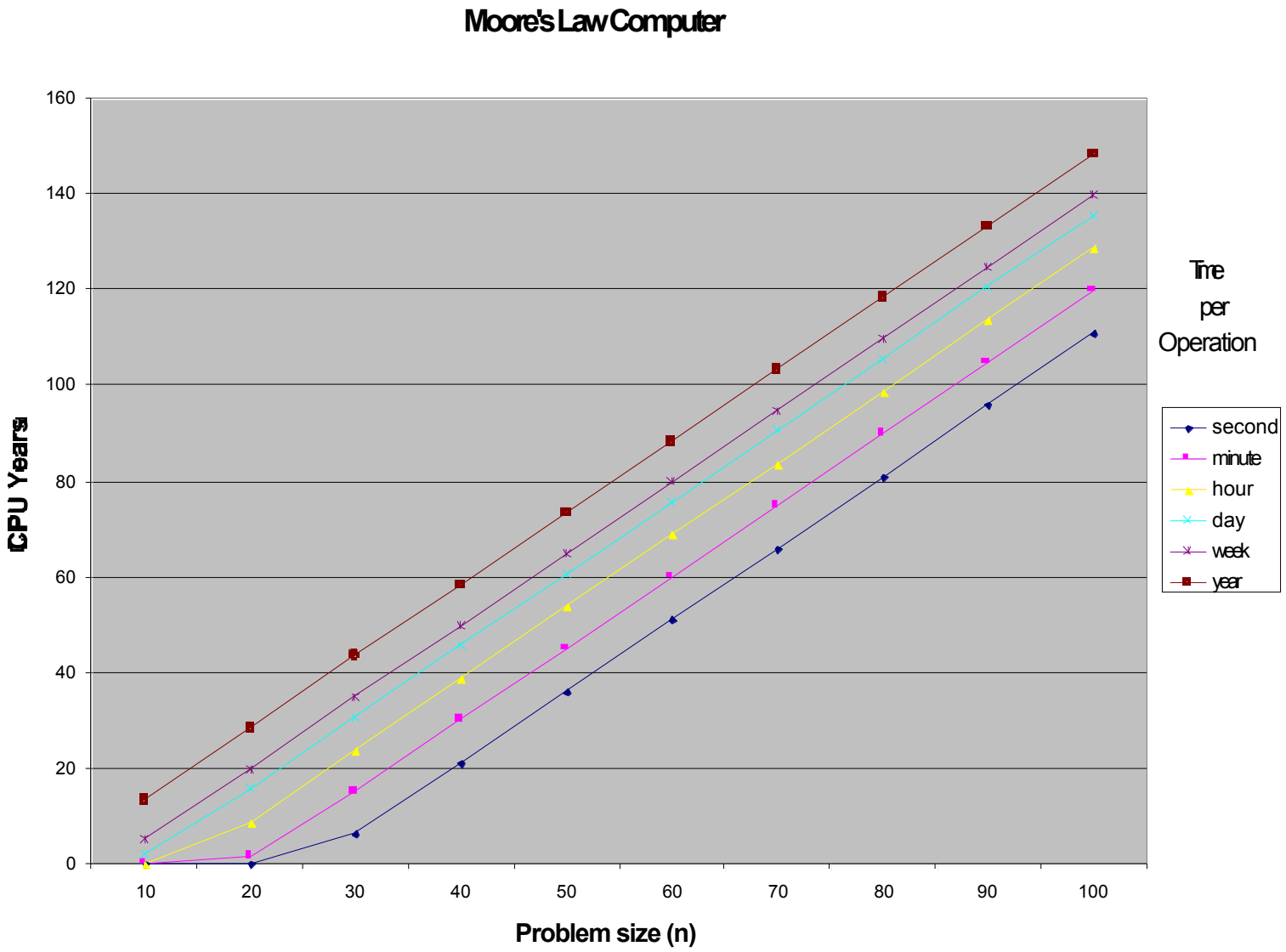
- For large values of *n*:

$$y \approx 1.5\, n + 2.16 \ln k - 1.67$$
$$= O(n)$$

Complexity

- Linear in problem size

Solving with a Moore's Law Computer



Effect of Step Complexity

Observe

- **Step complexity *k* adds only additive factor of** $2.16 \ln k$ **to running time**

