Representing Data

15-213: Introduction to Computer Systems
Recitation 3: Monday, Sept. 9th, 2013

Marjorie Carlson
Section A

Welcome to 15-213 (Belatedly!)

- Yay 15-213!

- My advice on doing well in this course:
  - Labs: start early.
  - Exams: **you should already be doing practice exam questions**.
    - Previous exam questions and answers are all online.
    - Questions don't change much from semester to semester.
    - If you do the exam questions related to each week's topic as you go, you'll know all the material by exam time.
  - The textbook is actually really useful. (!)
  - General advice: this course is a good place to get more comfortable with UNIX and C, if you aren't already.

Welcome to Recitation

- Recitation is a place for interaction
  - If you have questions, please ask.
  - If you want to go over an example not planned for recitation, let me know.
- Each week we'll cover:
  - A quick recap of topics from class, especially ones we have found students struggled with in the past.
  - Tips for labs.
  - Sample problems to reinforce the main ideas and prepare for exams.

Agenda

- How Do I Data Lab?
- Integers
  - Biasing division
- Floats
  - Binary fractions
  - IEEE standard
  - Example problem

How Do I Data Lab?

(due Thursday at 11:59 pm)

▪Step 1: Download lab files

- All lab files are on autolab
- Remember to also read the lab handout ("view writeup" link)

▪Step 2: Work on the right machines

- Remember to do all your lab work on Shark machines
- **This includes untarring the handout**. Otherwise, you may lose some permissions bits
- If you get a permission denied error, try "chmod +x *filename*"
- Do your work in `bits.c`

How Do I Data Lab?

- **Step 3: Test test test!**
  We have given you FOUR WAYS to test your code before submitting!
  - `./btest` lets you debug (printf, test single inputs).
    Type `make` before using it.
  - `./dlc bits.c` enforces the coding rules (number of operations).
  - `./bddcheck/check.pl` tests *definitively* for correctness.
  - `./driver.pl` uses both DLC and the BDD checker – this is what Autolab uses.

- **Code that passes btest will not necessarily pass autolab!**

How Do I Data Lab?

- **Step 4: Submit to Autolab**
  - Unlimited submissions, but please don't use autolab in place of `driver.pl`
  - Must submit via web form
  - To package/download files to your computer, use `tar -cvzf` *`out.tar.gz in1 in2`* … (if relevant) and your favorite file transfer protocol

How Do I Data Lab? – Tips!

- Write C like it's 1989 (for DLC – only used in data lab)
  - Declare variable at top of function
  - Make sure closing brace ("}") is in 1st column
- Be careful of operator precedence
  - Do you know what order `~a+1+b*c<<3*2` will execute in?
  - Neither do I. Use parentheses: `(~a)+1+(b*(c<<3)*2)`
- Take advantage of special values like 0, -1, and T_{min}
- Operations with undefined behavior in C may have defined behavior on our architecture. (Examples: addition overflow, bit-shifting by 32.) It's OK to use them.
- Reducing operations once you're under the threshold won't get you extra points (just more glory).

Where Can I Get Help?

- The assignment writeup
- The assignment writeup!
- The assignment writeup!!!!!!!!
- 15-213 FAQ: http://www.cs.cmu.edu/~213/faq.html
- Lecture notes and the textbook
- Staff email list: 15-213-staff@cs.cmu.edu
- Office hours: Sun-Thu, 5:30-8:30 pm, in Wean 5027
- Peer tutoring: Tue 8:30-11, Mudge Reading Room

