Bits and Bytes
CS 213
Aug. 27, 1998

Topics

- **Why bits?**
- **Representing information as bits**
  - Binary/Hexadecimal
  - Byte representations
    - numbers
    - characters and strings
    - Instructions
- **Bit-level manipulations**
  - Boolean algebra
  - Expressing in C

`class02.ppt`

Why Don’t Computers Use Base 10?

Base 10 Number Representation

- **That’s why fingers are known as “digits”**
- **Natural representation for financial transactions**
  - Floating point number cannot exactly represent $1.20
- **Even carries through in scientific notation**
  - $1.5213 \text{ X } 10^4$

Implementing Electronically

- **Hard to store**
  - ENIAC (First electronic computer) used 10 vacuum tubes / digit
- **Hard to transmit**
  - Need high precision to encode 10 signal levels on single wire
- **Messy to implement digital logic functions**
  - Addition, multiplication, etc.

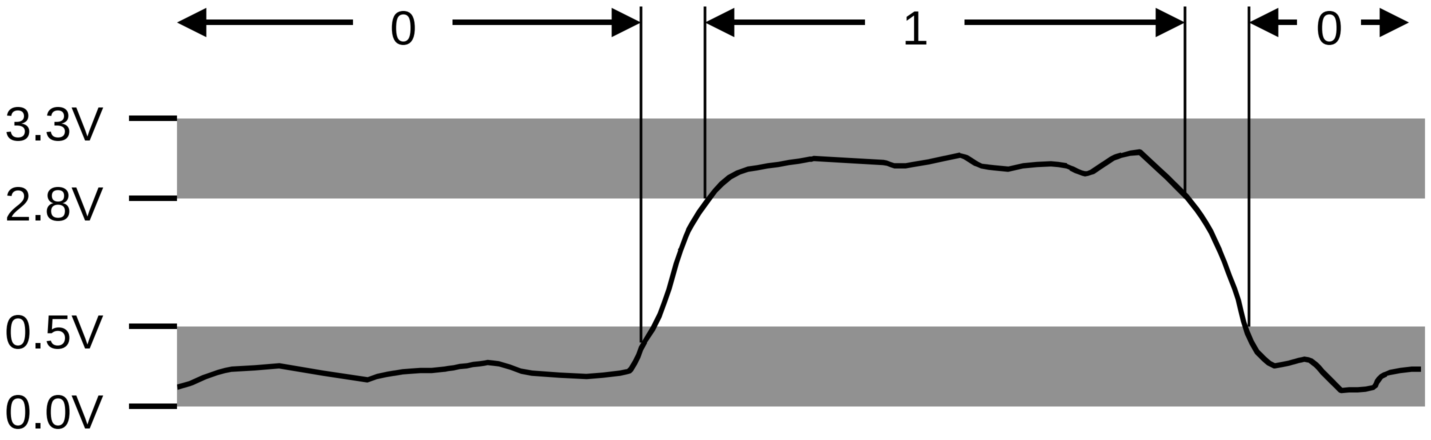
Binary Representations

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

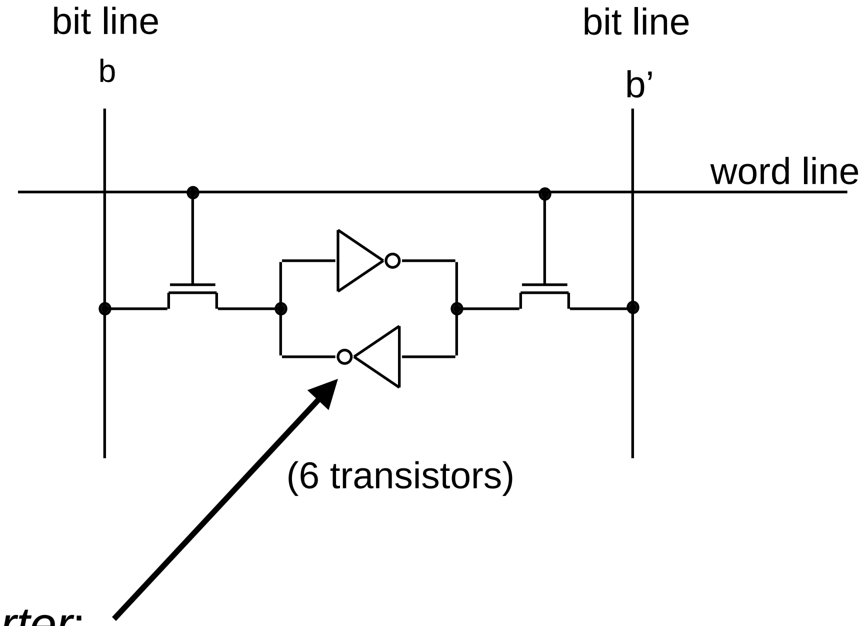
Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