Example

- **For *n* = 100**

k	*y*
1 second	111
1 minute	120
1 hour	129
1 day	136
1 week	140
1 year	148

Explanation

- **Final years of computation will be on exponentially faster machines**

Implications of Moore's Law

P=NP (Effectively)

- Problems of exponential complexity can be solved in linear time

Caveat

- Cannot hold forever

Fundamental Limit

- Argument due to Ed Fredkin
- Claim that ultimate limit to growth in memory capacity is cubic
- Cannot build storage device with less than one electron
- Assume consume all available material to build memories
  - Would soon exhaust planetary resources
  - Cannot travel into outer space faster than speed of light
- Total amount of material available at time t is $\Omega(t^3)$
- This limit will be hit in ~400 years

How to Be a Visionary

Pick a Really Hard Problem

- Sequencing of human genome
- Accurate weather prediction
- Flying helicopter autonomously

Make Proclamations

- "In 20 years, problem X will be solved"

Wait

- But make sure everyone credits you with the vision
- Maybe make a few contributions to technology

Amass Glory

- Turing Award Citation:
  - "He/She had the foresight to see that this problem could be solved."

Truly Hard Problems

Those That Get Harder over Time

- Track Moore's law growth
- How do I make sure my chip will operate correctly?
- How do I make sure my programs are correct?
- How do I manufacture state-of-the-art chips?

Highlight

- Research at CMU on formal verification of hardware

Motivation for Formal Verification

Intel's Challenge, (ca. 1992)

- Design a high performance, state of the art microprocessor to succeed the 486
- Maintain compatibility to 20-year old x86 product line
- Provide new levels of performance on floating point

Floating Point Divider

- Use radix-4, SRT algorithm developed in 1960's
- First time ever used by Intel

Validation

- Run lots of simulation tests
- Make sure it runs set of Windows applications

Manufacturing Environment

- Will produce millions of chips
- Cannot make any changes after manufacture

The Pentium Fiasco

Events

- **Prof. Thomas Nicely, Lynchburg College, VA**
  - Looking at properties of “twin primes”
  - Incorrect reciprocals for 824633702441 and 824633702443
    - » ~ Single precision accuracy (4×10^{-9})
  - Contacted others on Oct. 30, ‘94
- **Spreading of Information on Internet news group `comp.sys.intel`**
  - Terje Mathisen of Norway posts Nicely’s findings on Nov. 3
  - Andreas Kaiser of Germany finds 23 bad reciprocals, Nov. 10
- **Tim Coe, Vitesse Semiconductor, Nov. 16**
  - Created (good enough) software model of flawed divide algorithm
  - Discovered (nonreciprocal) cases with errror up to 6×10^{-5}
  - Later showed 1738 mantissa pairs with less than single precision accuracy
    - » out of 7.4×10^{13} single precision mantissa pairs

Resolution

Free Replacement Policy, Dec. 20

- No need to argue need
- Complex logistics
  - Many different versions
  - Actual replacement easy

Financial Impact

- Intel charged $475 *million* to it's 4Q94 earnings
- Still was 2nd most profitable year ever
- Few companies could survive such an expensive mistake
- In the end, generated lots of valuable PR for Intel

Is There a Better Way?