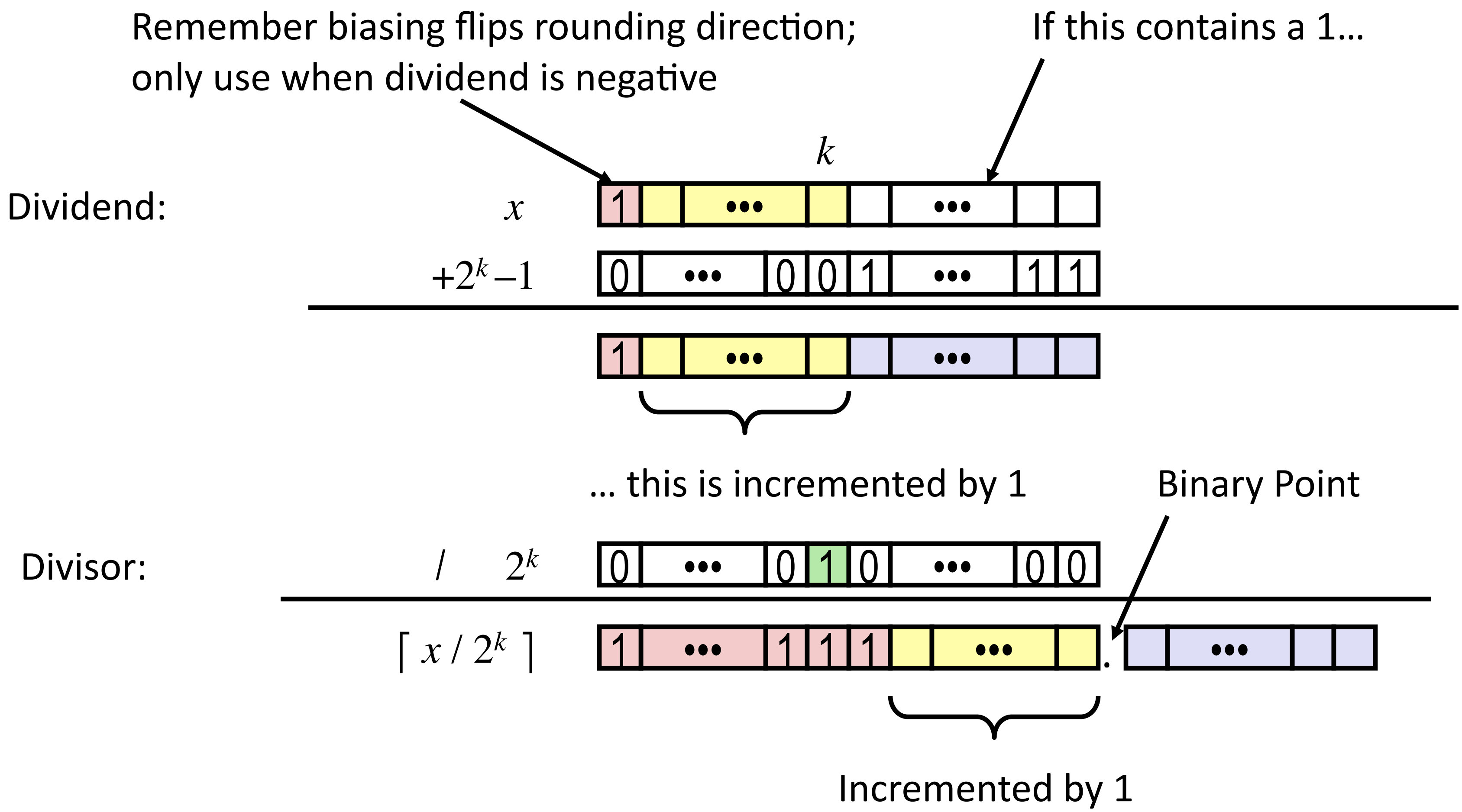
Agenda

- How Do I Data Lab?
- **Integers**
  - Biasing division
- Floats
  - Binary fractions
  - IEEE standard
  - Example problem

Integers – Biasing

- You can multiply and divide by powers of 2 with bitshifts
- As you'll see when we learn assembly, your computer does this a lot!
  - Multiply:
    - Left shift by k to multiply by 2^k
    - Let's try this with binary `00010`
  - Divide:
    - Right shift by k to divide by 2^k… sort of
    - Let's try this with binary `01111`
    - How about binary `10001`
    - Uh-oh!
    - Shifting rounds *down*, but we want to round *toward zero.*
    - Solution: biasing when the number is negative