- Straightforward implementation of arithmetic functions

Anatomy of an SRAM Cell



(6 transistors)

Inverter:
High input --> Low Output
Low input --> High Output

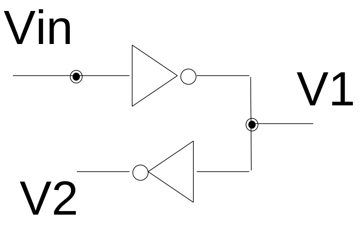
SRAM Cell Principle

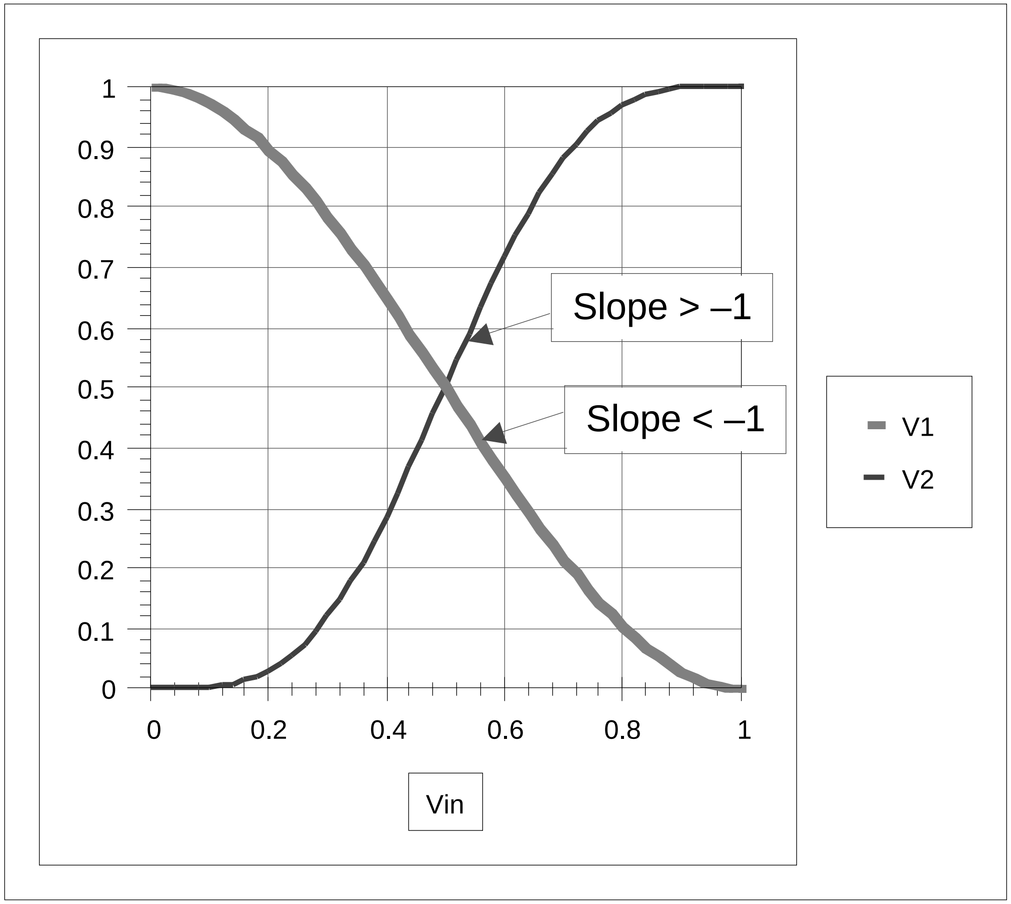
Inverter Amplifies

- Negative gain
- Slope < –1 in middle
- Saturates at ends

Inverter Pair Amplifies

