

15-213

“The course that gives CMU its Zip!”

Cache Memories

October 7, 2004

Topics

- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance

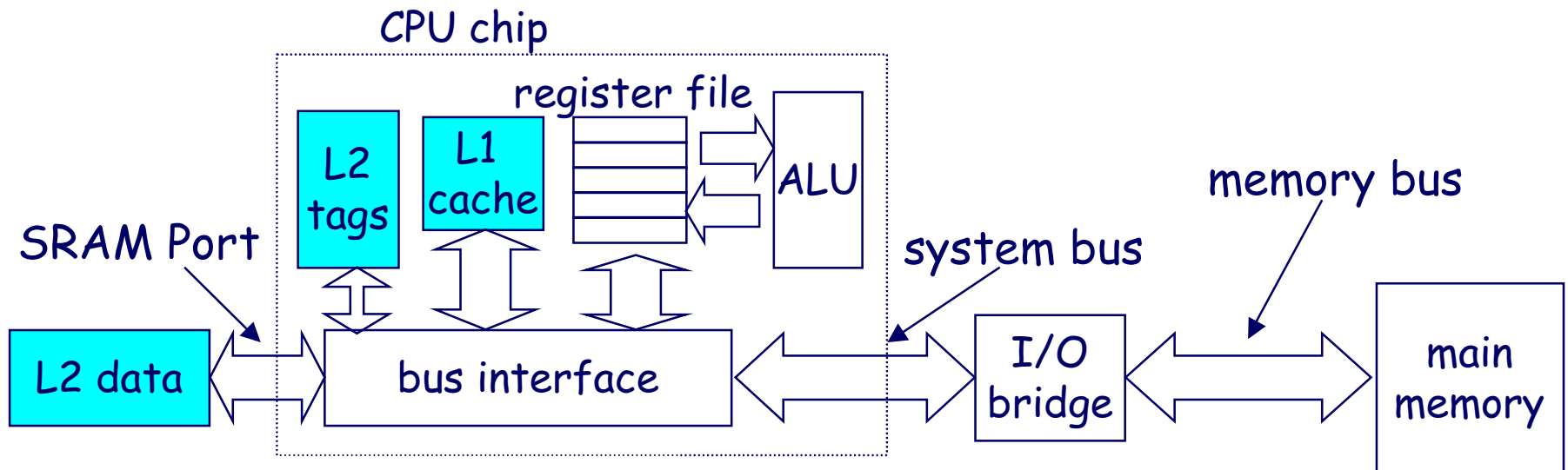
Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware.

- Hold frequently accessed blocks of main memory

CPU looks first for data in L1, then in L2, then in main memory.

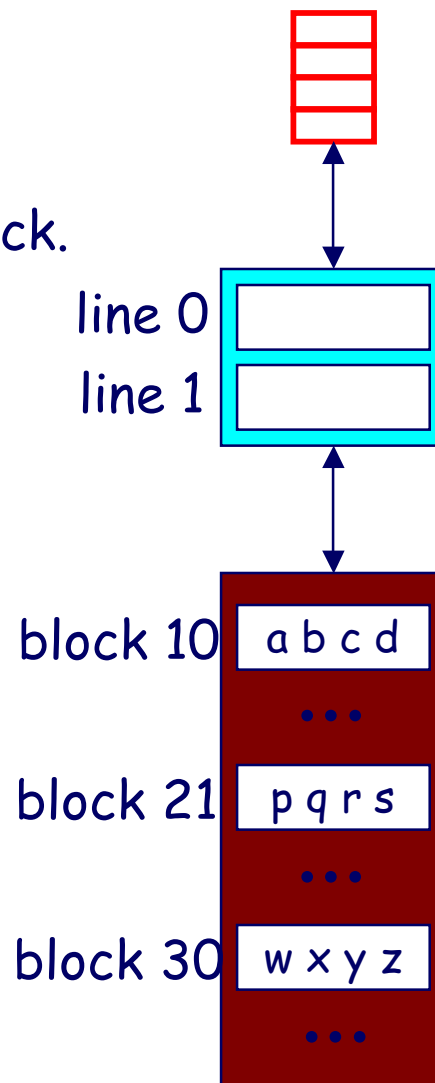
Typical system structure:



Inserting an L1 Cache Between the CPU and Main Memory

The transfer unit between the CPU **register file** and the **cache** is a 4-byte block.

The transfer unit between the **cache** and **main memory** is a 4-word block (16 bytes).



The tiny, very fast CPU **register file** has room for four 4-byte words.

The small fast **L1 cache** has room for two 4-word blocks.

The big slow **main memory** has room for many 4-word blocks.

General Organization of a Cache

Cache is an array of sets.

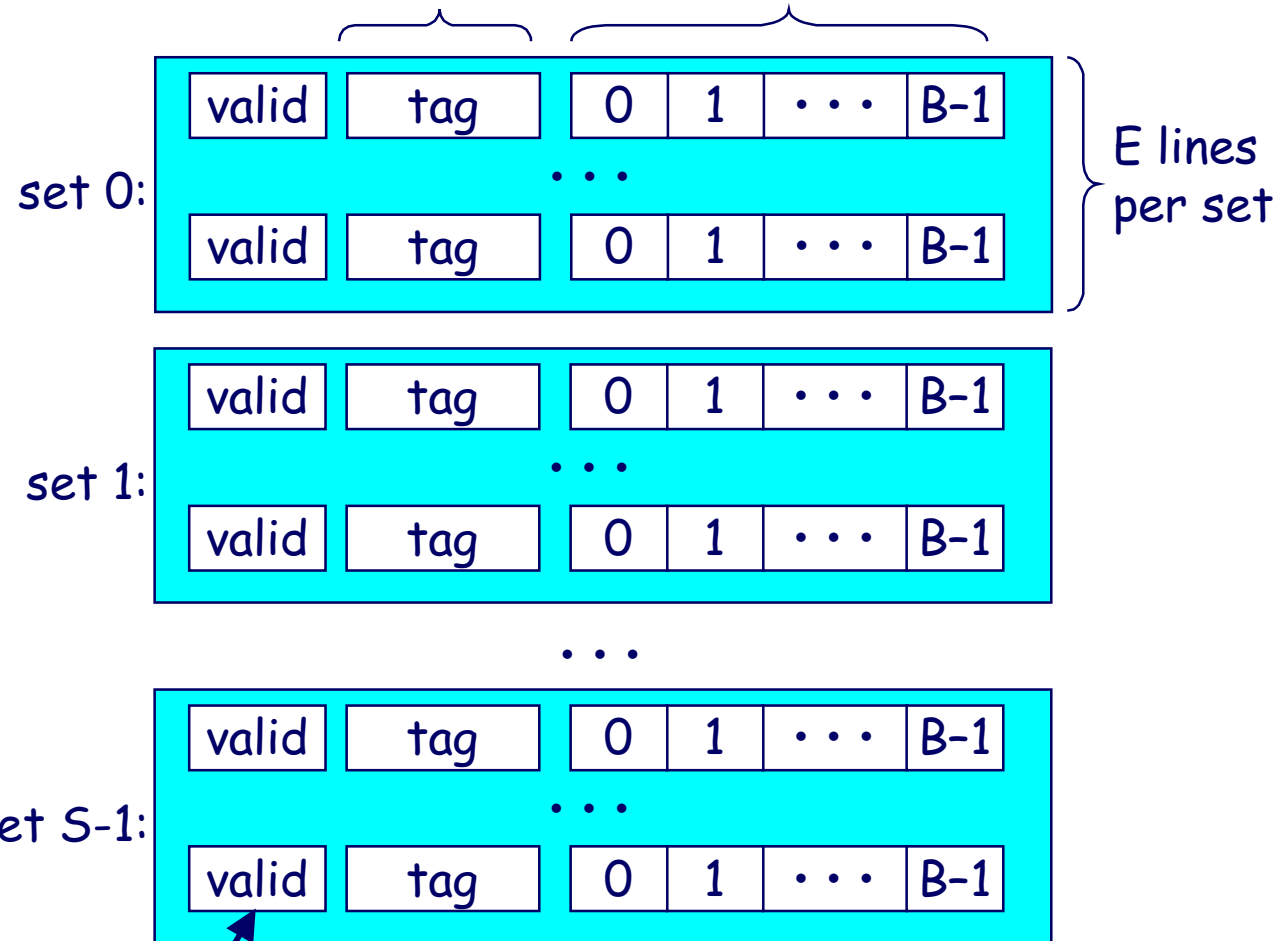
Each set contains one or more lines.

Each line holds a block of data.

$S = 2^s$ sets

t tag bits
per line

$B = 2^b$ bytes
per cache block



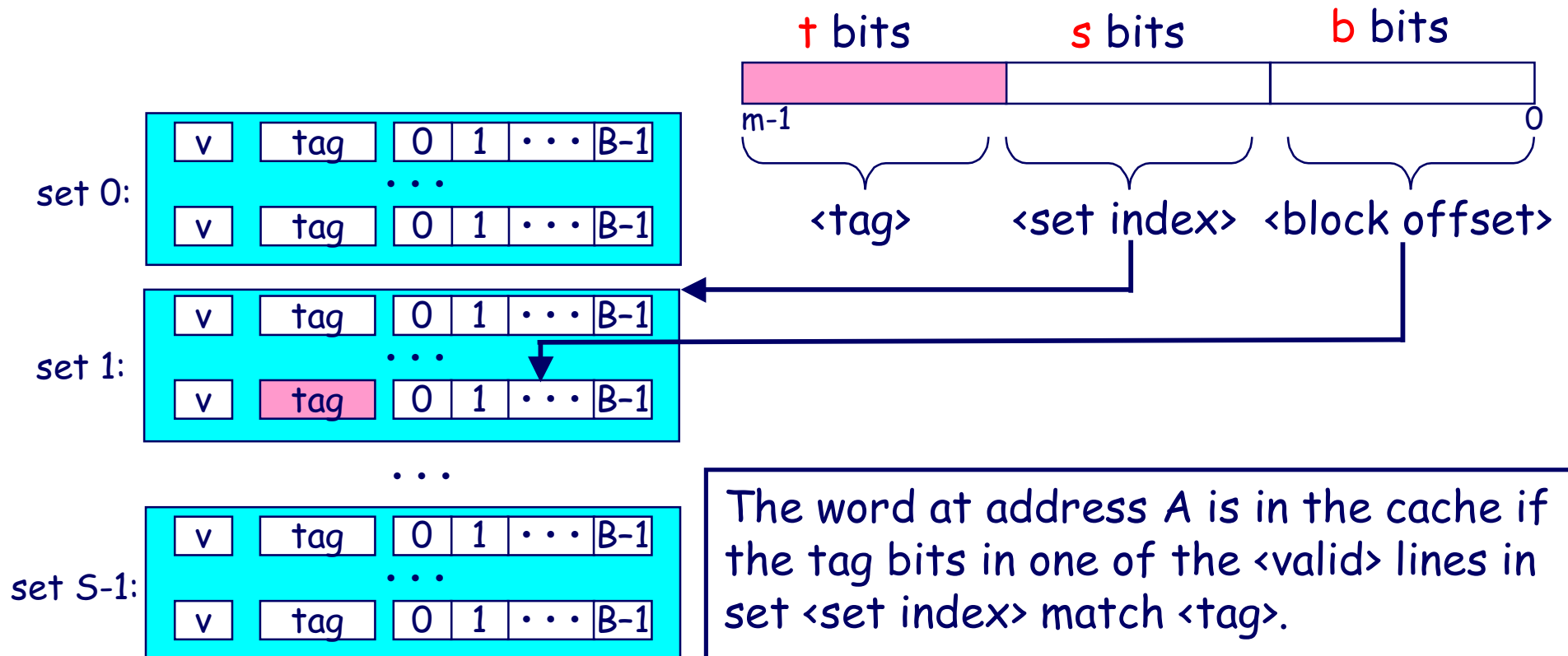
1 valid bit per line

Cache size: $C = B \times E \times S$ data bytes

15-213, F'04

Addressing Caches

Address A:

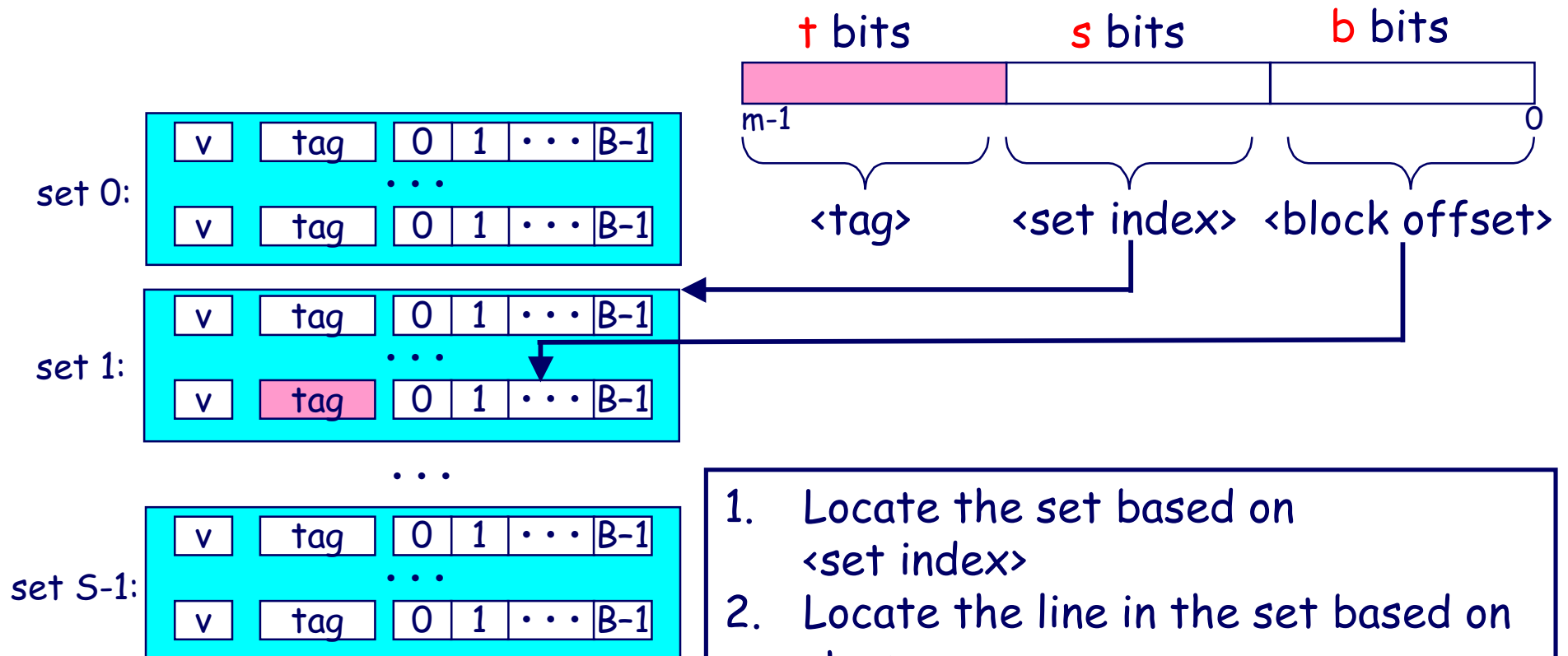


The word at address A is in the cache if the tag bits in one of the <valid> lines in set <set index> match <tag>.

The word contents begin at offset <block offset> bytes from the beginning of the block.

Addressing Caches

Address A:

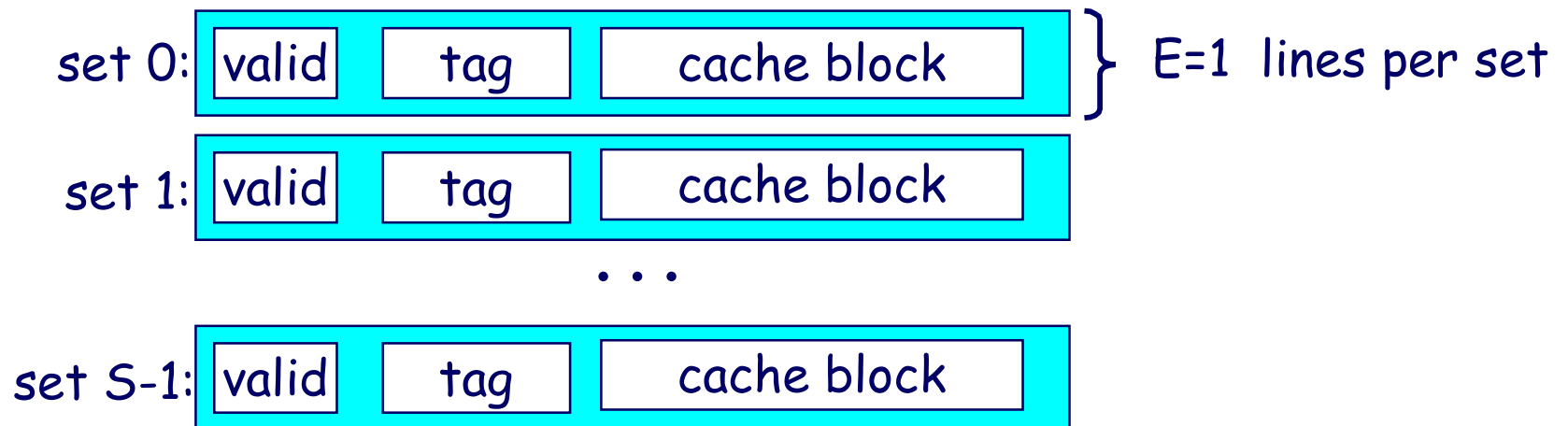


1. Locate the set based on **<set index>**
2. Locate the line in the set based on **<tag>**
3. Check that the line is valid
4. Locate the data in the line based on **<block offset>**

Direct-Mapped Cache

**Simplest kind of cache, easy to build
(only 1 tag compare required per access)**

Characterized by exactly one line per set.