Integers – Biasing



Biasing adds 1 to final result

Agenda

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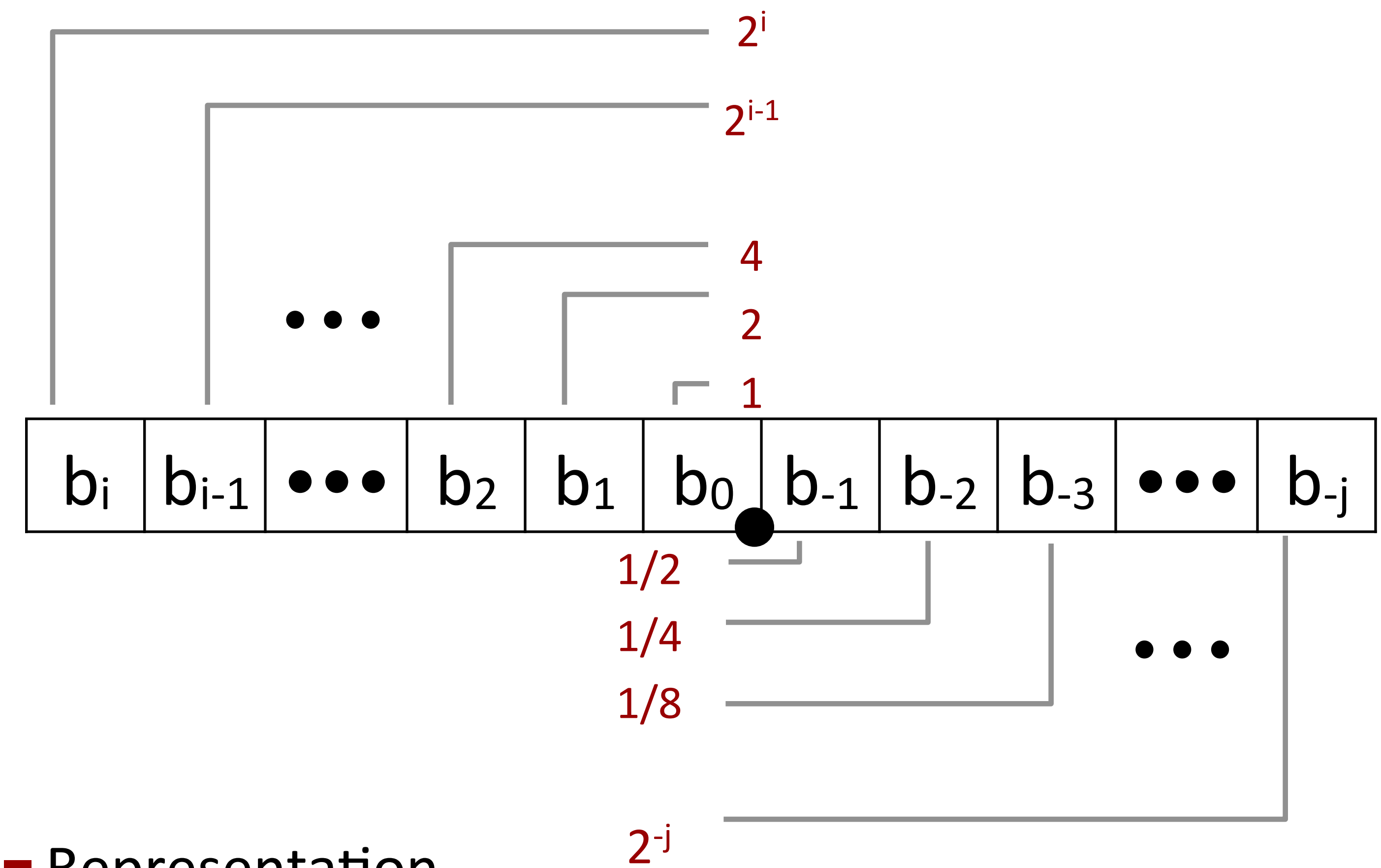
Floating Point – Fractions in Binary



- Representation
  - Bits to right of "binary point" represent fractional powers of 2
  - Represents rational number: $\sum_{k=-j}^{i} b_k \times 2^k$

Floating Point – Fractions in Binary

- Convert binary to decimal:
  - `1.1`
  - `0.0011`
  - `1010.00101`

- Convert decimal to binary:
  - `3 3/4`
  - `2 3/32`
  - `5.875`

How Can We Represent Numbers Efficiently?

- What do we do if we want to convey

-592349235823740180.3

in 10 digits?

How Can We Represent Numbers Efficiently?

- What do we do if we want to convey

-5923492358237401803

in 10 digits?

Hint:

$$-\_\,\_\,\_\,\_\,\_\,\_ \;*\; \_\,\_^{\_\,\_}$$

How Can We Represent Numbers Efficiently?

- What do we do if we want to convey

$$-59234923582374018 0.3$$

in 10 digits?

$$-\underbrace{5.92349}_{\text{mantissa}} * 10^{17}$$

sign exponent

Floating Point – Scientific Notation

- So, how can we put binary numbers into scientific notation?

$$101.111$$

$$\boxed{}\ \underbrace{1.01111}_{\text{mantissa (M)}} * 2^{2}$$

sign (S) mantissa (M) exponent (E)

- Numerical form: $(-1)^S\ M\ 2^E$

Floating Point – IEEE Standard

- Floating points are basically a way to **encode** binary scientific notation.

s	exp	frac
1	8 bits	23 bits

- But `exp` ≠ E and `frac` ≠ M, because IEEE format optimizes to increase the range of numbers that can be represented.
  - If numbers are always in the format 1.xxxx (we'll revisit this!), encoding the 1 is unnecessary. So `frac` is simply M without the leading 1. **`M = 1 + frac`**
  - `exp` is unsigned and can represent the numbers 0 to 255. We'd rather have it represent -127(ish) to 128(ish), so we subtract a **bias** of 127 (**2^{k-1}-1**) get from E to get exp. **`E = exp - bias`**