Provide Mechanism to "Patch" Functionality

- Make chips more "malleable" so that can update as would software
- Intel has started to incorporate such mechanisms
- Future technologies such as field-programmable logic could help (Seth Goldstein)

Make Sure Hardware is Really Correct

- Formal hardware verification
- Apply automated, mathematical techniques to prove properties about system
- Focus of this presentation

CMU's Research Contributions

Symbolic Model Checking

- **Developed by Ken McMillan while CMU PhD student**
  - Building on work by advisor Ed Clarke
- **Verify properties of finite state systems with 10^{20} or more states**

Binary Moment Diagrams

- **Developed by Bryant & Chen in 1994.**
- **Symbolic representation of functions having bit-level inputs and numeric outputs**
- **Compact for common logical and arithmetic operations**

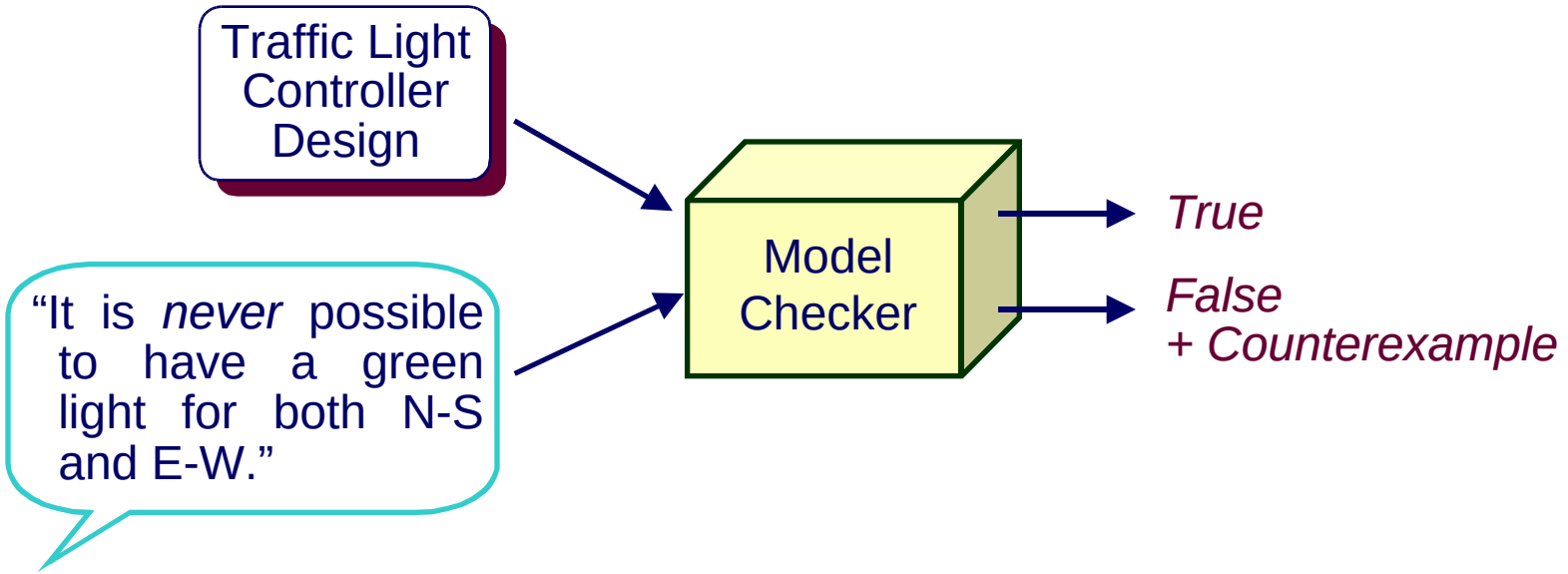
Word-Level Model Checking

- **Developed by Xudong Zhao while CMU PhD student**
  - Advisor Ed Clarke
- **Allow specification to contain arithmetic relations among words of data**