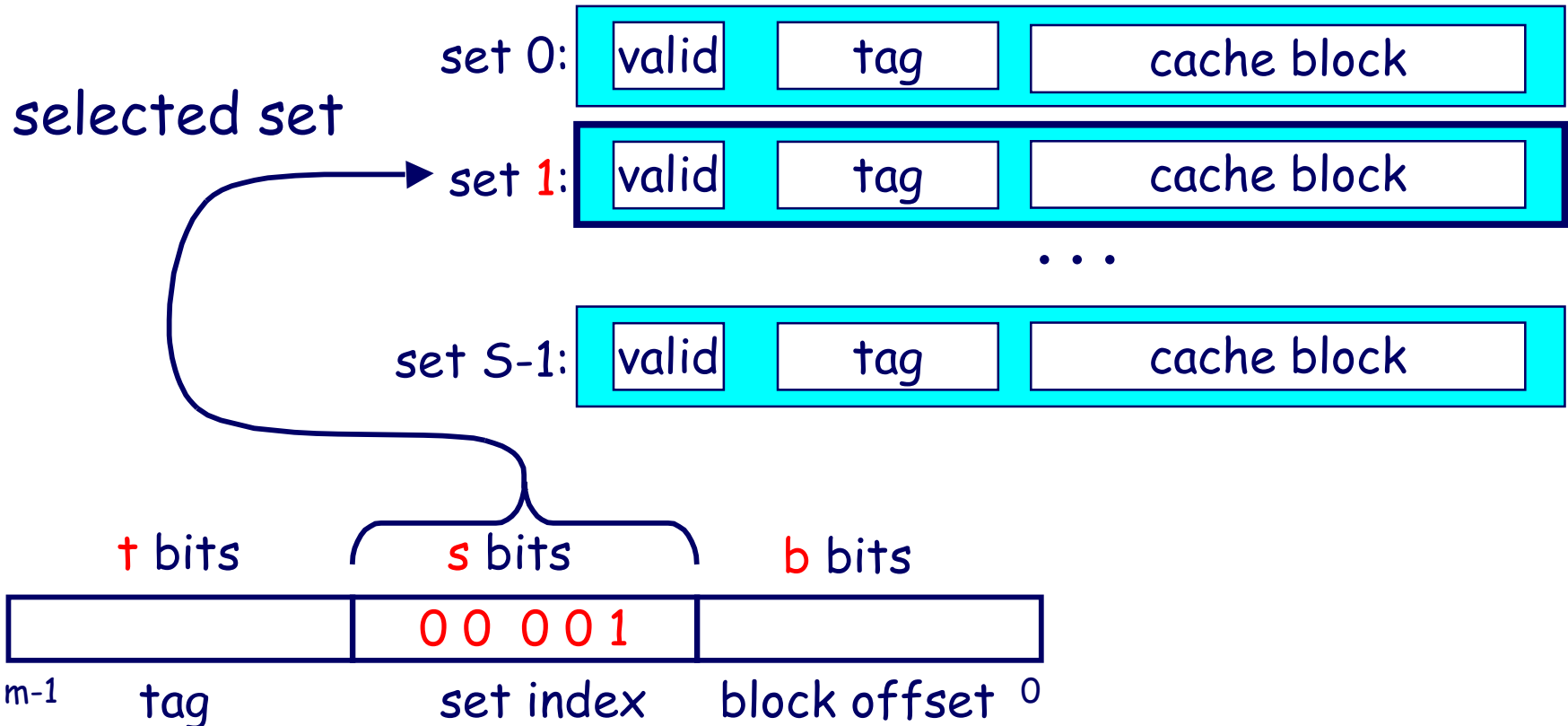


Cache size: $C = B \times S$ data bytes

Accessing Direct-Mapped Caches

Set selection

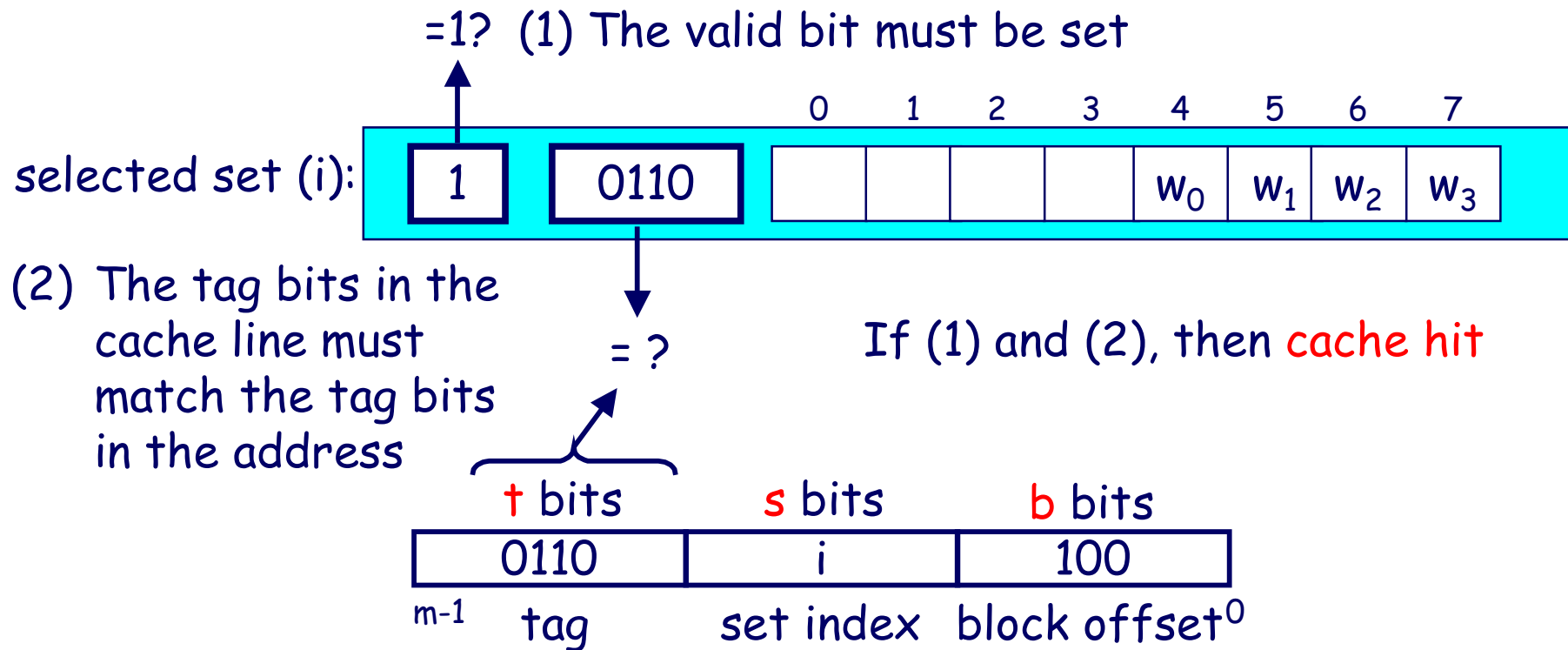
- Use the set index bits to determine the set of interest.



Accessing Direct-Mapped Caches

Line matching and word selection

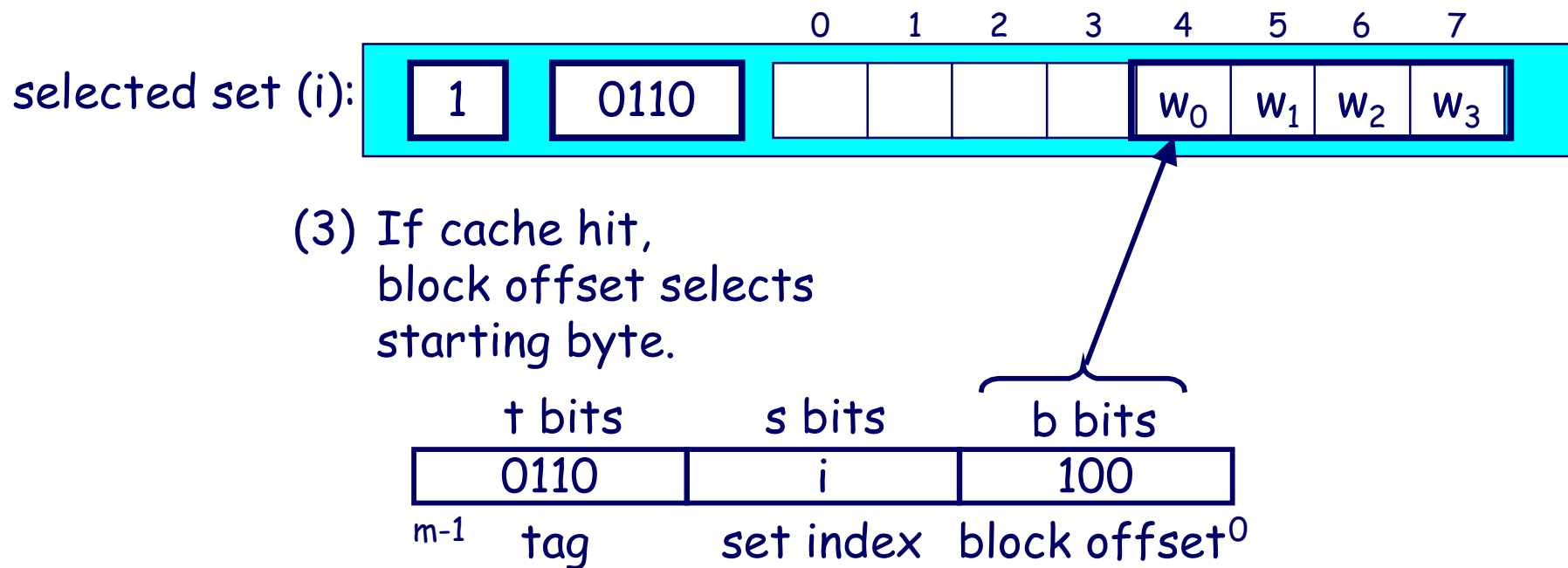
- **Line matching:** Find a valid line in the selected set with a matching tag
- **Word selection:** Then extract the word



Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching:** Find a valid line in the selected set with a matching tag
- **Word selection:** Then extract the word



Direct-Mapped Cache Simulation

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 entry/set

t=1 s=2 b=1

x	xx	x
---	----	---

Address trace (reads):

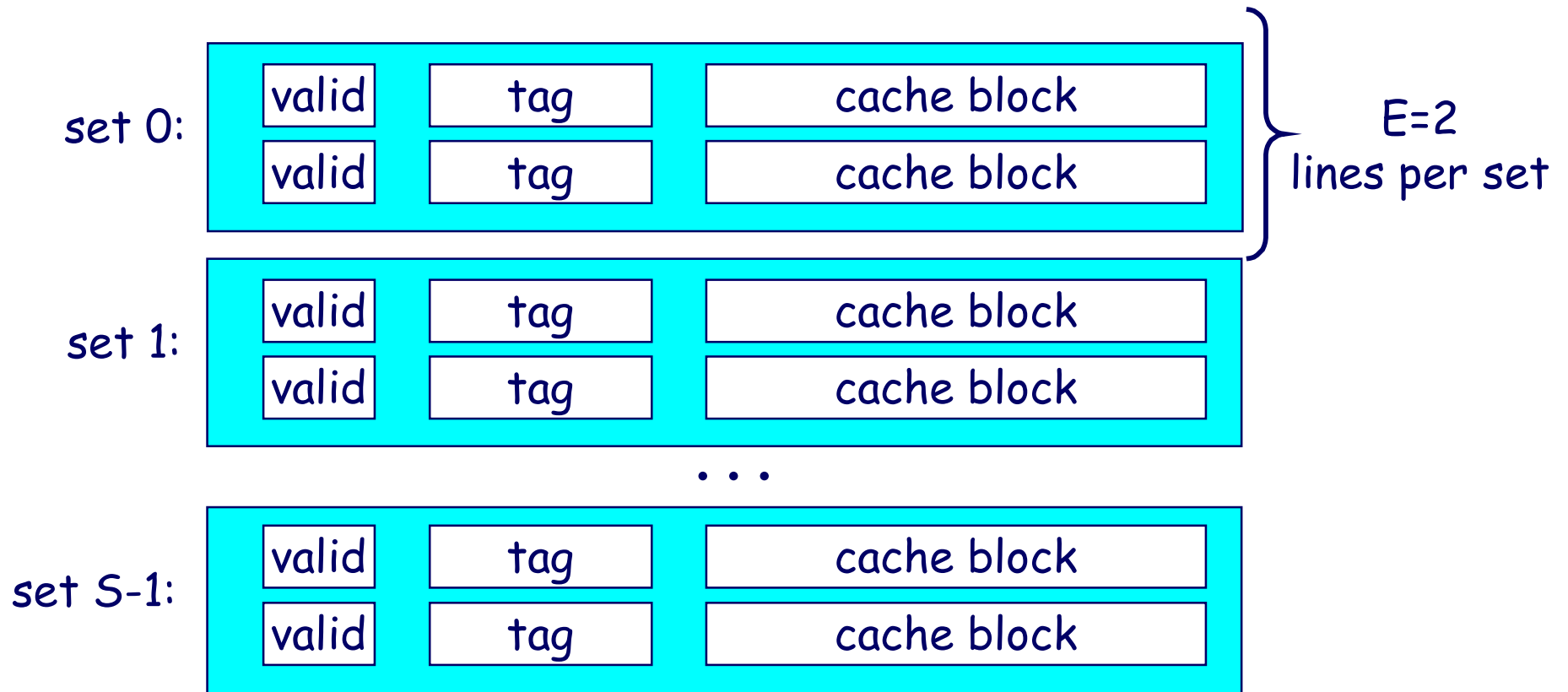
0	[0000 ₂]	miss
1	[0001 ₂]	hit
7	[0111 ₂]	miss
8	[1000 ₂]	miss
0	[0000 ₂]	miss

v	tag	data
---	-----	------

1	0	M[0-1]
1	0	M[6-7]