Floating Point – IEEE Standard

```
101.111
```

$$\boxed{}\ \underbrace{1.01111}_{\text{mantissa (M)}} * 2^{2}$$

sign (S) mantissa (M) exponent (E)

- S = + so s = 0
- E = 2 so exp =
- M = 1.01111 so frac =

```
Remember!
M = 1 + frac
E = exp – bias
Bias = 127
```

Floating Point – IEEE Standard

$$101.111$$

$$\boxed{}\;\underbrace{1.01111}_{\text{mantissa (M)}} * 2^{2}$$

sign (S)

mantissa (M)

exponent (E)

- S = + so s = 0
- E = 2 so exp = 129 (2 + bias)
- M = 1.01111 so frac =

Remember!

```
M = 1 + frac
E = exp - bias
Bias = 127
```

Floating Point – IEEE Standard

$$101.111$$

$$\boxed{}\;\underbrace{1.01111}_{\text{mantissa (M)}} * 2^{2}$$

sign (S) mantissa (M) exponent (E)

- S = + so s = 0
- E = 2 so exp = 129 (2 + bias)
- M = 1.01111 so frac = 01111

```
Remember!
M = 1 + frac
E = exp – bias
Bias = 127
```

Floating Point – Example

- Consider the following 5-bit floating point representation based on the IEEE floating point format. This format does not have a sign bit – it can only represent nonnegative numbers.
  - There are k=3 exponent bits.
  - There are n=2 fraction bits.

4	3	2	1	0
exp			frac	

- What's the bias?
- What does `100 10` represent?
- What does `001 01` represent?
- How would you represent 6?
- How would you represent ¼?

Floating Point – Denormalized Range

- If that was all there was to it, the smallest number representable would be 2^{-bias}, which is not that small. And it would be represented by `000 00`. Hmm…
- IEEE uses a trick to give us numbers closer to 0: **drop the implied leading 1.**

Normalized	Denormalized
exp ≠ 0	exp = 0
implied leading 1	**no implied leading 1**
E = exp - bias	E = **1 – bias**
denser near origin	evenly spaced
represents most numbers	represents very small numbers

Floating Point – Special Cases

- Well, denormalizing got us our 0. Now how about infinity?

- The largest exponent is coopted to encode special cases:
  - `exp` = all 1s
    `frac` = all 0s
    represents infinity (+ or -)

  - `exp` = all 1s
    `frac` **isn't** all 0s
    represents NaN

Floating Point – Special Case Examples

- Back to our mini-floats:
  - There are k=3 exponent bits.
  - There are n=2 fraction bits.
  - Bias = 3

4	3	2	1	0
exp			frac	

- What does `000 10` represent?
- What's the smallest representable nonzero value?
- What's the largest representable finite number?
- What's the smallest normalized number?
- What's the largest denormalized number?

Two More Tips and You Can Convert Anything!

- Decimal → float is a little trickier because you have to figure out whether it has to be encoded as normalized or denormalized.
  - Strategy 1: compare your number to the smallest normalized number before converting it.
  - Strategy 2: try to encode it as normalized; if your exponent doesn't fit in exp, change exp to 0 and shift your decimal point accordingly.

- You need to know how to round!

Floating Point – Rounding

- Floats round to even
  - Why? Avoid statistical bias of always rounding up or down.
  - How? Like this:

1.0100_2	truncate	1.01_2
1.0101_2	below half; round down	1.01_2
1.0110_2	interesting case; round to even	1.10_2
1.0111_2	above half; round up	1.10_2
1.1000_2	truncate	1.10_2
1.1001_2	below half; round down	1.10_2
1.1010_2	Interesting case; round to even	1.10_2
1.1011_2	above half; round up	1.11_2
1.1100_2	truncate	1.11_2

Floating Point – Rounding Examples

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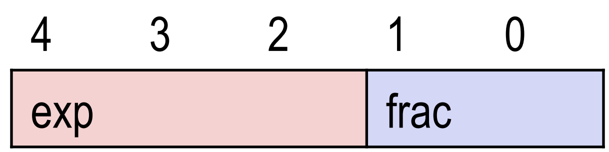

Value	Floating Point Bits	Rounded Value
9/32		
8		
9		
19		

Floating Point – Rounding Examples

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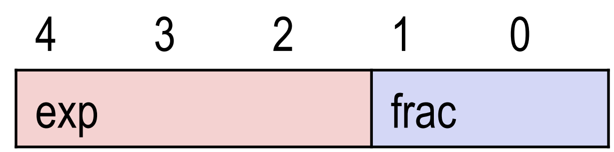

Value	Floating Point Bits	Rounded Value
9/32	**`001 00`**	**1/4**
8	**`110 00`**	**8**
9	**`110 00`**	**8**
19	**`111 00`**	**+ inf**

Questions?