Temporal Logic Model Checking

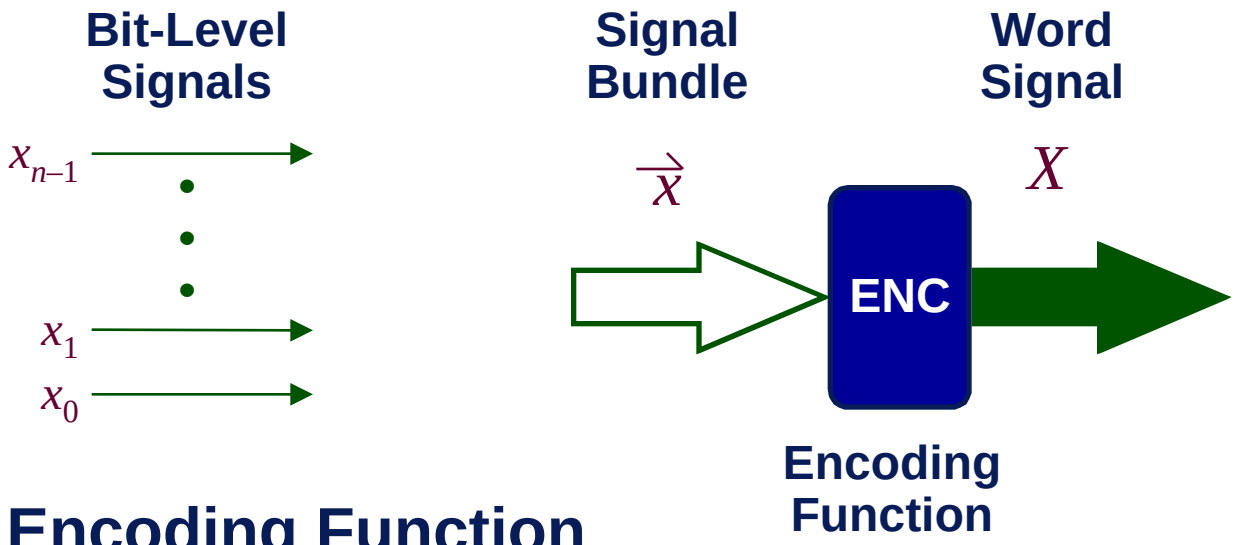
Verifying Reactive Systems

- **Construct state machine representation of reactive system**
  - Nondeterminism expresses range of possible behaviors
  - "Product" of component state machines
- **Express desired behavior as formula in temporal logic**
- **Determine whether or not property holds**



Word-Level Abstractions

- View bundle of wires as encoding numeric value
- Represent as function
  - Over Boolean variables
  - Yielding numeric value



Example Encoding Function

- Unsigned binary

$$X = x_0 + 2\,x_1 + 4\,x_2 + \ldots + 2^{n-1}\,x_{n-1}$$

Word-Level Verification

- Lai [USC], Vrudhula [Arizona]

Given

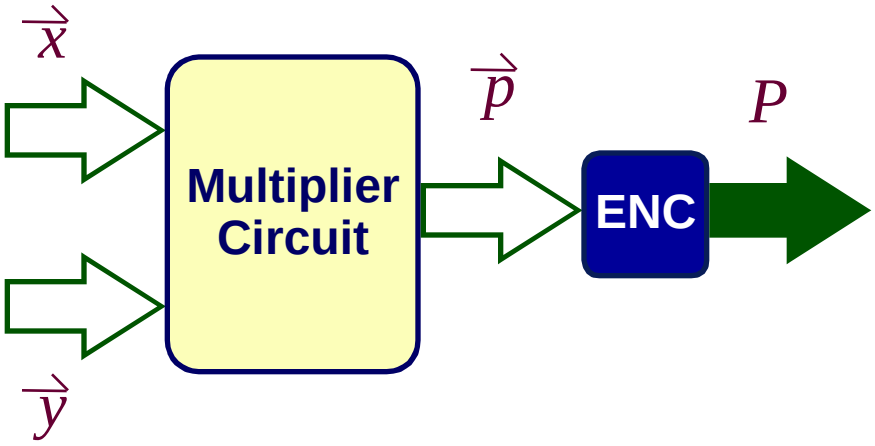
- Bit-level circuit representation
- Encodings of inputs and outputs
- Word-level specification