Set Associative Caches

Characterized by more than one line per set

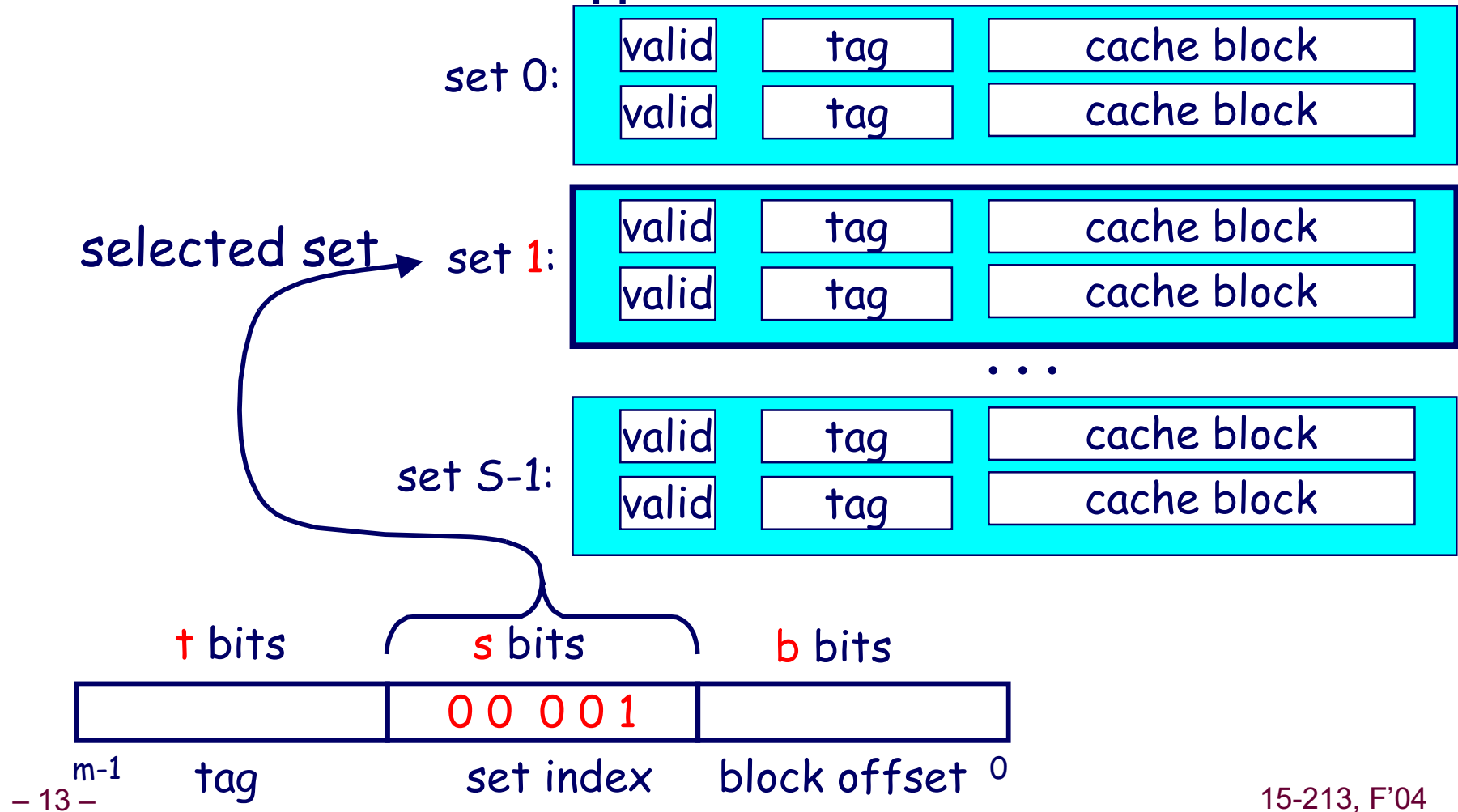


E-way associative cache

Accessing Set Associative Caches

Set selection

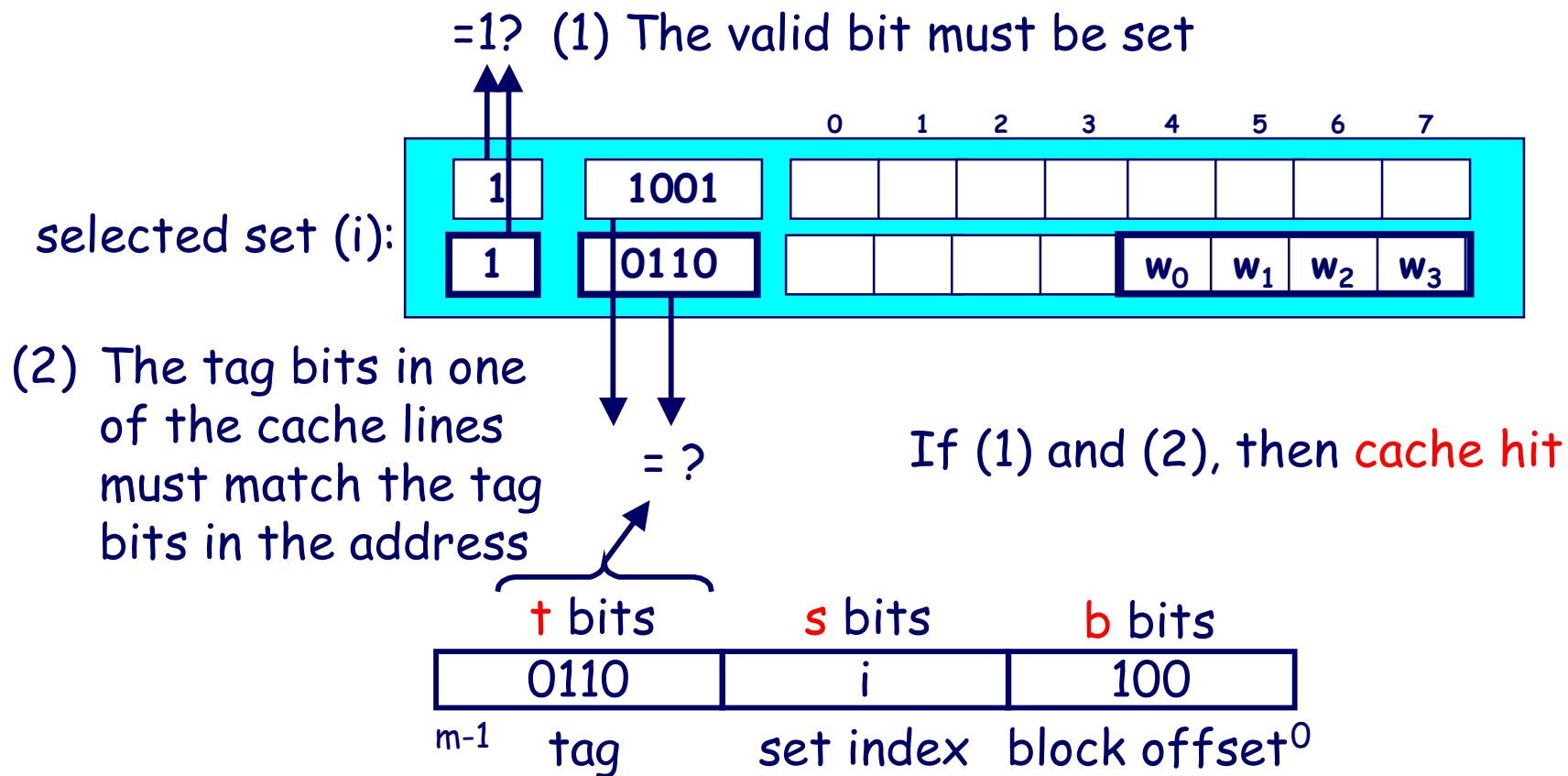
- identical to direct-mapped cache



Accessing Set Associative Caches

Line matching and word selection

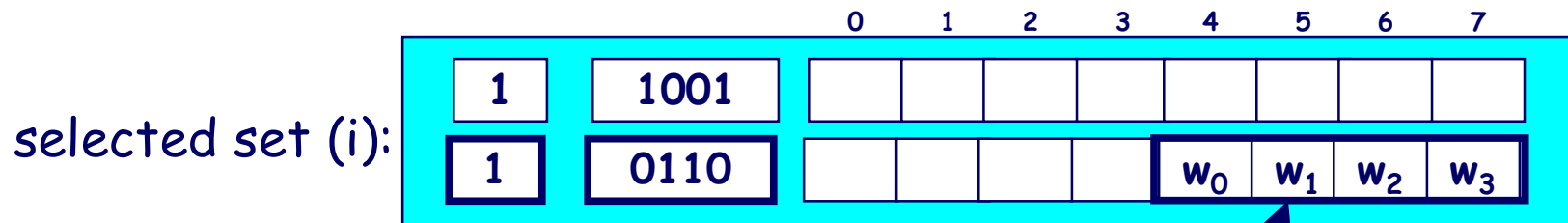
- must compare the tag in each valid line in the selected set.



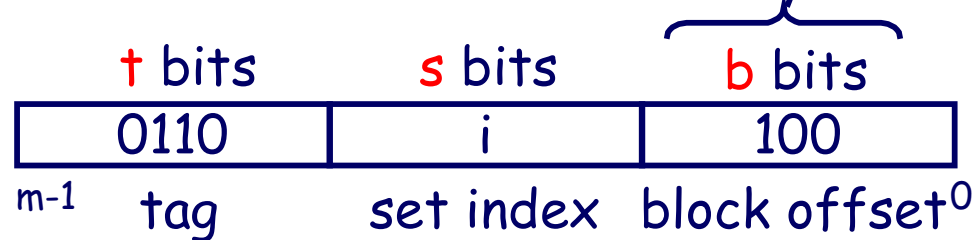
Accessing Set Associative Caches

Line matching and word selection

- Word selection is the same as in a direct mapped cache



- (3) If cache hit,
block offset selects
starting byte.



2-Way Associative Cache Simulation

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 entry/set

t=2 s=1 b=1

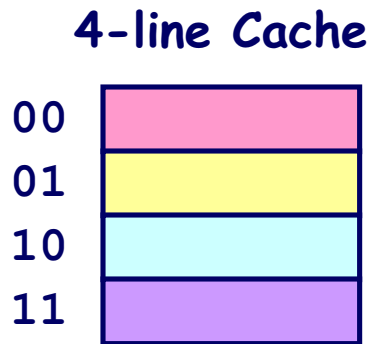
xx	x	x
----	---	---

Address trace (reads):

0	[0000 ₂]	miss
1	[0001 ₂]	hit
7	[0111 ₂]	miss
8	[1000 ₂]	miss
0	[0000 ₂]	hit

v	tag	data
1	00	M[0-1]
1	10	M[8-9]
1	01	M[6-7]
0		

Why Use Middle Bits as Index?