- Positive gain
- Slope > 1 in middle
- Saturates at ends

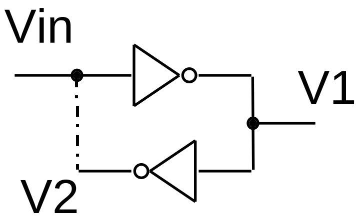




Slope > –1

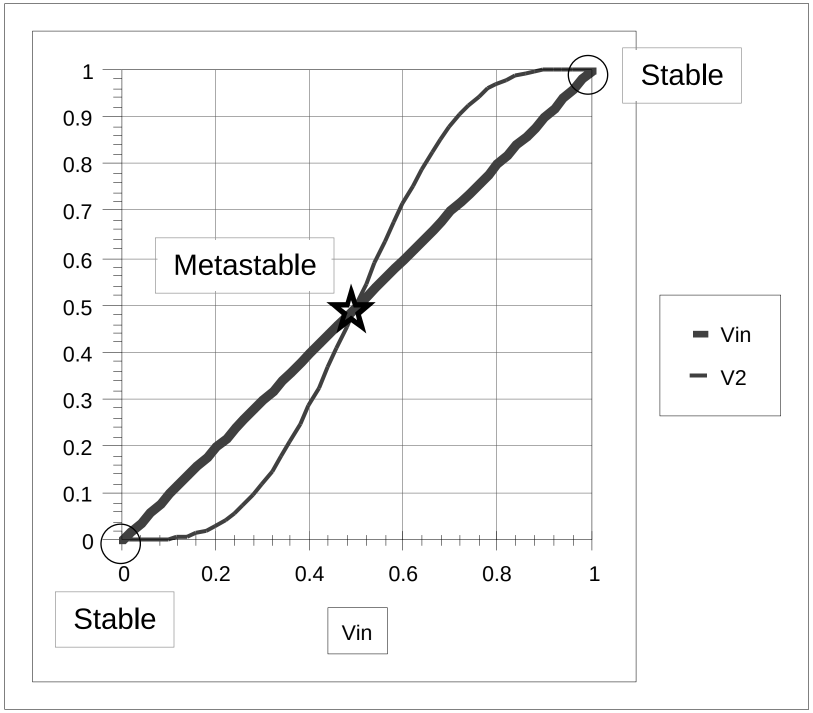
Slope < –1

Bistable Element

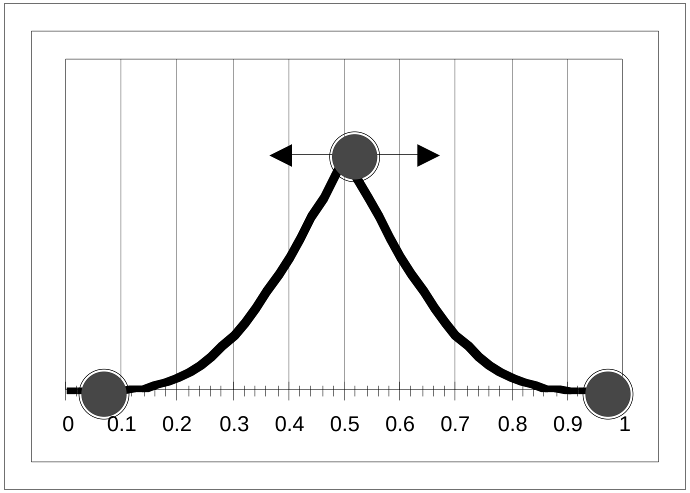


Stability

- **Require Vin = V2**
- **Stable at endpoints**
  - recover from pertubation
- **Metastable in middle**
  - Fall out when perturbed