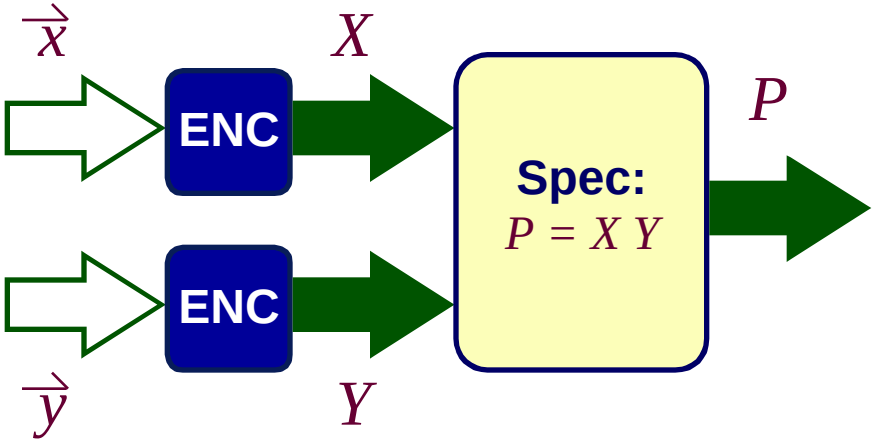
Compare

- Correspondence between two representations
- Under I/O encoding

Observation

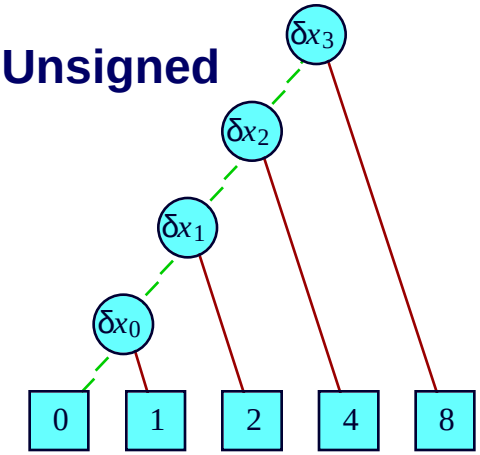
- Crossing abstraction boundary





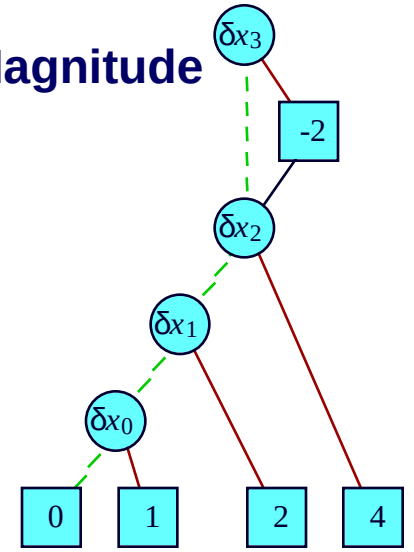
*BMD Representations of Integers

Unsigned



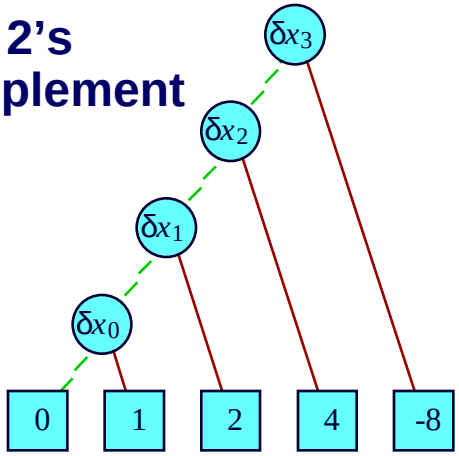
$x_0 + 2\,x_1 + 4\,x_2 + 8\,x_3$

Sign-Magnitude



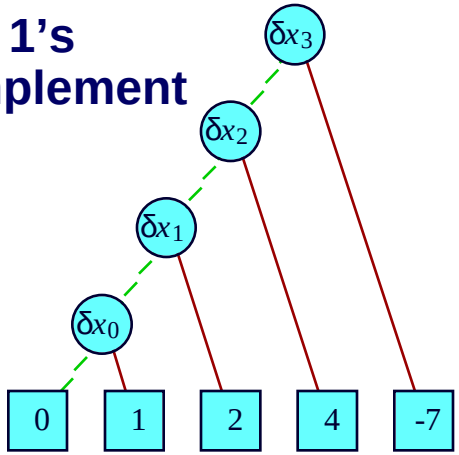
$(x_0 + 2\,x_1 + 4\,x_2)\,(1 - 2\,x_3)$

2's Complement



$x_0 + 2\,x_1 + 4\,x_2 - 8\,x_3$

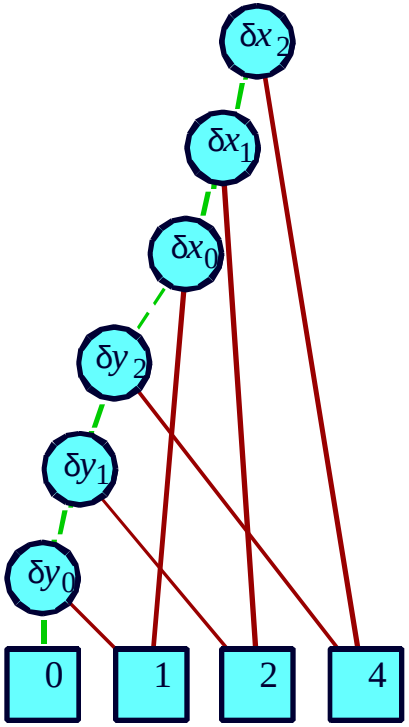
1's Complement



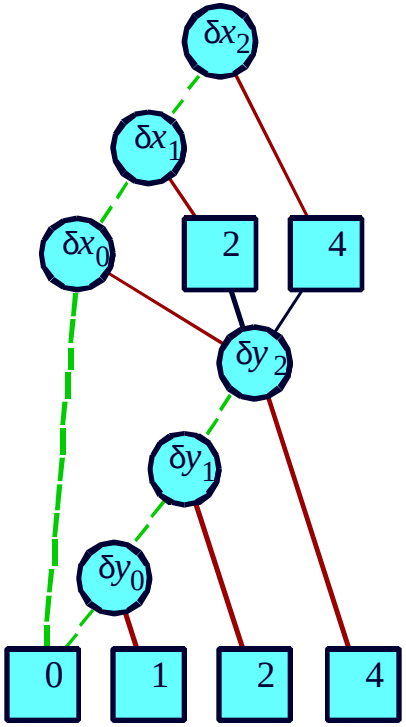
$x_0 + 2\,x_1 + 4\,x_2 - 7\,x_3$

Word-Level *BMDs

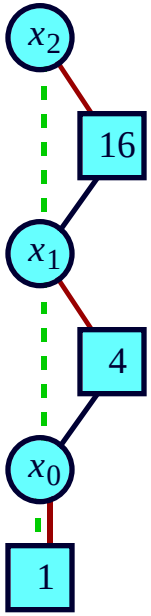
$X + Y$



$X \quad Y$



2^X



Using Word-Level Verification

Word-Level Model Checking

- Xudong Zhao, CMU PhD ‘97

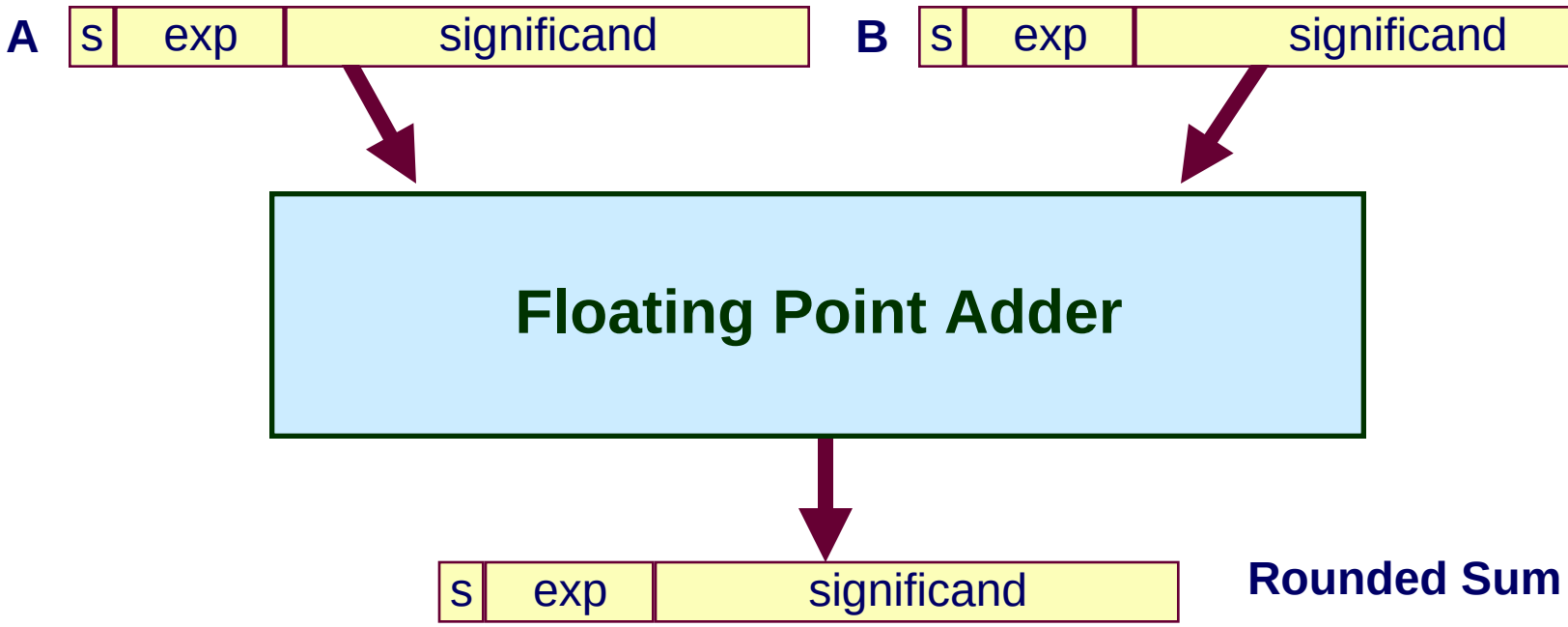
Idea