High-Order Bit Indexing

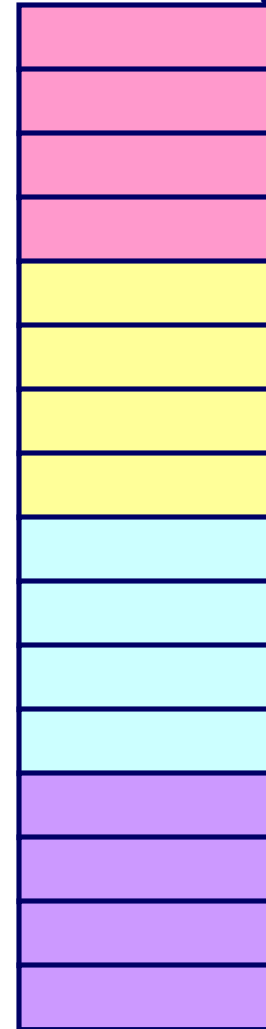
- Adjacent memory lines would map to same cache entry
- Poor use of spatial locality

Middle-Order Bit Indexing

- Consecutive memory lines map to different cache lines
- Can hold $S*B*E$ -byte region of address space in cache at one time

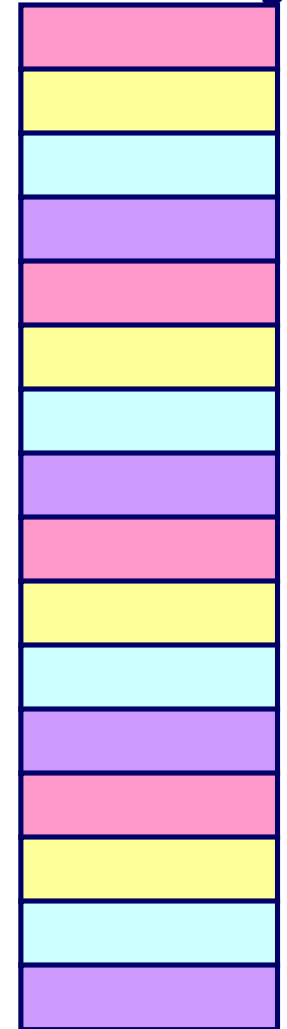
High-Order
Bit Indexing

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111



Middle-Order
Bit Indexing

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

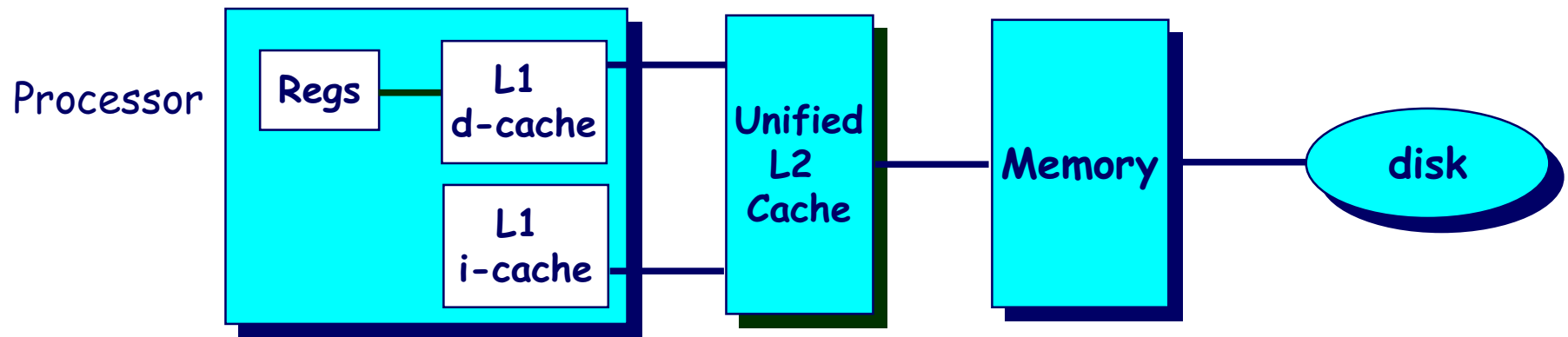


Maintaining a Set-Associate Cache

- **How to decide which cache line to use in a set?**
 - Least Recently Used (LRU), Requires $\lceil \lg_2(E!) \rceil$ extra bits
 - Not recently Used (NRU)
 - Random
- **Virtual vs. Physical addresses:**
 - The memory system works with physical addresses, but it takes time to translate a virtual to a physical address. So most L1 caches are virtually indexed, but physically tagged.

Multi-Level Caches

Options: separate **data** and **instruction caches**, or a **unified cache**



size:	200 B	8-64 KB	1-4MB SRAM	128 MB DRAM	30 GB
speed:	3 ns	3 ns	6 ns	60 ns	8 ms
\$/Mbyte:			\$100/MB	\$1.50/MB	\$0.05/MB
line size:	8 B	32 B	32 B	8 KB	

larger, slower, cheaper



What about writes?

Multiple copies of data exist:

- L1
- L2
- Main Memory
- Disk

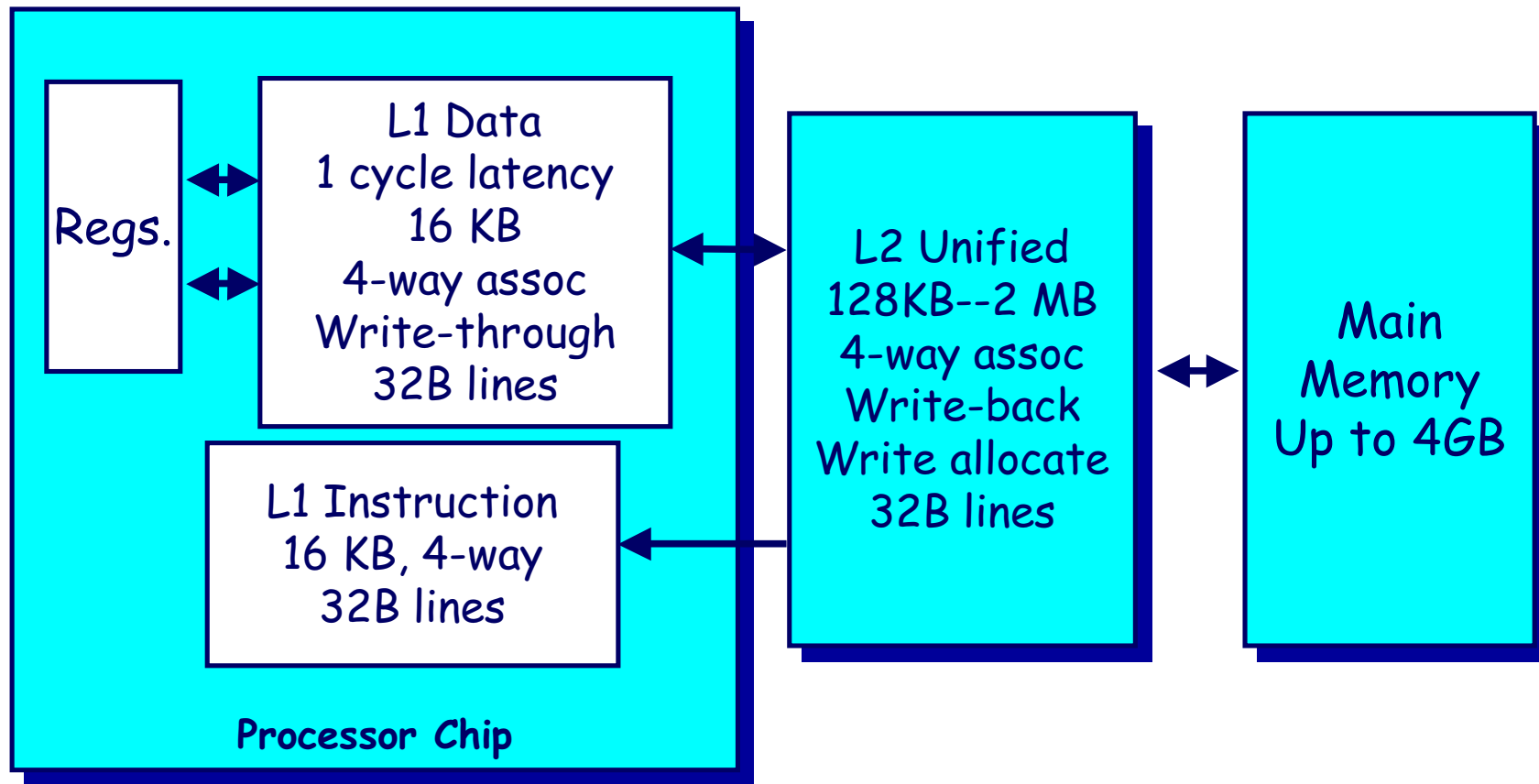
What to do when we write?

- Write-through
- Write-back
 - need a dirty bit
 - What to do on a write-miss?

What to do on a replacement?

- Depends on whether it is write through or write back

Intel Pentium III Cache Hierarchy



Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / references)
- Typical numbers:
 - 3-10% for L1
 - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
 - 1-2 clock cycle for L1
 - 5-20 clock cycles for L2

Aside for architects:
-Increasing cache size?
-Increasing block size?
-Increasing associativity?

Miss Penalty

- Additional time required because of a miss
 - Typically 50-200 cycles for main memory (Trend: increasing!)