Ball on Ramp Analogy



Byte-Oriented Memory Organization

Programs Refer to Virtual Addresses

- **Conceptually very large array of bytes**
- **Actually implemented with hierarchy of different memory types**
  - SRAM, DRAM, disk
  - Only allocate for regions actually used by program
- **In Unix and Windows NT, address space private to particular "process"**
  - Program being executed
  - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation

- **Where different program objects should be stored**
- **Multiple mechanisms: static, stack, and heap**
- **In any case, all allocation within single virtual address space**

Encoding Byte Values

Byte = 8 bits

- **Binary** **00000000_2** **to** **11111111_2**
- **Decimal:** **0_{10}** **to** **255_{10}**
- **Hexadecimal** **00_{16}** **to** **FF_{16}**
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Write FA1D37B_{16} in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Machine Words

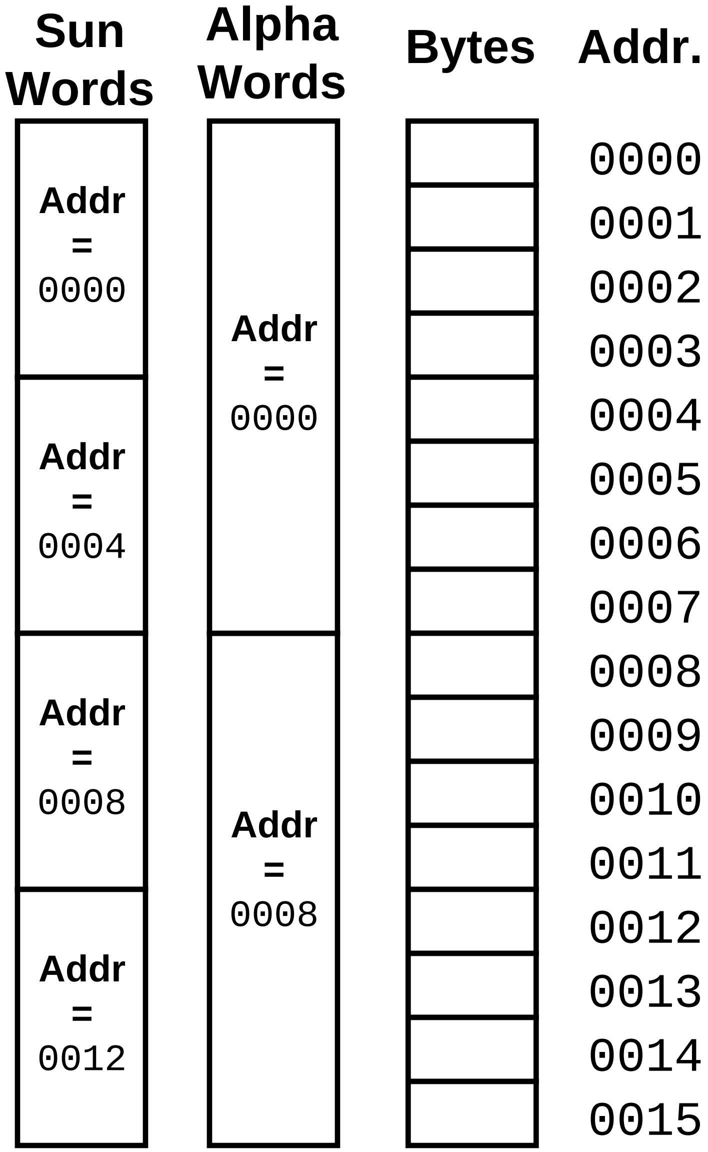
Machine Has “Word Size”