- Introduce word-level specifications into model checker’s specification language
- Implement with combination of BDDs and BMDs

Applying to Intel’s Circuits

- Verified that each iteration of SRT divider is correct
- Major breakthrough for Intel
- Still cannot do “end-to-end” verification of divider

Recent Result on Arithmetic Circuits



- **Yirng-An Chen, PhD '98**

Verifying Floating Point Adders

- **Able to automatic verify complete behavior, including rounding**
- **That it realizes IEEE FP standard**
- **Completely "hands-off"**
  - No guidance from user on how circuit really works

Formal Verification Tasks

Digital Circuits

- **Arithmetic circuits**
  - "Does this circuit compute the specified mathematical operation?"
- **Pipelined processors**
  - "Does this circuit implement the specified instruction set?"

Reactive Systems

- **Cache protocols**
  - "Is it possible for 2 processors to have write access to a single block?"
- **Controllers**
  - "If a car approaches the traffic light, will it eventually turn green?"

Software Systems

- **Operating Systems**
  - "Is it possible for the scheduler to exclude a process indefinitely?"

Long Term Verification Challenges

Processor Verification

- **Verification becoming much more difficult**
  - Out of order & speculative execution
  - Widening gap between abstract specification & actual system operation
- **Are we catching up or falling behind?**

Reactive Systems

- **Still requires reducing system complexity by abstraction**
  - E.g., reduce to single cache line, make buffers nondeterministic
  - Limited success with automated techniques

Software Systems

- **Can only deal with highly simplified system models**
- **Possible to catch bugs, but not to guarantee correctness**