Writing Cache Friendly Code

- Repeated references to variables are good (**temporal locality**)
- Stride-1 reference patterns are good (**spatial locality**)
- Examples:
 - cold cache, 4-byte words, 4-word cache blocks

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = **1/4 = 25%**

```
int sum_array_cols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = **100%**

Detecting the Cache Parameters

How can one determine the cache parameters?

- Size of cache?
- Size of cache block?
- Hit time?
- Miss penalty?
- Associatively?
- Number of levels in memory hierarchy?

Complicating factors

- Prefetch support (hardware and software)
- Non-blocking caches (“Hit-under-Miss” support)
- Superscalar processors with multiple, concurrent memory operations
- Victim caches, stream buffers, line-reservation

The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)

Memory mountain

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.

Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems, stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

Memory Mountain Main Routine

```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10) /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23) /* ... up to 8 MB */
#define MAXSTRIDE 16 /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS]; /* The array we'll be traversing */

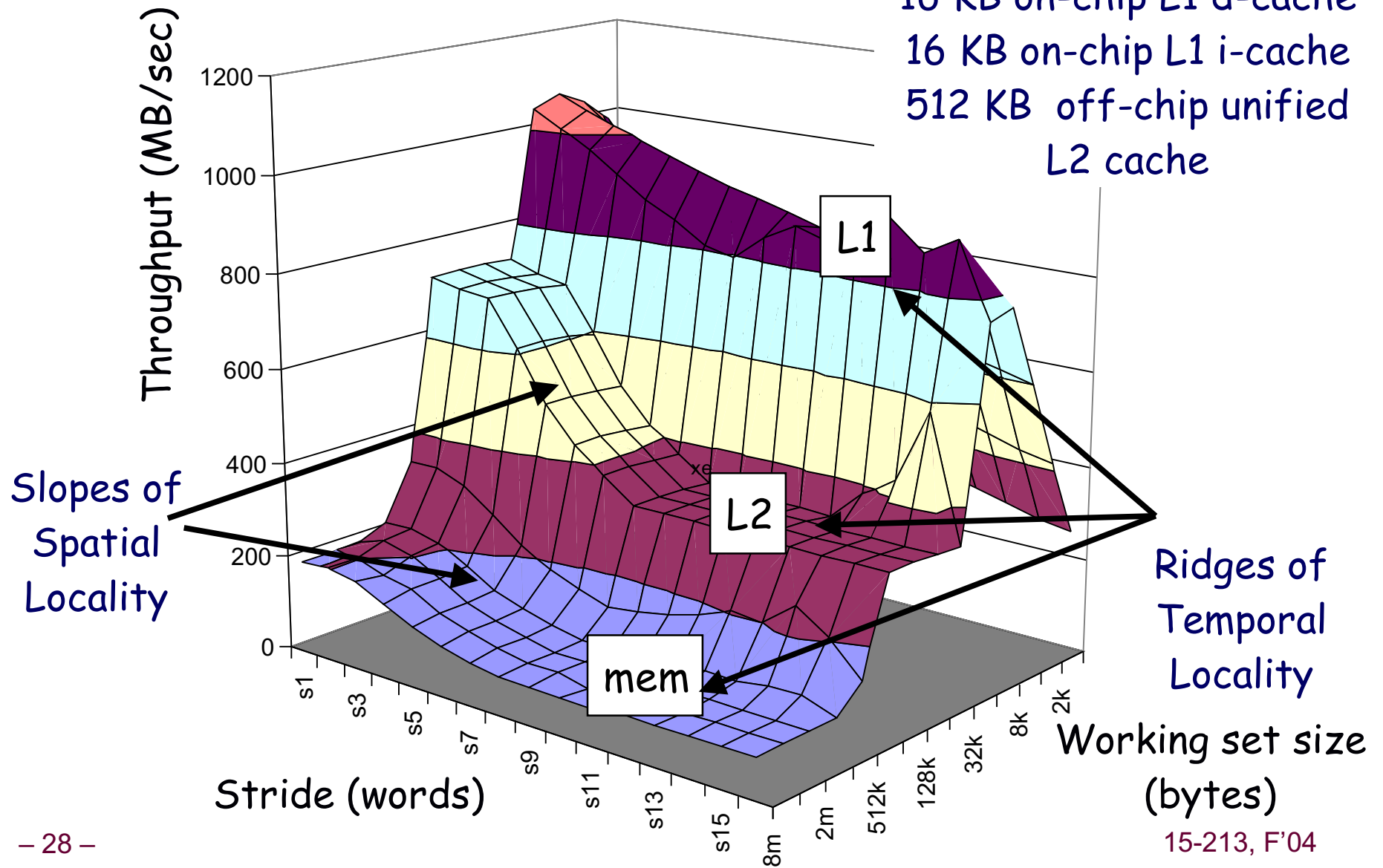
int main()
{
    int size; /* Working set size (in bytes) */
    int stride; /* Stride (in array elements) */
    double Mhz; /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0); /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

The Memory Mountain

Pentium III
550 MHz

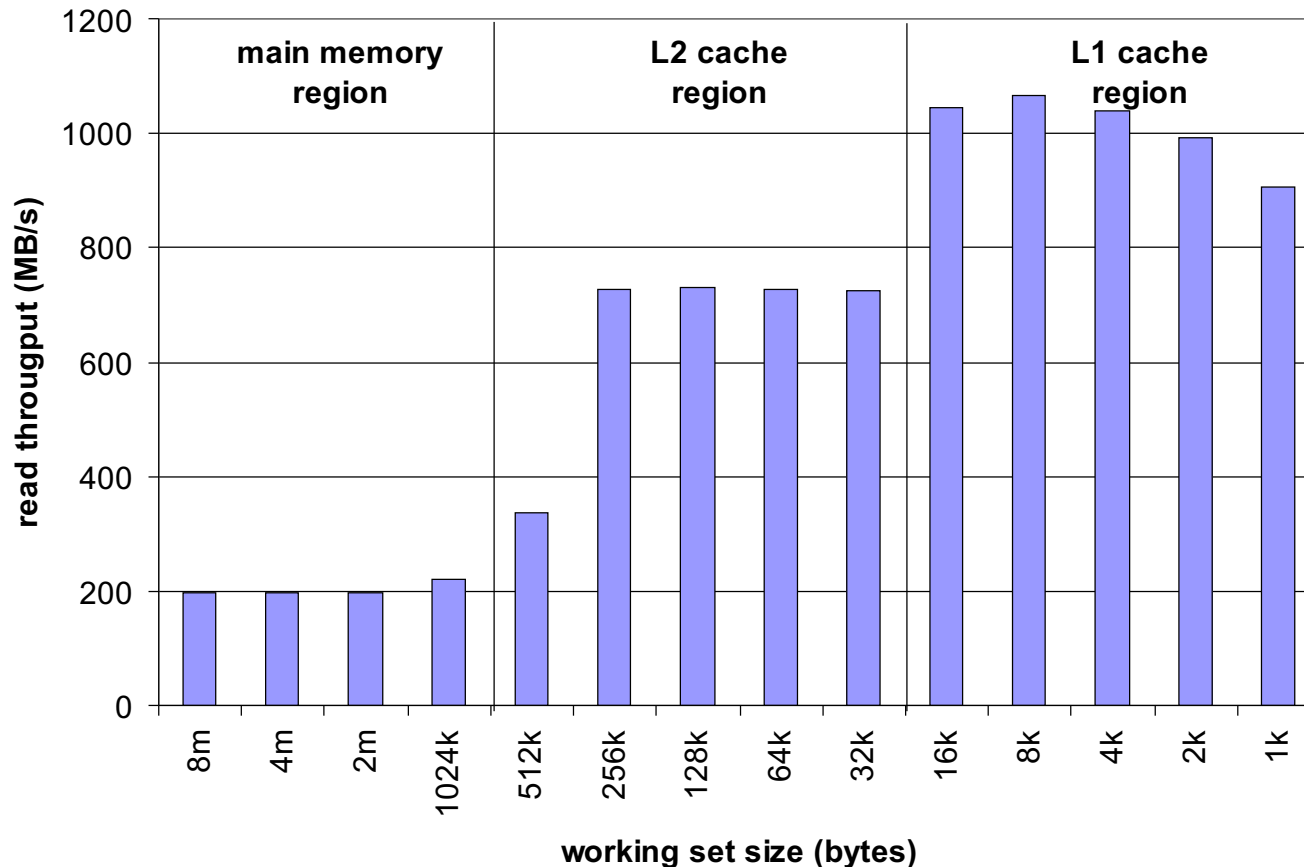
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified
L2 cache



Ridges of Temporal Locality

Slice through the memory mountain with stride=1

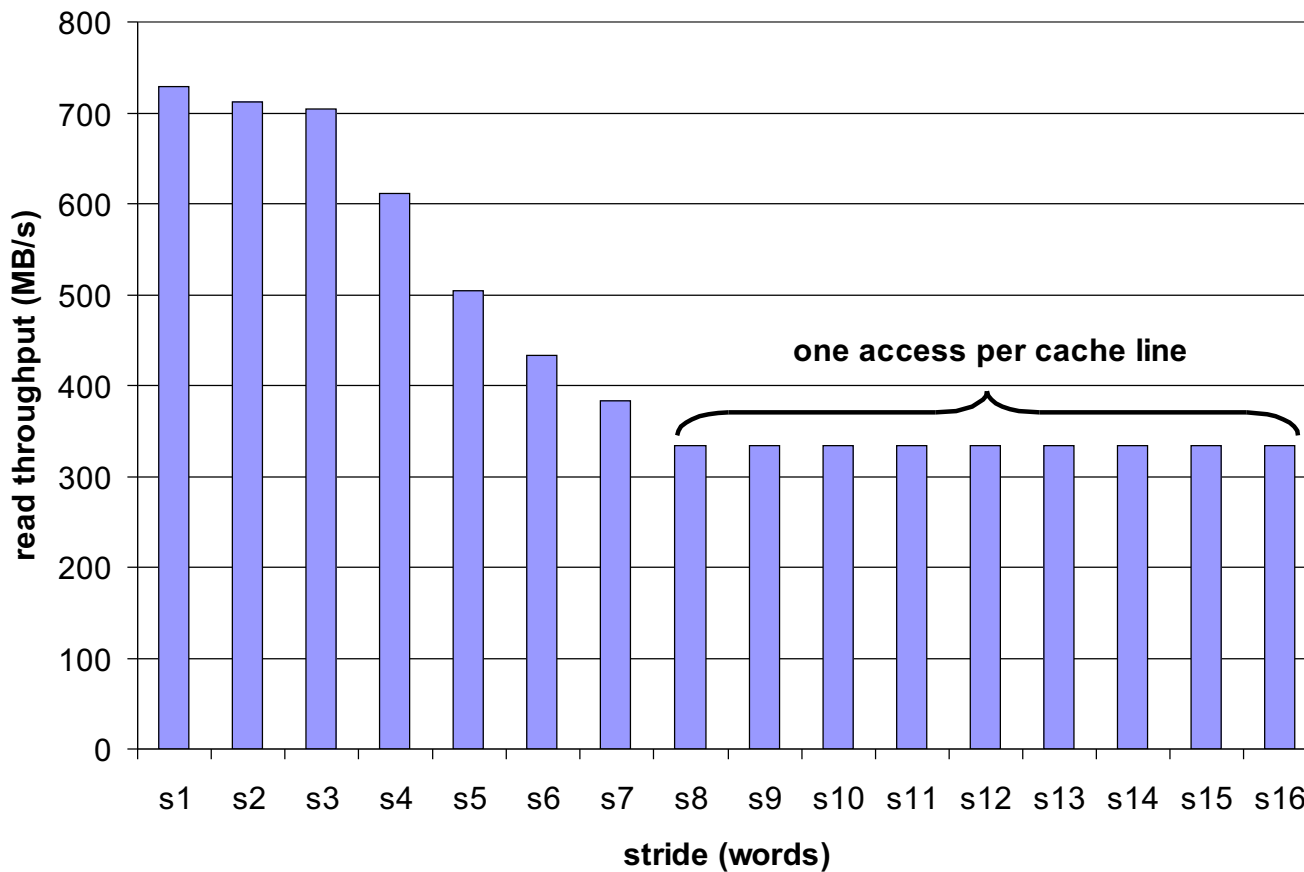
- illuminates read throughputs of different caches and memory