- **Nominal size of integer-valued data**
  - Including addresses
- **Most current machines are 32 bits (4 bytes)**
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- **Our Alphas are 64 bits (8 bytes)**
  - Potentially address 1.8 X 10^{19} bytes
  - Although current machines cannot do this
    - Limit is 4Terabytes
- **Machines support multiple data formats**
  - Fractions or multiples of word size
  - Always integral number of bytes

Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (Sun) or 8 (Alpha)



Sun Words	Alpha Words	Bytes	Addr.
Addr = `0000`	**Addr** = `0000`		`0000`
			`0001`
			`0002`
			`0003`
Addr = `0004`			`0004`
			`0005`
			`0006`
			`0007`
Addr = `0008`	**Addr** = `0008`		`0008`
			`0009`
			`0010`
			`0011`
Addr = `0012`			`0012`
			`0013`
			`0014`
			`0015`

Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Alpha	Sun, PC, Mac, etc.
`int`	4	4
`long int`	8	4
`char`	1	1
`short`	2	2
`float`	4	4
`double`	8	8
`char *`	8	4

» Or any other pointer

Byte Ordering

- **Alphas, PC's are "Little Endian" machines**
  - Least significant byte has lowest address
- **Sun's, Mac's are "Big Endian" machines**
  - Least significant byte has highest address

Examining Data Representations

Code to Print Byte Representation of Data

- **Cast pointer to unsigned char * creates byte array**

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer
%x: Print Hexadecimal

`show_bytes` Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result:

```
int a = 15213;
0x11ffffcb8  0x6d
0x11ffffcb9  0x3b
0x11ffffcba  0x0
0x11ffffcbb  0x0
```

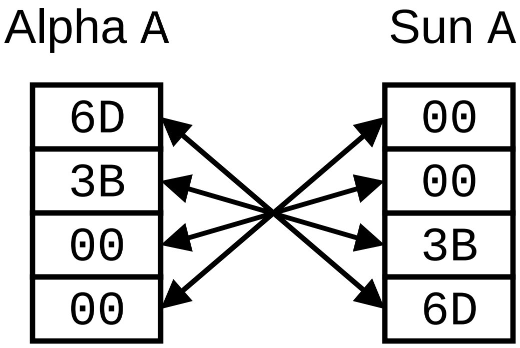
Representing Integers

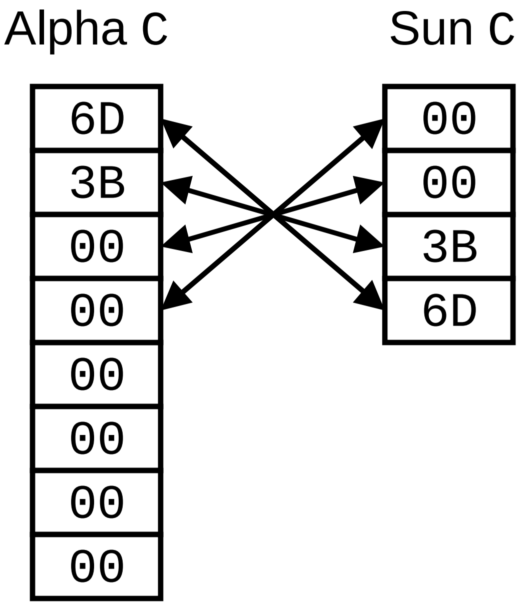
```
int A = 15213;
int B = -15213;
long int C = 15213;
```

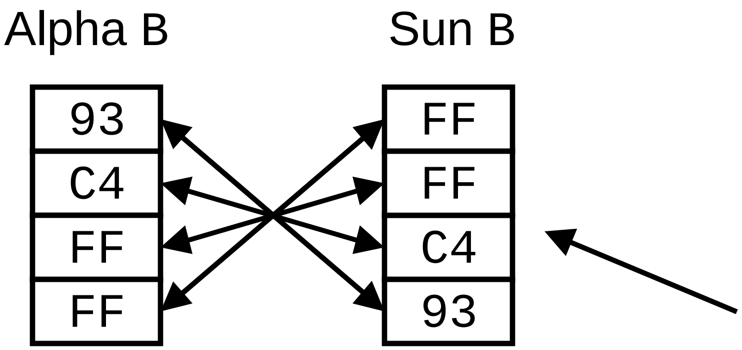
Decimal: 15213

Binary: 0011 1011 0110 1101

Hex: 3 B 6 D







Two’s complement representation
(Covered next lecture)