A Slope of Spatial Locality

Slice through memory mountain with size=256KB

■ shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider

- Total cache size
 - Exploit temporal locality and keep the working set small (e.g., use blocking)
- Block size
 - Exploit spatial locality

Variable *sum*
held in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Description:

- Multiply $N \times N$ matrices
- $O(N^3)$ total operations
- Accesses
 - N reads per source element
 - N values summed per destination

» but may be able to hold in register

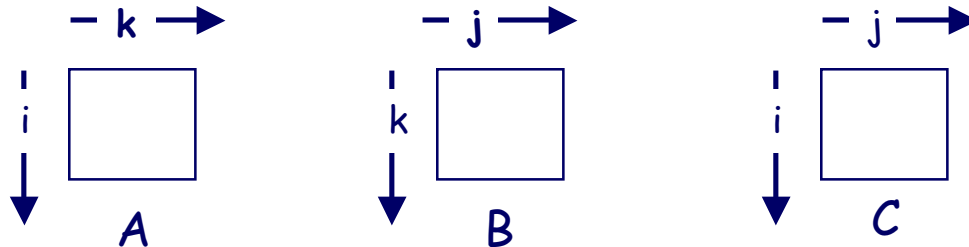
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = $32B$ (big enough for four 64-bit words)
- Matrix dimension (N) is very large
 - Approximate $1/N$ as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

C arrays allocated in row-major order

- each row in contiguous memory locations

Stepping through columns in one row:

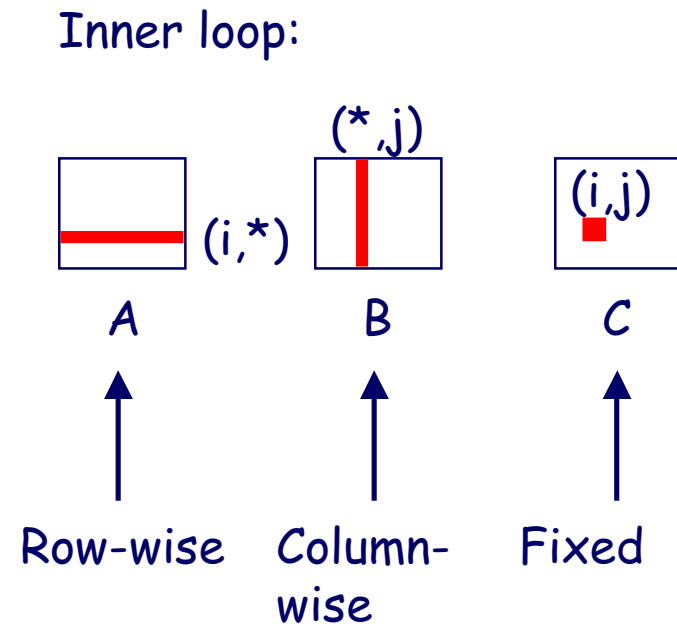
- ```
for (i = 0; i < N; i++)
 sum += a[0][i];
```
- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 4 bytes / B

## Stepping through rows in one column:

- ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```
- accesses distant elements
- no spatial locality!
 - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```



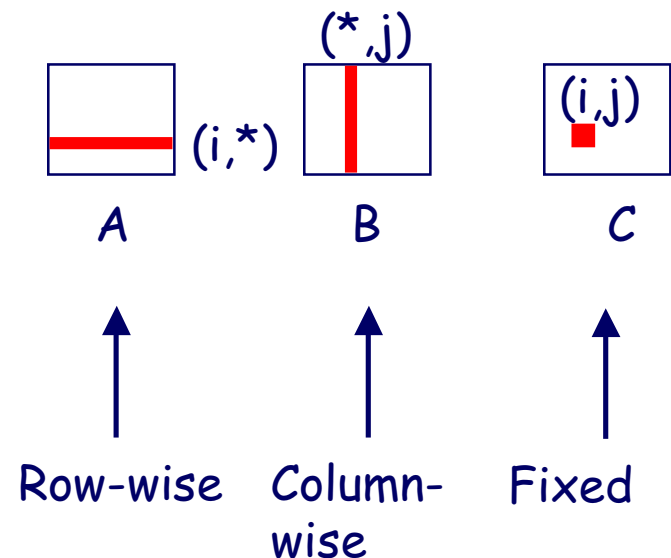
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

Inner loop:

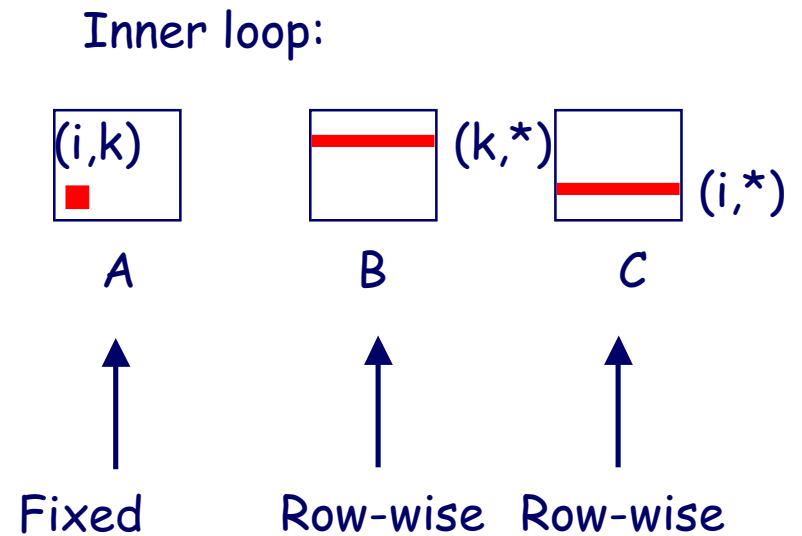


Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```



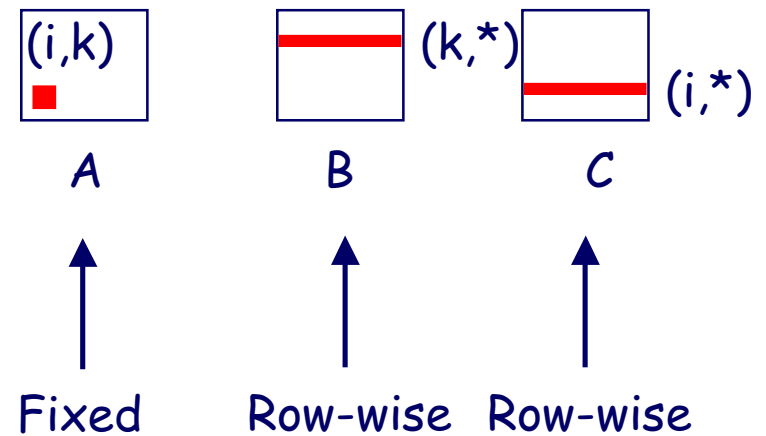
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

Inner loop:



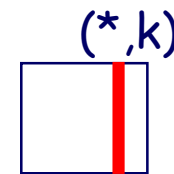
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (jki)

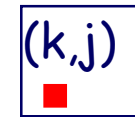
```
/* jki */  
for (j=0; j<n; j++) {  
    for (k=0; k<n; k++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

Inner loop:



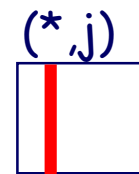
A

Column -
wise



B

Fixed



C

Column-
wise

Misses per Inner Loop Iteration:

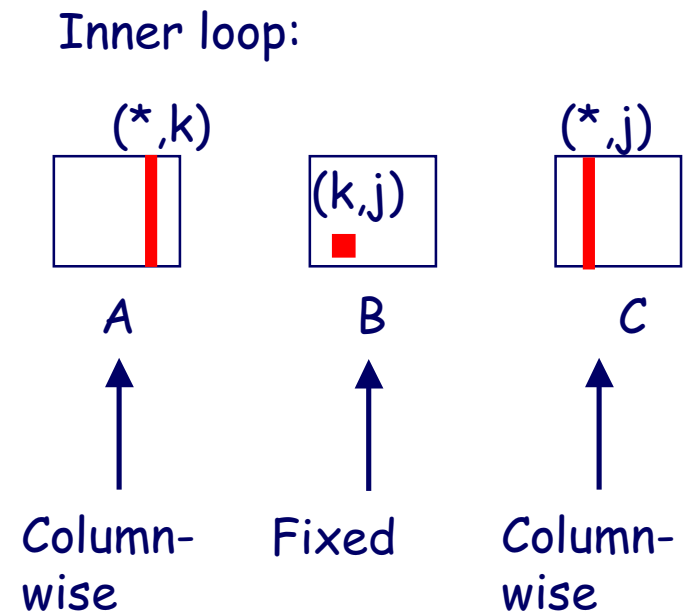
A
1.0

B
0.0

C
1.0

Matrix Multiplication (kji)

```
/* kji */  
for (k=0; k<n; k++) {  
    for (j=0; j<n; j++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```



Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {  
  for (j=0; j<n; j++) {  
    sum = 0.0;  
    for (k=0; k<n; k++)  
      sum += a[i][k] * b[k][j];  
    c[i][j] = sum;  
  }  
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

```
for (k=0; k<n; k++) {  
  for (i=0; i<n; i++) {  
    r = a[i][k];  
    for (j=0; j<n; j++)  
      c[i][j] += r * b[k][j];  
  }  
}
```

kij (& ikj):

- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {  
  for (k=0; k<n; k++) {  
    r = b[k][j];  
    for (i=0; i<n; i++)  
      c[i][j] += a[i][k] * r;  
  }  
}
```

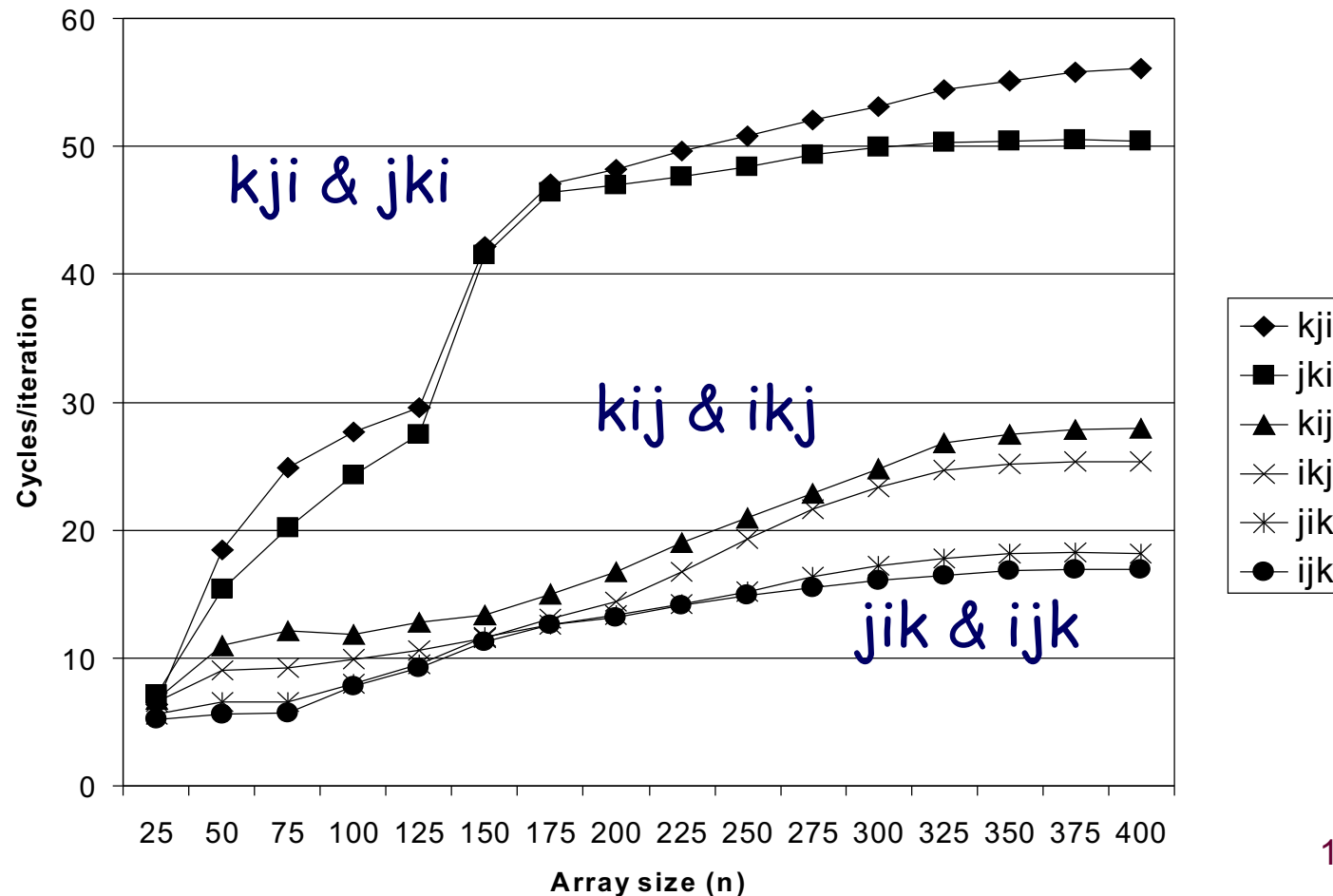
jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

Pentium Matrix Multiply Performance

Miss rates are helpful but not perfect predictors.

- Code scheduling matters, too.



Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication

- “block” (in this context) does not mean “cache block”.
- Instead, it mean a sub-block within the matrix.
- Example: $N = 8$; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars.

$$\begin{aligned} C_{11} &= A_{11}B_{11} + A_{12}B_{21} & C_{12} &= A_{11}B_{12} + A_{12}B_{22} \\ C_{21} &= A_{21}B_{11} + A_{22}B_{21} & C_{22} &= A_{21}B_{12} + A_{22}B_{22} \end{aligned}$$

Blocked Matrix Multiply (bijk)

```
for (jj=0; jj<n; jj+=bsize) {  
  
    for (i=0; i<n; i++)  
        for (j=jj; j < min(jj+bsize,n); j++)  
            c[i][j] = 0.0;  
  
    for (kk=0; kk<n; kk+=bsize) {  
        for (i=0; i<n; i++) {  
            for (j=jj; j < min(jj+bsize,n); j++) {  
                sum = 0.0  
                for (k=kk; k < min(kk+bsize,n); k++) {  
                    sum += a[i][k] * b[k][j];  
                }  
                c[i][j] += sum;  
            }  
        }  
    }  
}
```

Blocked Matrix Multiply Analysis

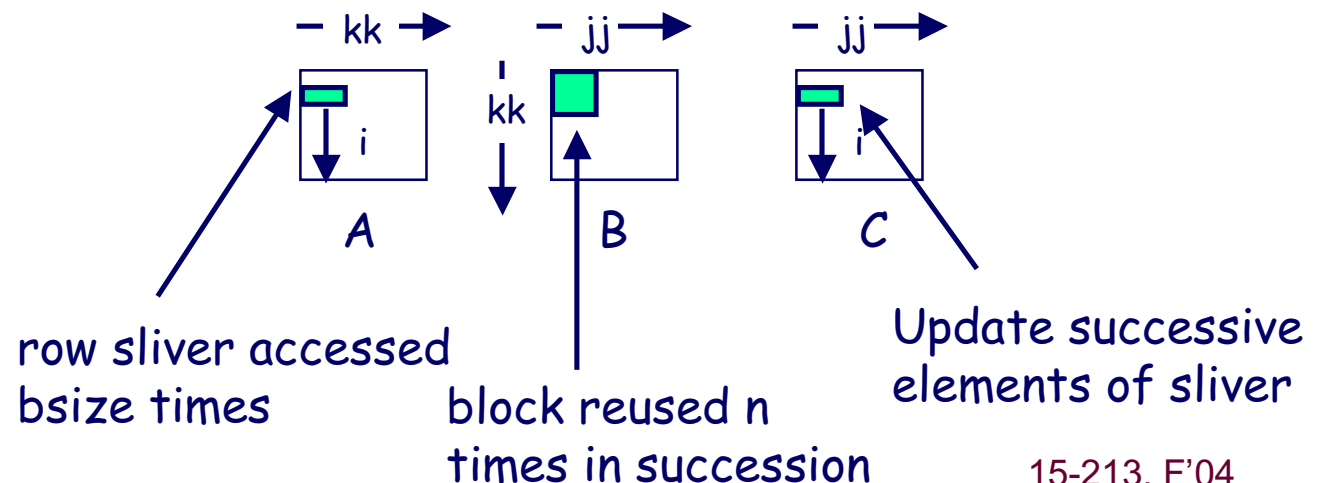
- Innermost loop pair multiplies a $1 \times bsize$ sliver of A by a $bsize \times bsize$ block of B and accumulates into $1 \times bsize$ sliver of C
- Loop over i steps through n row slivers of A & C , using same B

```

for (i=0; i<n; i++) {
    for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
            sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
    }
}

```

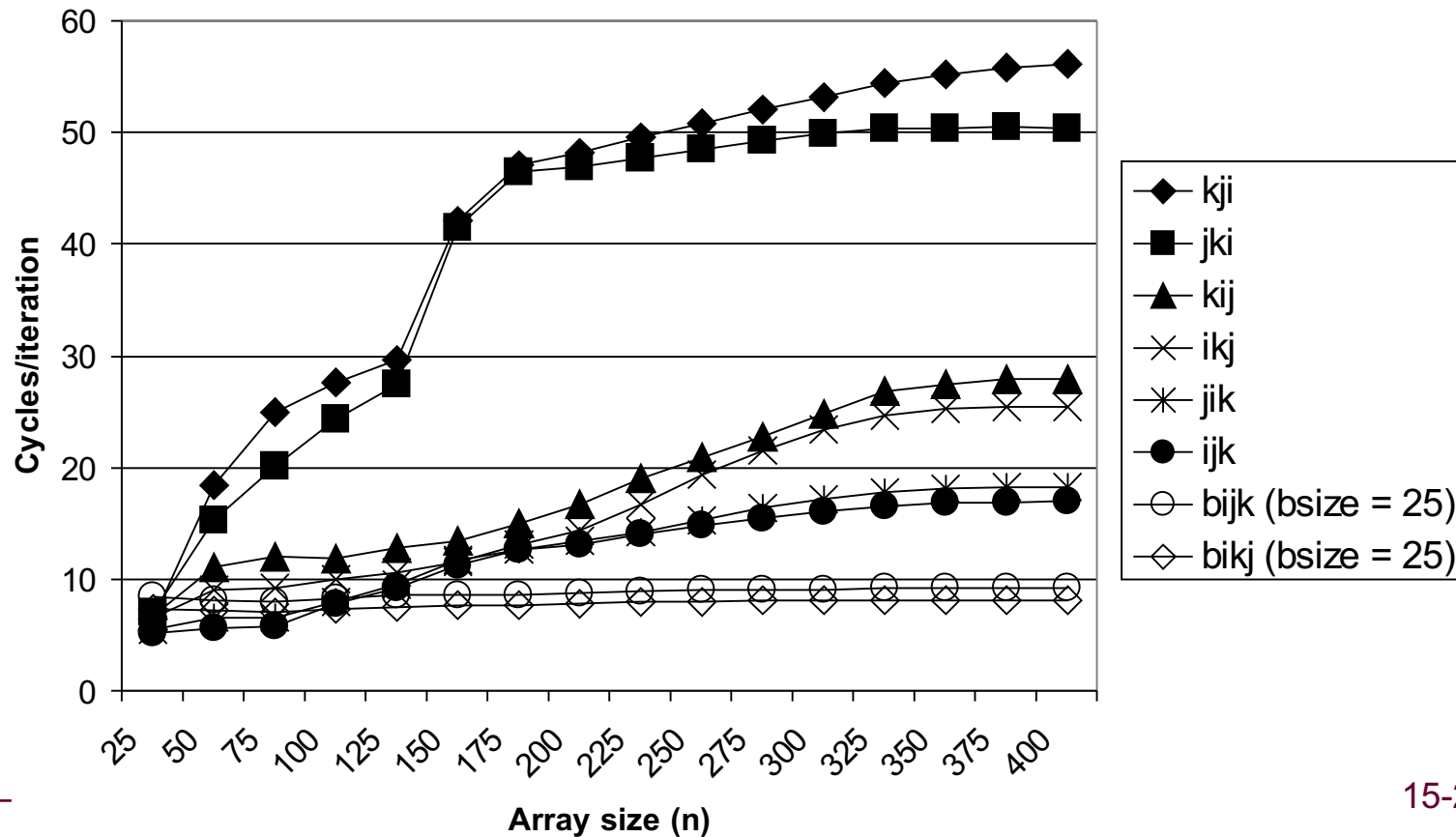
Innermost
Loop Pair



Pentium Blocked Matrix Multiply Performance

Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- relatively insensitive to array size.



Concluding Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
 - Nested loop structure
 - Blocking is a general technique

All systems favor “cache friendly code”

- Getting absolute optimum performance is very platform specific
 - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
 - Keep working set reasonably small (temporal locality)
 - Use small strides (spatial locality)