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Alpha P

A0
FC
FF
FF
01
00
00
00

Alpha Address

Hex:	1	F	F	F	F	F	C	A	0
Binary:	0001	1111	1111	1111	1111	1111	1100	1010	0000

Sun P

EF
FF
FB
2C

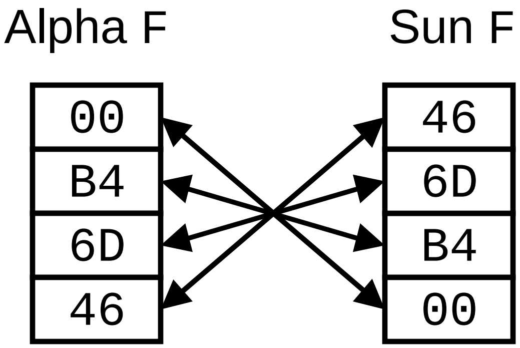
Sun Address

Hex:	E	F	F	F	F	B	2	C
Binary:	1110	1111	1111	1111	1111	1011	0010	1100

Different compilers & machines assign different locations to objects

Representing Floats

`Float F = 15213.0;`



IIEEE Single Precision Floating Point Representation

Hex:	4	6	6	D	B	4	0	0
Binary:	0100	0110	0110	1101	1011	0100	0000	0000
15213:			1110	1101	1011	01		

Not same as integer representation, but consistent across machines

Representing Strings

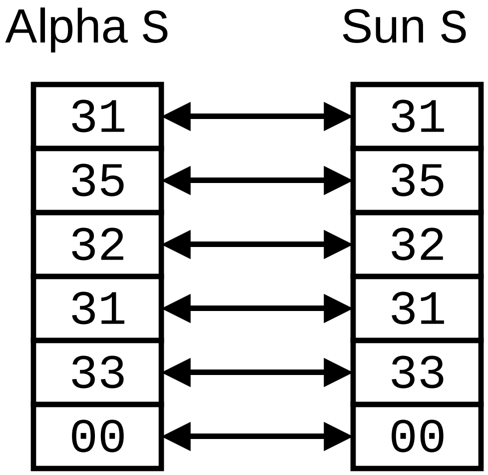
Strings in C

- **Represented by array of characters**
- **Each character encoded in ASCII format**
  - Standard 7-bit encoding of character set
  - Other encodings exist, but uncommon
  - Character “0” has code `0x30`
    - Digit *i* has code `0x30`+*i*
- **String should be null-terminated**
  - Final character = 0

Compatibility

- **Byte ordering not an issue**
  - Data are single byte quantities
- **Text files generally platform independent**
  - Except for different conventions of line termination character!

```
char S[6] = "15213";
```



Alpha S	Sun S
31	31
35	35
32	32
31	31
33	33
00	00

Machine-Level Code Representation

Encode Program as Sequence of Instructions

- **Each simple operation**
  - Arithmetic operation
  - Read or write memory
  - Conditional branch
- **Instructions encoded as bytes**
  - Alpha’s, Sun’s, Mac’s use 4 byte instructions
    - Reduced Instruction Set Computer (RISC)
  - PC’s use variable length instructions
    - Complex Instruction Set Computer (CISC)
- **Different instruction types and encodings for different machines**
  - Most code not binary compatible

Programs are Byte Sequences Too!

Representing Instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

Alpha `sum`	Sun `sum`
00	81
00	C3
30	E0
42	08
01	90
80	02
FA	00
6B	09

- **For this example, both use two 4-byte instructions**
  - Use differing numbers of instructions in other cases

Different machines use totally different instructions and encodings

Boolean Algebra

Developed by George Boole in 19th Century

- **Algebraic representation of logic**
  - Encode “True” as 1 and “False” as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Exclusive-Or (Xor)

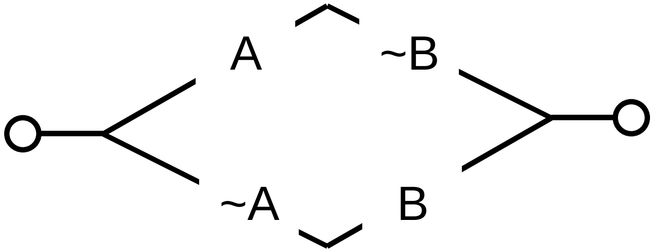
- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

Applied to Digital Systems by Claude Shannon

- **1937 MIT Master's Thesis**
- **Reason about networks of relay switches**
  - Encode closed switch as 1, open switch as 0



Connection when
A&~B | ~A&B
= A^B

Properties of & and | Operations

Integer Arithmetic

- Z, +, *, –, 0, 1 **forms a “ring”**
- **Addition is “sum” operation**
- **Multiplication is “product” operation**
- **– is additive inverse**
- **0 is identity for sum**
- **1 is identity for product**

Boolean Algebra

- {0,1}, |, &, ~, 0, 1 **forms a “Boolean algebra”**
- **Or is “sum” operation**
- **And is “product” operation**
- **~ is “complement” operation (not additive inverse)**
- **0 is identity for sum**
- **1 is identity for product**

Properties of Rings & Boolean Algebras

	Boolean Algebra	Integer Ring
• *Commutativity*		
	$A \mid B = B \mid A$	$A + B = B + A$
	$A \,\&\, B = B \,\&\, A$	$A * B = B * A$
• *Associativity*		
	$(A \mid B) \mid C = A \mid (B \mid C)$	$(A + B) + C = A + (B + C)$
	$(A \,\&\, B) \,\&\, C = A \,\&\, (B \,\&\, C)$	$(A * B) * C = A * (B * C)$
• *Product distributes over sum*		
	$A \,\&\, (B \mid C) = (A \,\&\, B) \mid (A \,\&\, C)$	$A * (B + C) = A * B + B * C$
• *Sum and product identities*		
	$A \mid 0 = A$	$A + 0 = A$
	$A \,\&\, 1 = A$	$A * 1 = A$
• *Zero is product annihilator*		
	$A \,\&\, 0 = 0$	$A * 0 = 0$
• *Cancellation of negation*		
	$\sim(\sim A) = A$	$-(-A) = A$

Ring ≠ Boolean Algebra

Boolean Algebra	Integer Ring
• Boolean: *Sum distributes over product*	
$A \mid (B \,\&\, C) = (A \mid B) \,\&\, (A \mid C)$	$A + (B * C) \neq (A + B) * (B + C)$
• Boolean: *Idempotency*	
$A \mid A = A$	$A + A \neq A$
– "A is true" or "A is true" = "A is true"	
$A \,\&\, A = A$	$A * A \neq A$
• Boolean: *Absorption*	
$A \mid (A \,\&\, B) = A$	$A + (A * B) \neq A$
– "A is true" or "A is true and B is true" = "A is true"	
$A \,\&\, (A \mid B) = A$	$A * (A + B) \neq A$
• Boolean: *Laws of Complements*	
$A \mid {\sim}A = 1$	$A + –A \neq 1$
– "A is true" or "A is false"	
• Ring: *Every element has additive inverse*	
$A \mid {\sim}A \neq 0$	$A + –A = 0$

Properties of & and ^

Boolean Ring

- {0,1}, ^, &, I, 0, 1
- **Identical to integers mod 2**
- I **is identity operation:** I **(A) = A**
  - A ^ A = 0

Property	Boolean Ring
Commutative sum	A ^ B = B ^ A
Commutative product	A & B = B & A
Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
Associative product	(A & B) & C = A & (B & C)
Prod. over sum	A & (B ^ C) = (A & B) ^ (B & C)
0 is sum identity	A ^ 0 = A
1 is prod. identity	A & 1 = A
0 is product annihilator	A & 0 = 0
Additive inverse	A ^ A = 0

Relations Between Operations

DeMorgan’s Laws

- **Express & in terms of |, and vice-versa**

  A & B = ~(~A | ~B)

  » A and B are true if and only if neither A nor B is false

  A | B = ~(~A & ~B)

  » A or B are true if and only if neither A is false nor B is false

Exclusive-Or using Inclusive Or

A ^ B = (~A & B) | (A & ~B)

» Exactly one of A and B is true

A ^ B = (A | B) & ~(A & B)

» Either A is true, or B is true, but not both

General Boolean Algebras

Operate on Bit Vectors

- **Operations applied bitwise**

```
  01101001       01101001       01101001
& 01010101     | 01010101     ^ 01010101      ~ 01010101
  01000001       01111101       00111100        10101010
```

Representation of Sets

- **Width** *w* **bit vector represents subsets of** {0, …, *w*–1}
- a_j = 1 **if** *j*   *A*
  - `01101001`   { 0, 3, 5, 6 }
  - `01010101`   { 0, 2, 4, 6 }
- **&** **Intersection** **`01000001`** { 0, 6 }
- **|** **Union** **`01111101`** { 0, 2, 3, 4, 5, 6 }
- **^** **Symmetric difference** **`00111100`** { 2, 3, 4, 5 }
- **~** **Complement** **`10101010`** { 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- **Apply to any “integer” data type**
  - `long int`, `int`, `short`, `char`
- **View arguments as bit vectors**
- **Arguments applied bit-wise**

Examples (Char data type)

- **`~0x41 --> 0xBE`**

  `~`01000001_2 `-->` 10111110_2
- **`~0x00 --> 0xFF`**

  `~`00000000_2 `-->` 11111111_2

- **`0x69 & 0x55 --> 0x41`**

  01101001_2 `&` 01010101_2 `-->` 01000001_2
- **`0x69 | 0x55 --> 0x7D`**

  01101001_2 `|` 01010101_2 `-->` 01111101_2

Contrast: Logic Operations in C

Contrast to Logical Operators

- **&&, ||, !**
  - View 0 as “False”
  - Anything nonzero as “True”
  - Always return 0 or 1

Examples (char data type)

- `!0x41  -->  0x00`
- `!0x00  -->  0x01`
- `!!0x41 -->  0x01`

- `0x69 && 0x55  -->  0x01`
- `0x69 || 0x55  -->  0x01`

Shift Operations

Left Shift: x << y

- **Shift bit-vector x left y positions**
    - – Throw away extra bits on left
    - – Fill with 0’s on right

Right Shift: x >> y

- **Shift bit-vector x right y positions**
    - – Throw away extra bits on right
- **Logical shift**
    - – Fill with 0’s on left
- **Arithmetic shift**
    - – Replicate most significant bit on right
    - – Useful with two’s complement integer representation

Argument x	`01100010`
`<< 3`	00010*000*
Log. >> `2`	*00*011000
Arith. >> `2`	*00*011000

Argument x	`10100010`
`<< 3`	00010*000*
Log. >> `2`	*00*101000
Arith. >> `2`	*11*101000

Cool Stuff with Xor

- **Bitwise Xor is form of addition**
- **With extra property that every value is its own additive inverse**
  - A ^ A = 0

```
void funny(int *x, int *y)
{
   *x = *x ^ *y;    /* #1 */
   *y = *x ^ *y;    /* #2 */
   *x = *x ^ *y;    /* #3 */
}
```

Step	`*x`	`*y`
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A^(B^B) = A^0 = A
3	(A^B)^A = (B^A)^A = B^(A^A) = B^0 = B	A
End	B	A